Diurnal Oscillations of the Tropospheric Wind Field above a Low-Level Jet[1]

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ABSTRACT

An hypothesis is given for the cause of the observed diurnal oscillations in the mid- and upper-tropospheric wind field that occur in association with oscillations of the boundary layer wind field and low-level, nocturnal jet occurrences. A simple mathematical model of the middle and upper troposphere is solved using perturbation techniques subject to the condition that the three-dimensional velocity is continuous at a plane separating the boundary layer from the atmosphere above, and the vertical motion is zero at the tropopause height. Theoretical results are presented which show good agreement with previously published wind data from the surface to the region of the tropopause.

1. Introduction

Hering and Borden (1962) have shown the existence of prominent diurnal oscillations of the mean wind field over the central United States up to the 25-km level. These upper-level oscillations were observed during periods with pronounced boundary layer wind oscillations. It is the purpose of this paper to show that the upper-level oscillations may be ageostrophic oscillations propagated upward from the oscillating boundary layer.

The dynamics of the boundary layer oscillation and the details of its behavior have been discussed by Lettau (1954; 1964), Blackadar (1955; 1957), Buajitti and Blackadar (1957), Wexler (1961), Hoecker (1965), Bonner (1965), Kaimal and Izumi (1965) and by Holton[2] and MacKay[3] among others. The gross features of the boundary layer oscillation are that the horizontal wind speed at each level in the boundary layer frequently undergoes an oscillation in time and direction yielding maximum values in the early morning hours and minimum values during the day. At the time of maximum wind speed the wind speed is usually super-geostrophic and the vertical profile of horizontal wind speed shows a peak within the boundary layer usually between 0.5 and 1.5 km above the surface. The phenomenon in the United States most often occurs during the summertime in the Great Plains region under a situation of strong southerly flow.

Figs. 1 and 2, constructed from the original plots of Hering and Borden, show the west and south components of the departure of the mean wind at various elevations and hours from the overall mean wind at Fort Worth, Tex., for July 1958. The boundary layer oscillation is evident below 2 km. Both figures show levels of non-oscillation at about 2 and 8 km with maximum departure amplitudes occurring at about 4.5 and 12 km as well as in the boundary layer.

For the purposes of this paper an oscillating boundary layer wind will be assumed to exist and its effect on a simple model of a frictionless atmosphere above investigated. The approach to be taken is that of first-order perturbation theory by considering the geostrophic departure components to be neutrally stable perturbations superimposed on the geostrophic wind field. The atmosphere above the boundary layer is considered to be a homogeneous, incompressible fluid whose basic motion is unidirectional with constant speed. The effects of the boundary layer wind behavior are introduced into the upper fluid by requiring that the vertical motion field along the bottom surface of the frictionless fluid be identical to the vertical motion field along the top surface of the boundary layer.

It will be helpful at this point to give a brief description of the three phases of the analysis to be carried out. First, the model of the atmosphere above the boundary layer will be described and the appropriate perturbation equations written. The method of introducing the effects of the boundary layer wind to the lower surface of the upper fluid will be shown. The perturbation equations will be reduced to a simpler form by consideration of the vertical motion continuity condition at the top of the boundary layer and its implications. Second, it will be demonstrated that the

[1] Contribution from the Missouri Agricultural Experiment Station. Journal Series No. 5110.

[2] Holton, J. R., 1967: The diurnal boundary layer wind oscillation above sloping terrain. Paper presented at the Conf. on Physical Processes in the Lower Atmosphere, Amer. Meteor. Soc.

[3] Mackay, K. P., 1967: Geostrophic wind field applicable to the low-level jet. Paper presented at the Conf. on Physical Processes in the Lower Atmosphere, Amer. Meteor. Soc.

