Convection Below Cloud Base

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ABSTRACT

From observations made from the ground and from aircraft it is considered that clear air thermals are continuing plumes. They form above the forced convection region near the surface; initially their temperature excess is about 1C, their size 200 m, and upward velocity about 1 m sec^{-1}. The thermals rise through a neutral to slightly stable environment of thermally quiet, slowly descending air; however, the air both inside and outside the thermal is strongly turbulent. The thermal initially accelerates upwards and probably decreases in size. Air is continually mixing into and out of the thermal, and at levels above about 100 m the size remains sensibly constant. The whole field increases in temperature, partly as a result of the detrainment of warm air from the thermals and partly from downward motion of their stable environment.

1. Introduction

Convection, which results in the transfer of heat, water vapor and momentum from one level in the atmosphere to another, has been the subject of much study. Two broad, general lines have been followed: one dealing with average conditions and the other with the motion of an individual element under the influence of buoyancy.

Through the use of concepts such as the eddy diffusivities of heat and vapor and the eddy viscosity, it has been possible to study the turbulent transfer of some property in an atmosphere unvarying with time and horizontal position, and without any implications regarding the mechanism of transfer itself. Experimental studies of the eddy fluxes under different conditions of atmospheric stability and wind shear have enabled the identification of two regimes under which buoyancy either plays no role or dominates the motion. Theories have been developed which allow deductions to be made regarding the temperature-height profile and the heat and vapor fluxes; they are based largely on similarity considerations and dimensional analysis, and their main justification is their accord with experimental evidence.

Any more fundamental approach must involve a study of the motion of an individual buoyant element under defined environmental conditions. Priestley (1953) derived equations for the principal types of elemental motion which occur under differing buoyancy and environmental stability without specifying in detail the structure of the element itself, and he was able to deduce the importance of the physical size of an element in determining its behavior. Morton *et al.* (1956) considered the behavior of an element which remains spherical and progressively adds to its mass by entrainment of environmental air and deduced its final equilibrium height as a function of initial buoyancy and stratification of the environment. Their treatment, in common with that of other authors such as Scorer (1957), Woodward (1959), Levine (1959), and Turner (1963a), differs from that of Priestley in the relative importance given to mixing induced by the element's own motion and that due to motions already existing in the environment. Turner (1963b) recognized that both mixing processes are important and derived equations in which they are treated separately.

When we consider all these ideas in the light of observations in the atmosphere we find at best only partial agreement and in some aspects direct conflict. Furthermore, too great an emphasis has been given to the visual similarity in outline of clouds in the atmosphere and buoyant "thermals" in water tanks, upon whose behavior rests the only real test of many of the models proposed. In this paper we examine the most important features of thermals observed in the free atmosphere and their implications. In a separate paper one of us (Telford, 1966) has developed a model taking many of these features into account and has examined its predictions.

2. Observations

The observations discussed below relate almost entirely to conditions above extensive, uniform, horizontal surfaces on which there were no identifiable local sources of heat. The aircraft used in the observations was fitted with two sets of recording instruments, i.e., relatively slowly responding elements for measuring temperature and humidity during spiral soundings, and equipment of fast response for measuring these same parameters, as well as the vertical velocity of the air, during horizontal traverses. These horizontal runs were generally made along the line of the wind; however, since our previous observations seemed to indicate little difference in thermal structure along or across the wind, the direction of the runs in relation to the wind was not closely controlled and considerable differences may have existed in some cases.

a. Average properties of the atmosphere. It is convenient to examine first the average potential temperature and mixing ratio lapse rates found in the atmosphere when active convection is occurring. It is important to note, however, that these lapse rates may be somewhat misleading when applied to buoyant elements, since these elements themselves contribute to the average but are affected only by their own environment; nevertheless, this is the structure which is commonly discussed. Using instruments accurate to 0.05C and having a response time of about 5 sec, we find that the horizontal space average of potential temperature increases only slightly with height or is constant, and the mixing ratio decreases with height or is constant. The density gradient is thus slightly stable to neutral. These remarks apply within the active convection layer at heights above about 100 m; below that level a superadiabatic layer is commonly observed. The results of a number of soundings in strongly convective situations over flat grassland in cloudless conditions are shown in Fig. 1.