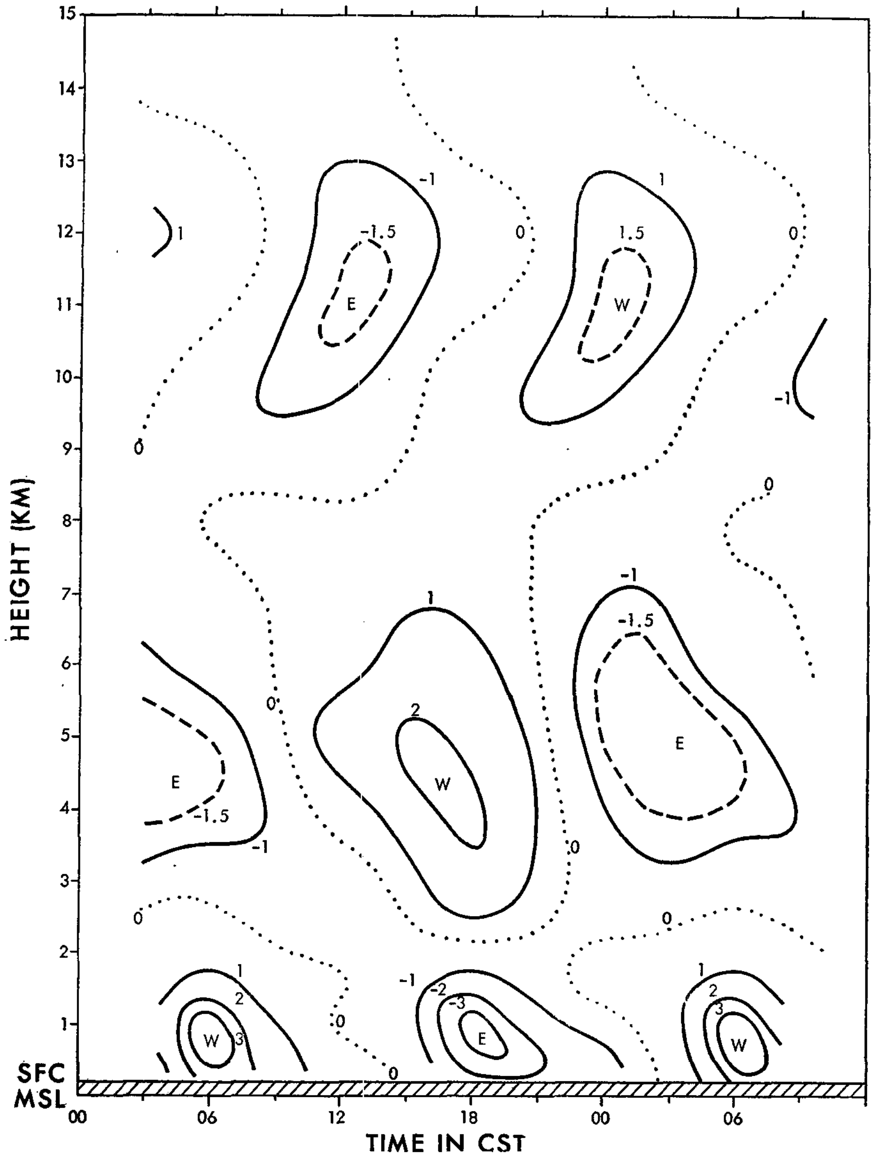

FIG. 1. The westerly component of the mean departure vector as a function of altitude and time for July 1958 at Fort Worth, Tex., with values in m sec^{-1} (after Hering and Borden, 1962).

model will admit perturbation solutions with a periodic dependence on height above the boundary layer for any integrable departure profile in a convergent boundary layer. Finally, an example of the results possible through this approach will be presented for a particular profile of wind departures from the mean within the boundary layer.

2. The model and boundary conditions

As a mathematical model for the phenomenon under consideration, assume that the atmosphere above the boundary layer is frictionless, homogeneous and incompressible. Assume also that the mean or basic flow is geostrophic, oriented in the y direction and constant in time and space. The perturbation components are assumed independent of y, and latitudinal variations of f, the Coriolis parameter, are neglected. The perturbation equations of motion and contintuity are then as follows:

$$(\partial u/\partial t)+(1/\bar{\rho})(\partial p/\partial x)-fv=0, \quad (1)$$

$$(\partial v/\partial t)+fu=0, \quad (2)$$

$$(\partial w/\partial t)+(1/\bar{\rho})(\partial p/\partial z)=0, \quad (3)$$

$$(\partial u/\partial x)+(\partial w/\partial z)=0, \quad (4)$$

where u, v and w are the perturbation velocity components in the x, y and z directions, and p is the perturbation pressure.

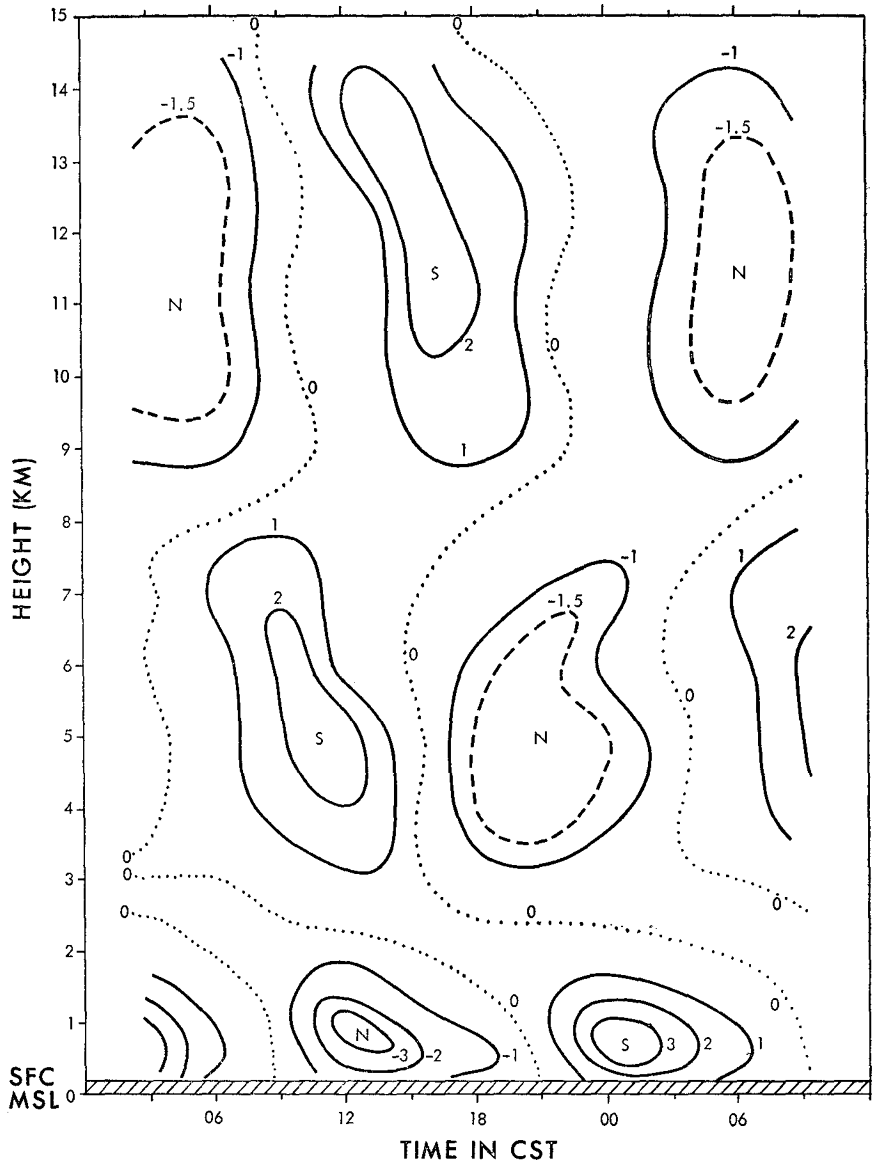

FIG. 2. The southerly component of the mean departure vector as a function of altitude and time for July 1958 at Fort Worth, Tex., with values in m sec^{-1} (after Hering and Borden, 1962).

The boundary layer of the atmosphere will be introduced into this model by specifying the behavior of the wind components in the boundary layer. That is, rather than including the dynamics of the boundary layer into the model, the kinematical behavior of the total wind components will be specified by writing

$$u_-^* = u_-(x,z,t)$$
$$v_-^* = V(z) + v_-(x,z,t)$$

where the functional forms can be determined from theory or observations. The subscripts are used to refer to the atmosphere below the gradient wind level. These functions represent a boundary layer wind that has mean or basic flow $V(z)$ in the y direction and a vector departure from the mean wind which has time and spatial variation.

In this initial analysis of the phenomenon, let us assume that the boundary layer departure oscillation is similar to that proposed by Blackadar (1957) but with arbitrary frequency ν and that the horizontal distribution of the amplitude is simple, periodic so as to simulate the low-level, horizontal jet profile. Accordingly, one may write

$$u_-(x,z,t) = a(z)\cos(\mu x)\cos(\nu t),$$
$$v_-(x,z,t) = a(z)\cos(\mu x)\sin(\nu t),$$

both for $z \leq H$, where H is the gradient wind level. Integration of the equation of continuity through the boundary layer to its top at $z=H$ with the kinematic boundary condition $w=0$ at $z=0$ yields

$$w_-(x,H,t) = -\mu \sin(\mu x) \cos(\nu t) \int_0^H a(z)dz.$$

Boundary conditions for the problem will be continuity of the velocity components at the gradient wind level, $z=H$, and zero vertical motion at the tropopause, $z=Z$.