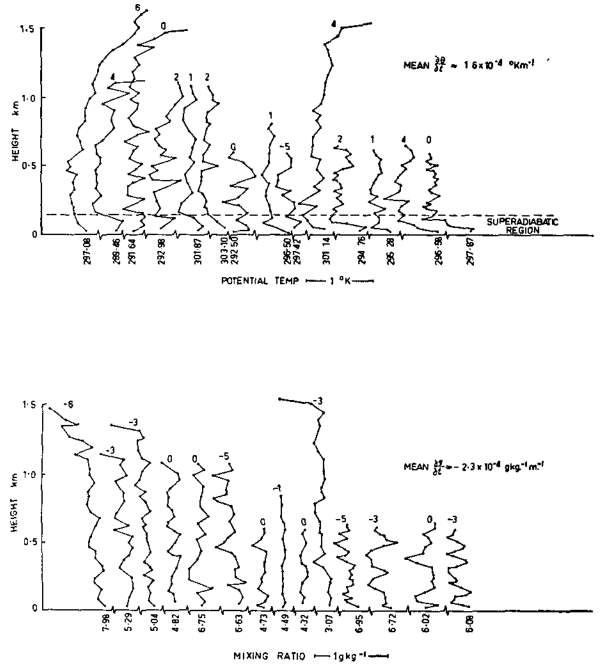

Fig. 1. The potential temperature and mixing ratio gradients observed in strongly convective situations over flat grassland in cloudless conditions. The figures at the top of each curve represent its average lapse rate in 10^{-4}K m^{-1} or 10^{-4} gm kg^{-1} m^{-1}, and those at the bottom represent its initial value in °K or gm kg^{-1}.

An important point to remember is that the soundings we have discussed represent conditions essentially at one specific time. However, during the morning and early afternoon, as surface heating increases, the air temperature rises; typically, we find that at every level up to some maximum the potential temperature itself increases with time but the gradient stays sensibly constant. The level at which there is an abrupt increase in potential temperature gradient rises progressively. Provided that there is adequate moisture in the air and surface heating proceeds long enough, cloud ultimately forms, and commonly at a level slightly above the abrupt increase in potential temperature gradient. A typical sequence of events is shown in Fig. 2.

b. Properties of thermals. The thermal structures in the lower atmosphere have been described in some detail by, for example, James (1953, 1954), Priestley (1959), Vul'fson (1961), Grant (1965) and the present authors (Warner and Telford, 1963). These authors employed sensitive and rapidly responding thermometer elements capable of showing the structure over distances

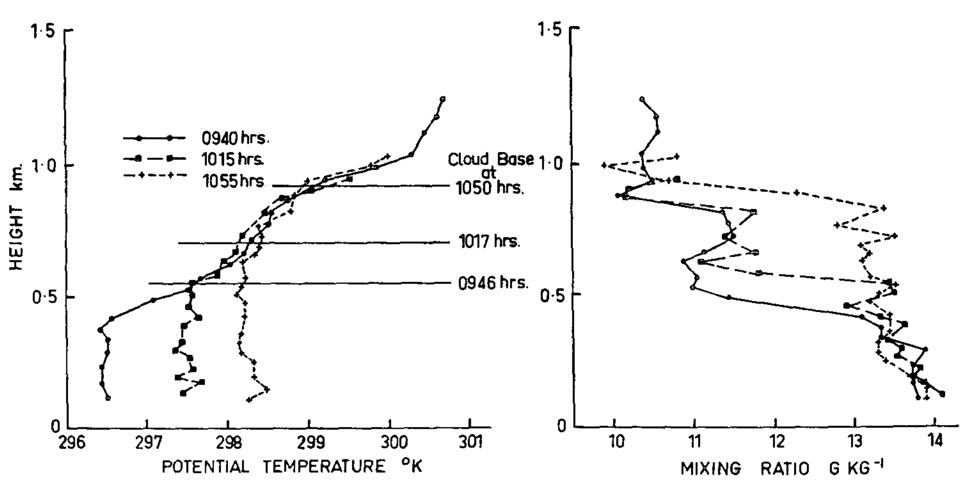

Fig. 2. Changes with time of potential temperature and mixing ratio. Cloud base was observed during each sounding and in each case can be seen to lie slightly above the height at which there is an abrupt change in potential temperature and mixing ratio gradients.