For the atmosphere above the gradient wind level H, if the vertical motion is assumed separable in the independent variables, i.e.,

$$w_+(x,z,t) = C(z)g(x)h(t), \quad z \geq H,$$

the continuity of the vertical motion at $z=H$ implies that

$$C(H)g(x)h(t) = -\mu \sin(\mu x) \cos(\nu t) \int_0^H a(z)dz.$$

Hence, one can write

$$g(x) = \sin(\mu x),$$

$$h(t) = \cos(\nu t),$$

$$C(H) = -\mu \int_0^H a(z)dz,$$

and the vertical motion perturbation for $z \geq H$ can be written (suppressing the + subscript), in the form

$$w(x,z,t) = C(z) \sin(\mu x) \cos(\nu t).$$

Substituting this vertical motion function in the equation of continuity (4) would imply that $u(x,z,t)$ vary as $\cos(\nu t)$ and $\cos(\mu x)$; since this variation of $u(x,z,t)$ and Eq. (2) implies that $v(x,z,t)$ vary as $\sin(\nu t)$ and $\cos(\mu x)$, then either (1) or (3) implies that $p(x,z,t)$ varies as $\sin(\nu t)$ and $\sin(\mu x)$. Hence, an examination of the perturbation equations for the atmosphere above $z=H$ and the boundary condition of continuous vertical motion at this level leads one to assume trial solutions, for $z \geq H$, of the following form:

$$u(x,z,t) = A(z) \cos(\mu x) \cos(\nu t),$$

$$v(x,z,t) = B(z) \cos(\mu x) \sin(\nu t),$$

$$w(x,z,t) = C(z) \sin(\mu x) \cos(\nu t),$$

$$p(x,z,t) = D(z) \sin(\mu x) \sin(\nu t).$$

Since H is the gradient wind level (by definition), the boundary conditions are (the prime representing differentiation):

$$A(H) = B(H) = 0 \rightarrow C'(H) = 0,$$

$$C(H) = -\mu \int_0^H a(z)dz,$$

$$C(Z) = 0,$$

where Z is the tropopause height and for the basic flow

$$V(H) = V_g = \text{constant}.$$

Substituting the above functions into (1)–(4) leads to the set:

$$-\nu A(z) + (\mu/\bar{\rho})D(z) - fB(z) = 0, \quad (5)$$

$$\nu B(z) + fA(z) = 0, \quad (6)$$

$$-\nu C(z) + (1/\bar{\rho})D'(z) = 0, \quad (7)$$

$$-\mu A(z) + C'(z) = 0. \quad (8)$$

Eliminating $B(z)$ between (5) and (6) yields

$$D(z) = -[\bar{\rho}(f^2 - \nu^2)/\mu\nu]A(z). \quad (9)$$

Differentiating (7) and using the result to eliminate $C'(z)$ from (8) one obtains

$$D''(z) = (\nu\bar{\rho}\mu)A(z). \quad (10)$$

Now differentiating (9) twice to eliminate $D''(z)$ from (10) leads to

$$A''(z) + [\mu^2\nu^2/(f^2 - \nu^2)]A(z) = 0. \quad (11)$$

For $\nu^2 < f^2$, or oscillation period τ greater than inertial period τ_i, the vertical dependence of the perturbation solutions will be periodic. Also, the following relationship exists between the period and the horizontal and vertical wavelengths L_x and L_z:

$$\tau = \tau_i[1 + (L_z/L_x)^2]^{\frac{1}{2}}. \quad (12)$$

From (12) it is seen that for $L_x \gg L_z$, the oscillation period approaches an inertial period which, at Fort Worth, Tex., is nearly 24 hr.

To satisfy the boundary conditions, from (11) let us write

$$A(z) = u_0 \sin[(2\pi z/L_z) + \psi].$$

Then, from (8)

$$C(z) = -(\mu L_z u_0/2\pi) \cos[(2\pi z/L_z) + \psi].$$

The boundary conditions on the vertical motion will be satisfied for the following choice of the parameters:

$$u_0 = (2\pi/L_z) \int_0^H a(z)dz,$$

$$\psi = -(2\pi H/L_z),$$

$$L_z = 2(Z - H)/(n + \tfrac{1}{2}), \quad (n = 0, 1, 2, \cdots).$$

One may now use (5), (6) and (7) to determine the remaining z-dependent functions. The perturbation solutions are then given by:

$$u=\{(2\pi/L_z)\int_0^H a(z)dz\}\cos(2\pi x/L_x)\times\sin[2\pi(z-H)/L_z]\cos(\nu t),$$

$$v=\{-(2\pi f/\nu L_z)\int_0^H a(z)dz\}\cos(2\pi x/L_x)\times\sin[2\pi(z-H)/L_z]\sin(\nu t),$$

$$w=\{-(2\pi/L_x)\int_0^H a(z)dz\}\sin(2\pi x/L_x)\times\cos[2\pi(z-H)/L_z]\cos(\nu t),$$

$$p=\{-(\nu\bar{\rho}L_z/L_x)\int_0^H a(z)dz\}\sin(2\pi x/L_x)\times\sin[2\pi(z-H)/L_z]\sin(\nu t),$$

where L_z is given above, L_x and $a(z)$ are to be determined from the boundary layer wind field, and $\nu=2\pi/\tau$ is to be determined from (12).