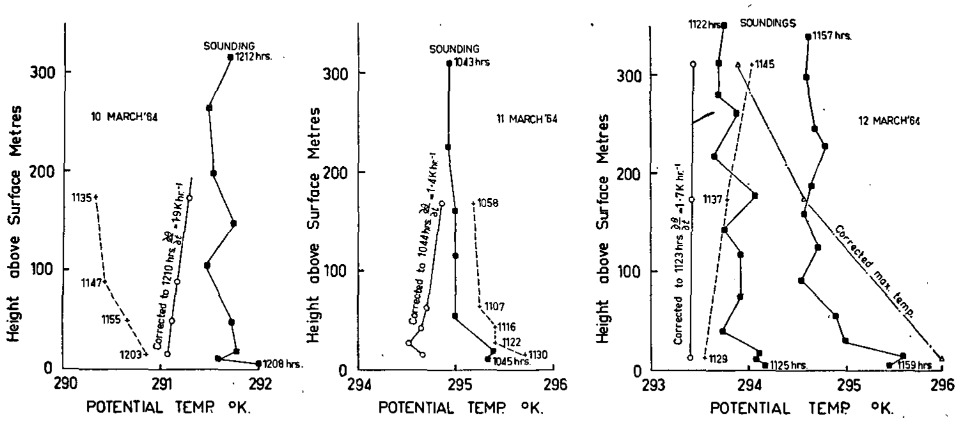

FIG. 3. The potential temperature in the thermally quiet regions as observed, and after correction to the time at which a sounding was made. In one case the mean maximum temperature during the run, i.e., the temperature within the thermals, is shown; it also has been corrected to the time of the sounding. The figure also shows the "average" potential temperature measured with slowly responding instruments during spiral ascent or descent.

of a few meters, and, in some cases, corresponding measurements of vertical air velocity were made. They agree that at levels above a few meters or some tens of meters, depending on surface roughness, there are regions of elevated temperature, seldom exceeding 2C, embedded in regions of nearly uniform base-level temperature. Within the thermal itself there are rapid temperature fluctuations often decreasing to near the surrounding base-level temperature, the latter being remarkably free from temperature fluctuations. In this paper we describe in more detail the results of our own measurements.

It is of considerable interest to examine the way in which the temperature in the thermally quiet regions changes with height. However, since periods in excess of 5 min elapse between successive aircraft traverses at different levels and since the temperature of the whole field is changing with time, it is necessary to correct for this factor before any meaning can be attached to the measurements. It is further necessary that the site and observing procedure be such that the effects of advection can be properly accounted for. The results of three series of measurements made under such conditions over extensive flat grasslands and corrected for time on the basis of soundings are shown in Fig. 3. Thus, while the mean temperature displays the familiar superadiabatic layer near the surface surmounted by a deep neutral or stable region, the thermally quiet region, which is the environment through which the thermals are rising, is seen to be neutral to slightly stable throughout the whole depth which the aircraft could sample and within which thermals are recognizable entities. In some cases, it should be remarked, as we progress downwards into the superadiabatic layer the thermally quiet regions disappear and we can no longer observe organized thermals.

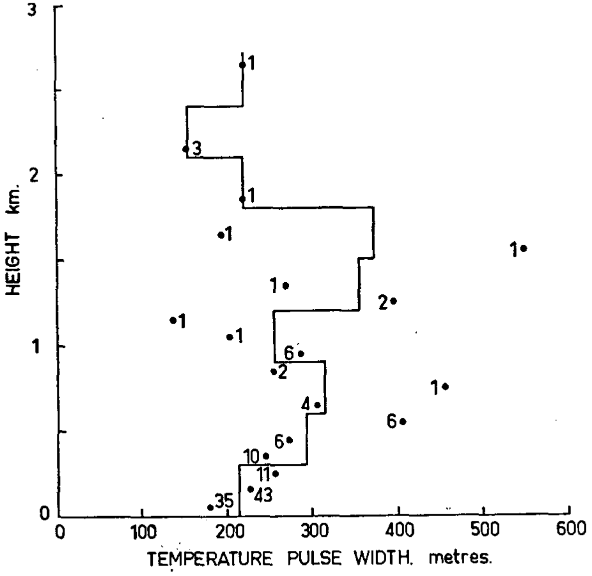

FIG. 4. The average width of the temperature pulses, which define the boundaries of thermals, as a function of height. The points represent averages over a height interval of 100 m, the number alongside being the number of sampling runs involved. The line represents the same data averaged over 300-m height intervals.

The irregular outlines of a thermal, as we have already reported, together with the slowly decreasing temperature excess often observed on its downwind side, make it inherently difficult to assess its size. There is also the difficulty associated with sampling along a given line involved in either ground or aircraft observation. However, as is seen in Fig. 4, we find no pronounced systematic change in size with height for levels above about 100 m. Nor is there any indication that nearly all thermals do not penetrate the full depth of the convective layer since the ratio of thermal width to spacing between adjacent thermals, as shown in Fig. 5, also remains sensibly constant with height. These two figures display the results of all our observations made over uniform level surfaces under conditions of active convection with little or no cloud coverage. Data from observations above the level at which the potential temperature gradient began to increase sharply with height are not included, and, thus, most data relate to the lower levels. The "number of observations" noted on the figures relates to the number of approximately 2-min sampling runs made by the aircraft in the height range, each of which, on the average, encountered about 10 thermals. Fig. 6 shows the close relationship between thermal size and spacing, each point representing one sampling run. It is likely that the points nearest the origin represent elements that do not persist much above the levels close to the surface at which they were observed. The points furthest from the origin may not be significant since their small temperature excess made recognition difficult.

Our observations thus show a thermal to be 200 to 300 m across and separated from its neighbors by a distance only slightly in excess of its own size. James' (1954) data for heights up to 900 m are in substantial agreement with these values; his figures for thermal size in the sub-cloud layer may well involve different considerations since here the thermals are probably entering a more stable region. Vul'fson (1961) also suggests that the size changes little with height, though the average dimension he observes is only 50–100 m. Grant (1965), however, finds that the mean size increases from about 200 m near the surface to 900 m at a height of 600 m, decreasing again as cloud base is approached. While the material presented by these three authors suggests, contrary to our own results, that more thermals are observed in the lower levels than higher up, the absence of information about average temperature lapse rates makes comparison with our own observations difficult. It seems likely that some of their data include observations in the surface layers where organization of the thermals is not fully developed.

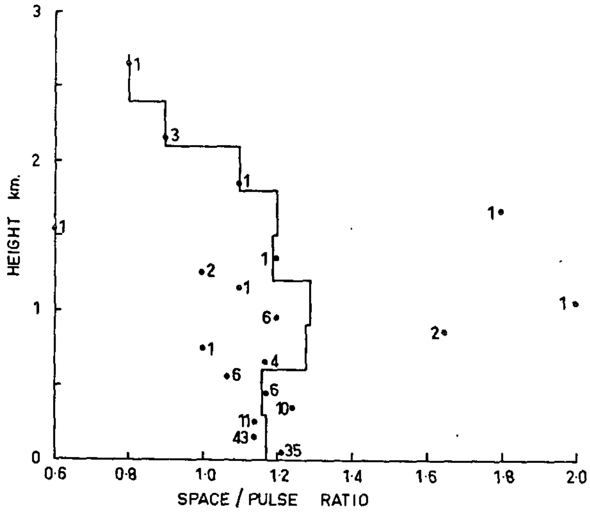

Fig. 5. The average ratio of the space between thermals to their horizontal dimension as a function of height. The numbered points have the same significance as in Fig. 4.

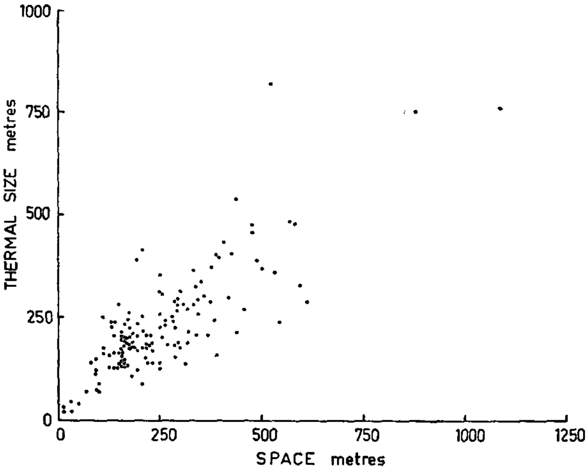

Fig. 6. Average thermal size vs. average spacing. Each point represents the mean value for a sampling run 5–10 km in length.

We have already noted that towards the surface the temperature structure becomes erratic, that there are no quiescent regions, and that we can no longer speak of organized thermals. The depth of this disorganized region can vary quite markedly—from under a meter to more than a hundred meters, depending on wind speed, surface roughness and heat flux. This is discussed by Webb (1964) in terms of the Obukov scale length. We have found that the size of thermals is sensibly independent of this length, differing little when over trees in strong winds, grasslands, or even over a calm warm water surface.

Our observations suggest that on a particular occasion, and at any given height above about the first hundred meters, temperature excess or buoyancy is remarkably similar for all thermals. This conclusion was also drawn with respect to lower levels by Priestley (1959), who remarked that "the minimum temperature remained constant from one quiet interval to another while the maximum temperature of the disturbed periods remained equally constant." The average temperature excess decreases progressively with height, there being some indication of an initial rapid decrease in the lower layers followed by a more gradual tapering-off.