3. Application to a particular boundary layer wind field

Let the vertical structure of the horizontal wind in the boundary layer be specified by the functions

$$V(z)=V_0[1-\exp(-\gamma z)\cos(\gamma z)],$$

$$a(z)=\alpha\exp(-\beta z)\sin(\pi z/H),$$

where α, β, γ, V_0 are constants. It might be noted at this point that the basic wind and the normal departure component are of a form similar to the Ekman spiral solutions. A smooth match of the mean wind at $z=H$ is accomplished by the values,

$$\gamma=7\pi/4H;\quad V_0=V_g/[1-\exp(-7\pi/4)\cos(7\pi/4)],$$

where V_g is the geostrophic wind for $z>H$, and its value is taken to be 10 m sec^{-1}. To aid in the choice of the boundary layer profile parameters α and β, the number β will be eliminated in favor of the ratio of the height of the boundary layer wind maximum, to be denoted by H^*, to the depth of the boundary layer H, since this ratio is more easily determined from observation than is the number β. The maximum value of the departure amplitude $a(z)$ can then be obtained in terms of this ratio and the parameter α by simply evaluating $a(H^*)$ and eliminating β as described.

In the boundary layer, if H^* is the height of the maximum departure amplitude, then the equation $a'(H^*)=0$ can be solved for β to obtain,

$$\beta=(\pi/H)\cot(\pi H^*/H).$$

With this β, the amplitude at the level H^* is given by

$$a(H^*)=\alpha\exp[-(\pi H^*/H)\cot(\pi H^*/H)]\sin(\pi H^*/H).$$

Since this represents the maximum amplitude of the departure vector in the boundary layer, values for H^*, H and $a(H^*)$ may be taken from Hering and Borden's (1962) data (Figs. 1 and 2) in order to determine α and also β. For the values $H=2$ km, $H^*=0.55$ km, $a(H^*)=3.75$ m sec^{-1}, α and β are 10.27 m sec^{-1} and 1.35 km^{-1}, respectively.

For the horizontal dependence of the amplitude of the boundary layer departure vector, choose $L_x=1200$ km.

For the vertical dependence of the solutions above $z=H=2$ km, choose the tropopause height $Z=17$ km and the integer $n=2$. Then $L_z=12$ km.

With these values, τ is essentially 24 hr at the latitude of Fort Worth. The origin of coordinates is chosen such that $x=0$ is the location of the horizontal wind