A study of the vertical velocity structure brings to light several important features. The most obvious, as Priestley (1959) noted, is that there is no cessation of turbulence during the periods when the temperature record is fluctuation-free; indeed, thermals can only be identified with any certainty from the temperature record. As the height of observation increases, longer wavelength components appear in the velocity records which have no counterpart in the temperature traces, and the higher frequency components generally decrease somewhat in importance. It is difficult to determine

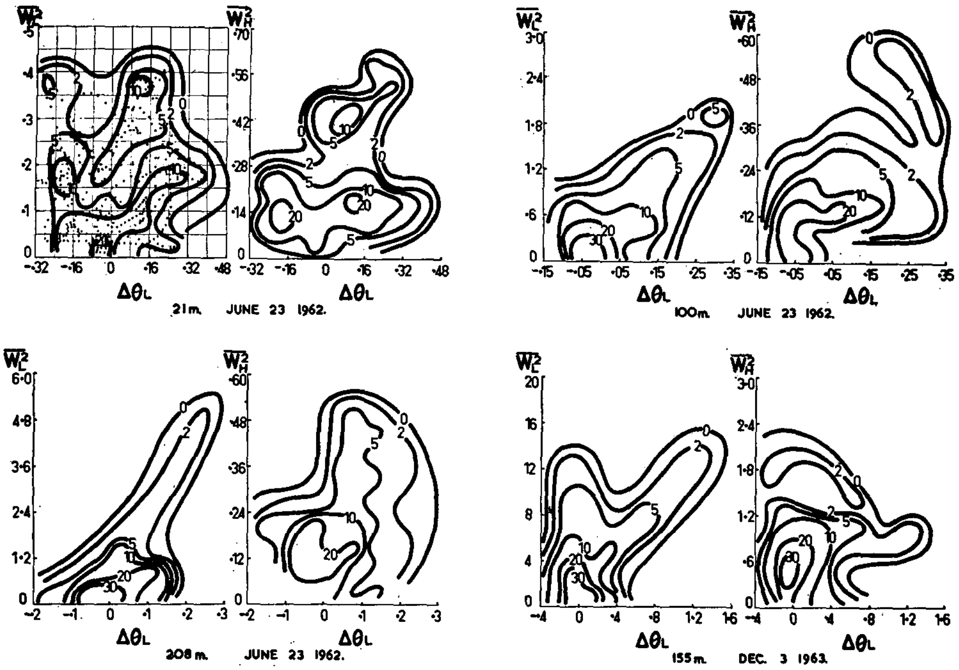

FIG. 7. The distribution of mean square velocity with temperature fluctuation about its mean for four horizontal traverses. Subscripts L and H denote the outputs from low-pass and high-pass numerical filters, respectively. One original scatter diagram is shown by the points in the top left-hand part of the figure. From the number of points in the marked squares, isopleths have been drawn representing the density of distribution of points. The remaining parts of the diagram show only the isopleths. No absolute significance should be attached to the numbers alongside the contours; they indicate the number of points in regions which differ in absolute magnitude from one to the other of the eight elements of the diagram. It should also be noted that the scales differ in different parts of the diagram.

from the observations what is the general upward motion of the buoyant thermal and what is turbulence, though it is clear that the more prominent updrafts are associated with the warmer air. That there is also a strong correlation between high-frequency components of temperature and of velocity, particularly in the lower levels, is clear from the results of Telford and Warner (1964), who showed that most of the heat flux at these heights is carried by elements of only a few tens of meters in size.

We have remarked in a previous paper (Warner and Telford, 1963) that the degree of turbulence differs little between the thermal and its environment. It seems desirable to examine this statement quantitatively, particularly in view of the opposite conclusion reached by Grant (1965). In Fig. 7 a number of scatter diagrams show the relationship between potential temperature and mean square vertical velocity for runs at fixed heights. The values of potential temperature have been put through a low-pass numerical filter designed in the manner described by Graham (1963). The use of upwards of 50 successive data points on each side of the center enables a sharp cut-off to be obtained; typically, a filter gives an output which is unity up to its design cut-off frequency of 0.3 cps, falling to zero at 0.5 cps, and remaining less than 0.01 thereafter. Thus, the filter can effectively separate the general structure, including the thermals themselves, from the fluctuations within the thermals and in the spaces between them. Prior to this filtering there is, of course, complete removal of any very low-frequency components associated with trends persisting over the whole length of the run. In the case of vertical velocity we have taken, in addition to the output from the low-pass filter, its high-pass complement to obtain frequency components above 0.3 cps. The squares of these filtered values of velocity have been obtained and a running mean taken over 19 observations (1.9 sec or about 120 m). For the high-pass output this smoothed mean square velocity is a measure of the turbulent fluctuations of wavelength less than 200 m (corresponding to this filter cut-off at 0.3 cps) either inside or outside the thermals, since the motion of the thermal itself has been largely removed. On the other hand, the mean square velocity derived from the low-pass filter will contain the thermal's upward velocity, and the potential temperature derived from the low-pass filter gives a measure of its temperature excess. The scatter diagrams in Fig. 7 enable us to examine the relation between this temperature excess and either the low-frequency components of velocity, amongst which is the thermal's upward motion, and the high-frequency components which are a measure of turbulence.

Since the warm thermals are generally moving upwards we should expect a correlation between the filtered potential temperature output and the mean square velocity derived from the low-pass filter output. In addition, if the turbulence (as defined by the higher frequency components of motion) is consistently different in the thermals from its value in the intervening thermally quiescent regions, we should expect a similar correlation with the mean square velocity derived from the high-pass filter output. While the material in Fig. 7 supports the former contention, it certainly does not support the latter, and we must conclude that there is little difference between the turbulence in the thermals and in the surrounding, thermally quiescent, descending air.

We will now consider the variation with height of turbulence and general upward motion by examining the filtered values of vertical air velocity described above. The features we are describing appear, of course, in the unfiltered material, but the rapidly fluctuating character of the records makes it difficult to appreciate their significance so readily. Figs. 8 and 9 show the results of flights at different levels on different days over uniform terrains. The low-frequency component of air velocity, which is associated with the thermal, clearly increases with height initially, thereafter changing little. The average rms turbulent velocity in the high-frequency components in general decreases with height, though not markedly so at heights above the superadiabatic layer.

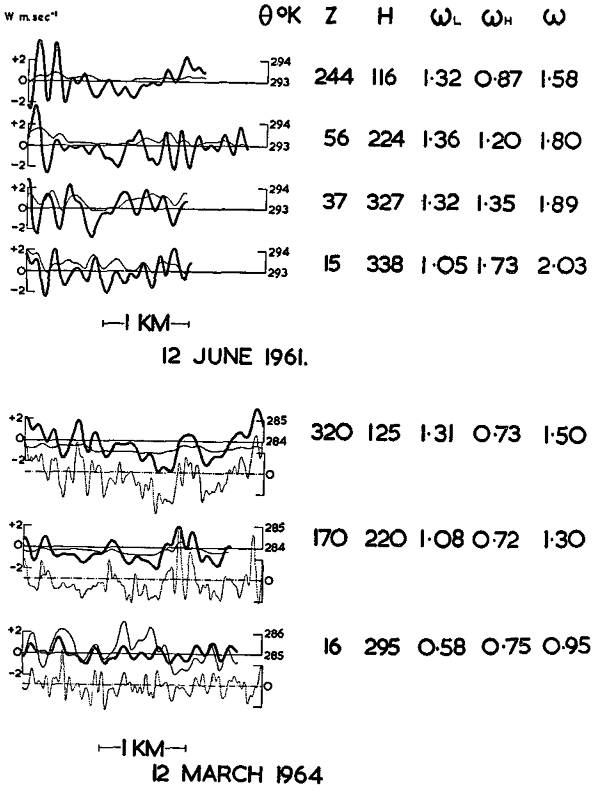

Fig. 8. The variations of filtered velocity and potential temperature for two series of runs at different heights. The heavy trace represents vertical air velocity and the light trace potential temperature, both of which have been passed through a low-pass numerical filter with a cut-off frequency of 0.3 cps. The dotted trace represents vertical velocity after passing through a filter of cut-off frequency 1 cps and is to the same scale as the heavy trace. The table to the right of the figure shows for each run the height in meters, the vertical heat flux in W m^{-2} and the rms vertical velocity in m sec^{-1}, for the low frequencies ω_L, high frequencies ω_H and for all components ω. The transition between low and high frequencies is sharply defined at 0.3 cps.

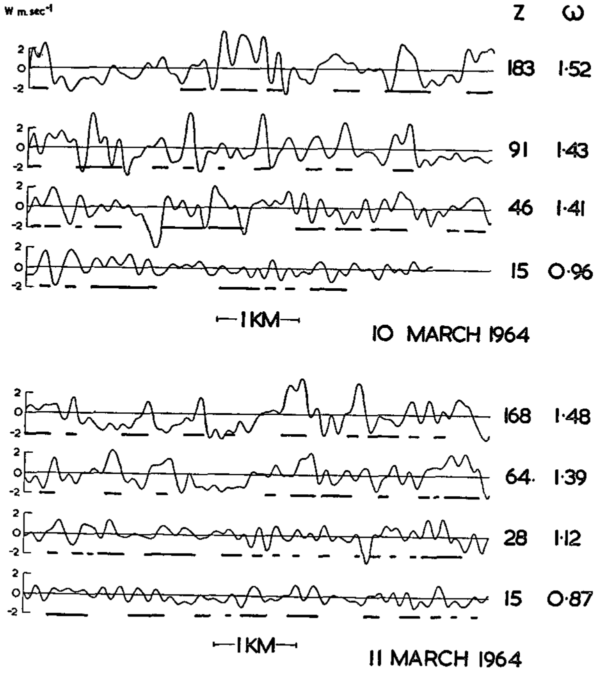

Fig. 9. Similar material to that of Fig. 8, except here the data has not been analyzed for potential temperature, so the regions where well-defined thermals appear are indicated by heavy lines. The cut-off frequency of the filter through which this velocity data were passed was 0.3 cps. To the right are given the height of the run in meters and the rms velocity in m sec^{-1} averaged over all frequencies.