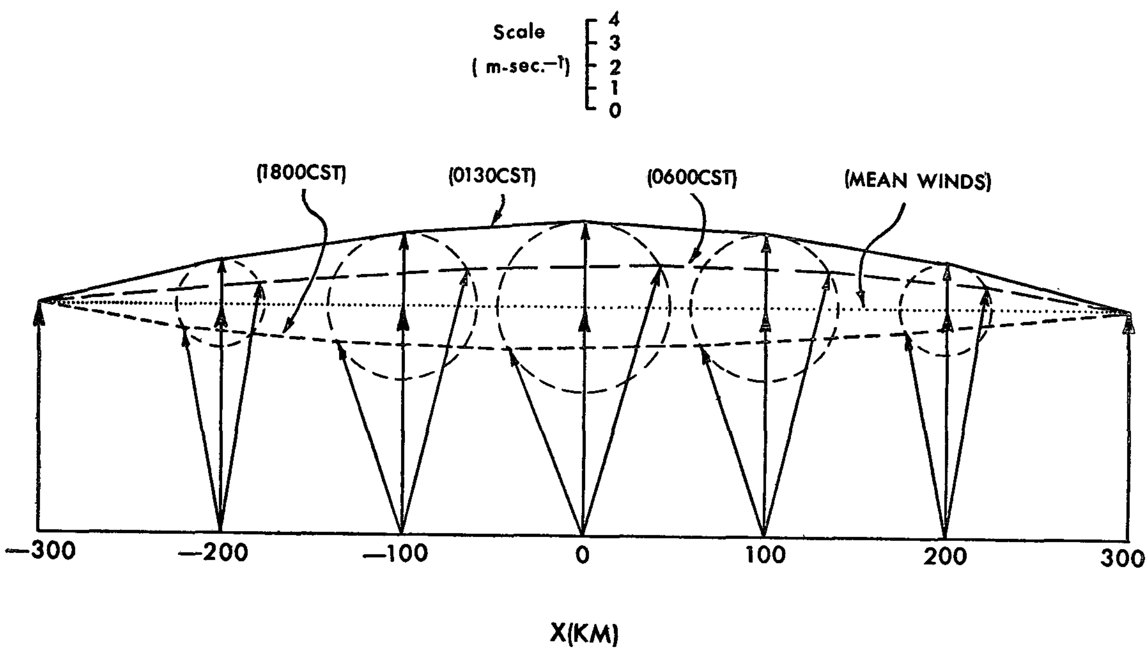

FIG. 3. Horizontal profile of the modeled, boundary layer wind vector at H^*, the height of maximum departure amplitude.

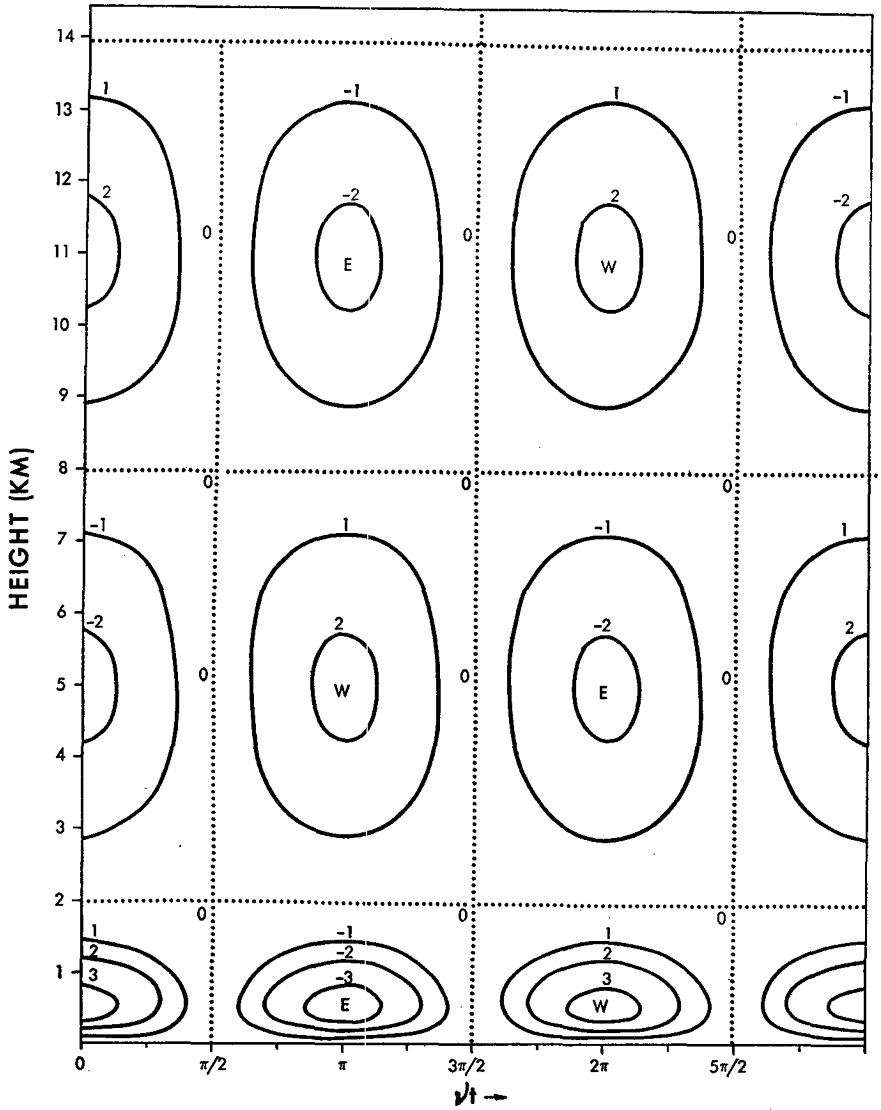

FIG. 4. The westerly component of the perturbation velocity as a function of altitude and time at the axis of the low-level jet. Values are in m sec^{-1} and ν is the oscillation frequency.

profile maximum and $t=0$ is the time of maximum easterly departure in the boundary layer (about 1930 CST).