In some cases the boundaries of the thermal, as defined by its temperature structure, are so well defined that an alternative procedure was adopted to examine the way in which its upward motion changed with height; in these cases a simple mean was taken over all the region within all thermals encountered at each level. Fig. 10 shows the unfiltered temperature records and the mean values of vertical velocity over the defined regions. Again we see that the upward motion increases with height or remains roughly constant. This approach implies a "top-hat" velocity profile for the thermal, which is of interest for comparison with theoretical models such as that of Telford (1966). However, as will appear below, the magnitude of the velocity obtained is of doubtful validity, and an alternative method of obtaining a top-hat velocity is desirable.

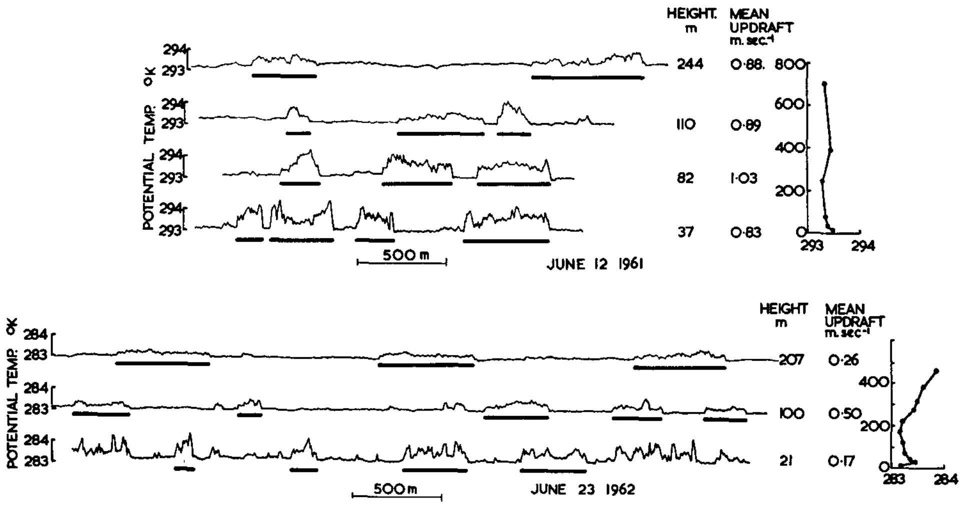

FIG. 10. Potential temperature records on two days when well-defined thermals were present. The average vertical velocity over the marked regions is tabulated at the right, as is the height of observation. Also to the right is the variation of mean potential temperature with height, indicating that the upper four runs were in an essentially neutral region, but the run at 207 m in the lower series was just above the boundary in a much more stable region.

One of the fundamental physical parameters in determining a convection field is clearly the heat flux; this is calculated from our measurements of instantaneous temperatures and velocity. If we obtain the mean temperature excess of the thermals, which can be readily done in cases where they have well-defined boundaries and the temperature trace between them is nearly flat, then we can deduce a top-hat velocity from this mean temperature excess and the heat flux. This will be greater than the simple mean upward velocity for two reasons: it gives the difference in velocity between the rising thermal and its downward moving environment, and it is increased by downward-moving air within the thermal, which correlates with decreasing temperature, whereas the simple mean is decreased. The top-hat velocity thus calculated is considered to be that which is relevant to the theory of an isolated plume. This method of calculation may have an added advantage in that it can be applied to on-line data processing in the observing aircraft to yield an average top-hat velocity at the height of the run from the measured temperature excess and heat flux. Table 1 shows the result of the use of this method of obtaining velocity when applied to some of the runs given in Figs. 8 and 10. Also shown in this table is the rms velocity output from the high-pass filter. When this is taken as a measure of turbulence, its ratio to updraft velocity is small enough to have only negligible effect on the shape of a plume computed on the model suggested by Telford (1966).

TABLE 1. Top-hat temperature excess and velocity.

Date	Height (m)	Temperature excess (°K)	Velocity (m sec^{-1})	ω_H (m sec^{-1})
12 March 1964	320	0.30	1.42	0.73
	170	0.38	2.15	0.72
	16	0.87	1.40	0.75
12 June 1961	82	0.44	2.17	1.01
	37	0.47	1.84	1.35

3. Implications of the observations

Our observations indicate that the atmosphere at all heights above about 100 m is neutral to slightly stable. We must conclude, therefore, that air at any level with a buoyancy excess must be within a thermal already in motion. It is thus most improbable that thermals originate anywhere except in the superadiabatic layer. At least part of this layer is superadiabatic in average properties owing to the large temperature excess of the embedded thermals whose environment is still a neutral to stable one.