Fig. 3 shows a horizontal profile of the boundary layer wind vector, as it has been described here, for the level $z=H^*$ and at the times corresponding to the maximum low level jet development, $\nu t=\pi/2$, and for $\nu t=-\pi/6$, $5\pi/6$. This graph not only shows the wind vector, at each location x, oscillating to super-geostrophic values at the time of maximum development, but the entire horizontal profile develops a jet-like appearance during the oscillation. Horizontal profiles in the middle and upper troposphere behave similarly, away from the nodal points, except for the phase of the oscillation.

Fig. 4 shows a plot of u vs. height and time at the axis of the low-level horizontal jet and can be compared with Fig. 1. It is seen that the centers of maximum departure are in roughly the same location on these graphs as are also the levels of non-oscillation. It is also seen that the maximum departure component in the layer immediately above the boundary layer is less, in magnitude, than the boundary layer maximum, and the ratio of these values compares favorably to the data of Hering and Borden. Notably missing in the theoretical graph is the tilt or time lag in the upper layer oscillation cell that is apparent in Fig. 1.

Fig. 5 shows the time-height cross section of the vertical velocity perturbation. The values given should be multiplied by $\sin(2\pi x/L_x)$ and are in cm sec^{-1}.

A plot of the total horizontal wind speed vs. height is shown in Fig. 6 at $x=0$ and for $\nu t=0$, $\pi/2$, π, $3\pi/2$. These profiles show the sub- and super-geostrophic

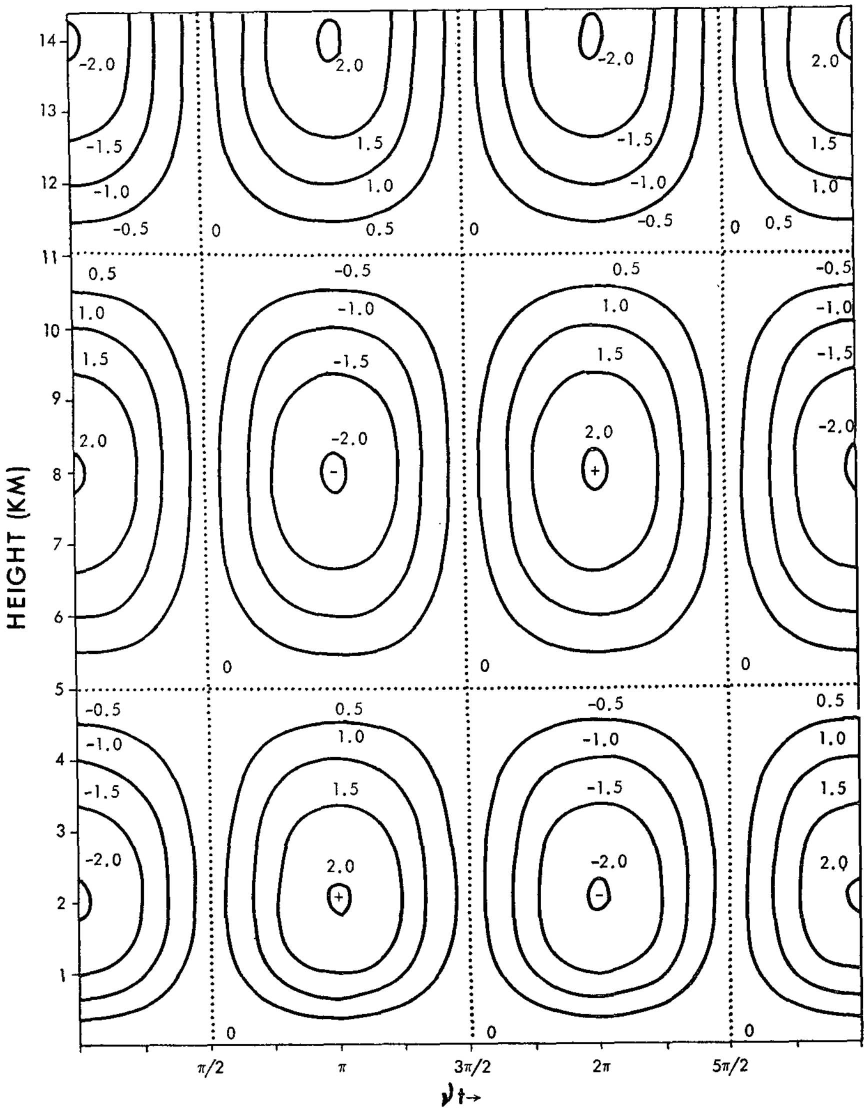

FIG. 5. The vertical motion perturbation as a function of altitude and time. Values are in cm sec^{-1} and are to be multiplied by $\sin(2\pi x/L_x)$.

winds in the upper levels that respond to the boundary layer oscillation.