Since the potential temperature gradient with height of the thermally quiet regions is never negative, the turbulent motions within them cannot transfer heat upwards. Furthermore, the downward transfer of heat by eddy diffusivity within the descending air, when this air is stable, is probably entirely negligible. In contrast, the heat transported through a given level by air descending from potentially hotter regions results in air at that level becoming warmer; this is equivalent to an upward heat flux decreasing with height.

In the nearly constant potential temperature region we find that the potential temperature gradient remains constant as the whole layer heats up. Thus, at any given time $\partial\theta/\partial t$ is independent of height z. Suppose we restrict our remarks to conditions such as those of the observations, that is, to extensive uniform surfaces where we can consider conditions to be quasi-stationary. If wind shear is small, then, since the heat loss from the air due to long-wave radiation is small and changes little with height, the heat flux gradient $\partial H/\partial z$ must be directly proportional to $\partial\theta/\partial t$ and is, consequently, also independent of height. Under these conditions, therefore, the heat flux can only be constant with height when the potential temperature of the layer is not changing with time. Over a considerable period of the day we find that the potential temperature increases uniformly with time, thus implying a linear gradient of the heat flux with height. Observed departures from such a gradient are most probably due to advection.

Coming now to the detailed structure of the thermals, we observe that at any one level all thermals have essentially the same buoyancy. Their size, at least above 100 m, seems little changed with height, remaining typically at 2–300 m in horizontal extent. It should perhaps be remarked that aircraft sampling precludes the possibility of following the development of an individual thermal and that we have only an average statistical picture of the course of events. Webb (1964), basing his remarks on simultaneous observations at different heights on fixed towers a few tens of meters in height, suggests that in this region there is an initial decrease with height of the horizontal dimension of the thermal along the line of the wind; this he associates with increasing upward speed.

The number of thermals, once they are fully organized and the thermally quiescent region is distinct, does not change significantly with height. It seems not unreasonable, therefore, to suggest that all thermals proceed to roughly the same height, that at which their upward velocity falls to zero.

The rapid fluctuations of temperature within the thermal suggest that the air carries traces of its origin in the forced convection region and that it continues to mix vigorously with its environment as it rises. The quiescent temperature recorded in the region between thermals is an indication of a positive potential temperature gradient; this must be associated with downward moving air when the whole field is increasing in temperature. Clearly-defined regions of thermally undisturbed air suggest that detrainment from the thermals takes away their least buoyant parts, those which are almost at their equilibrium level in the stable descending air.

The observation, previously reported, that thermals are sometimes hottest and with a sharp front on the upwind side can be easily explained only if the thermal is a column leaning downwind, when small, hot, internal elements could rise to the upper, upwind side. The extended downwind region with decreasing temperature excess and small fluctuations suggests that, here, part of the buoyant material is being eroded away into the surrounding air. In the lower levels it is possible to be more definite regarding the shape of thermals. Priestley (1959) examined the sequence of thermals passing a tower with thermometers mounted at heights between 1.5 and 32 m. He deduced that the thermal was here "a continuing plume originating at, and moving with the speed of the wind at some quite low level close to or below the lowest height of the observations"; this level corresponds clearly to the region of forced convection. It is important to note that although thermals start from within this region close to the surface they are not here recognizable entities, and there is no necessity at all that they should originate at specific surface features.

4. Conclusions

From the above a fairly definite picture of a clear air thermal emerges. Shortly after its emergence from the region of forced convection it is a mass of air about 200 m across with a temperature excess of about 1K. It is probably a continuing plume rather than a bubble, and its initial upward velocity is about 1 m $\sec^{-1}$. It rises in a neutral or very slightly stable environment of thermally quiet, slowly descending air; however, the air both inside and outside the thermal is strongly turbulent. The thermal initially accelerates upwards and may decrease in size. Mixing is to be expected, both into and out of the thermal; this entrainment and detrainment undoubtedly continue to the higher levels, probably at comparable rates, the size is maintained nearly constant, and the vertical velocity changes more slowly. The increase of air temperature with time results from detrainment of warm air from the thermals as well as downward motion of their stable environment. Over an extensive uniform surface, and in the absence of advection, the heat flux gradient is often constant and the whole field increases in temperature at a uniform rate.

This picture shows little resemblance to most existing models, and it is suggested that the basis of present theories needs considerable modification if an explanation is to be found for the features that are observed.

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