4. Discussion

It is hypothesized that the observed oscillations in the wind field above the boundary layer are caused by the oscillations within the boundary layer associated with low-level jet development. This hypothesis is based on the fact that reasonable theories, as cited in the introduction, have been proposed for the boundary layer oscillation that do not require a dynamic influence from the upper levels.

If one begins at roughly 1800 CST at a location where strong frictional retardation has occurred during the day, the total horizontal wind vector is sub-geostrophic with a cross-isobaric component to the left. If the departures are distributed so that low-level convergence exists within the boundary layer at this time, then the equation of continuity and kinematic boundary condition at the earth's surface imply that the vertical velocity achieves a local maximum at the level where no departures are induced by the frictional effects. This level is at height H in the model. Although the model presented here does not envelop the dynamics of the boundary layer, the assumption of low-level horizontal convergence is reasonable in light of the spatial variability in terrain features, eddy viscosity, heat balances and other pertinent factors in boundary layer theory. The periodic form of the distribution was chosen to simulate the gross features of the horizontal jet profile. With a maximum vertical velocity at height H and strong stability of the stratosphere overlaying the tropospheric region under study, there must be a vertical convergence above the boundary layer and a corre-

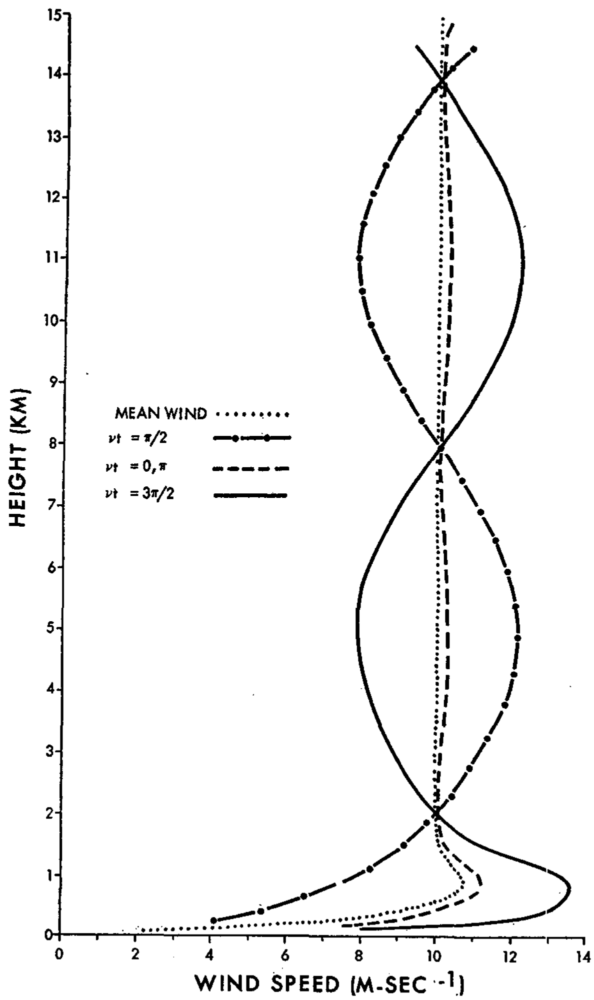

FIG. 6. Theoretical horizontal wind speed as a function of altitude and time at the axis of the low-level jet.

sponding horizontal divergence giving rise to the geostrophic departures aloft. As the low-level wind field oscillates, the convergence field also oscillates giving rise to the oscillatory nature of the geostrophic departures aloft.

It should be pointed out that the possibility exists of a more complicated horizontal distribution of the departure components than has been used here. The fact that solutions with simple periodicity in x satisfy the perturbation equations for the atmosphere above $z=H$ suggests that the boundary condition at the gradient wind level for a more general vertical motion function in the boundary layer could be satisfied by a Fourier series representation in x alone, for the simple time periodicity, or in x and t for an even more general space-time distribution. In fact, it is possible to reformulate this same mathematical model in a more general way to obtain eigenvalue problems in each independent variable whose eigenvalues (wavelengths, frequencies, etc.) and eigenfunctions are intercoupled in a way similar to Eq. (12) and the solution forms given here.

In this simple model of the atmosphere above the boundary layer, the geostrophic wind was assumed uniform in time and space. More general mathematical models and procedures for solution are presently under study to determine the effects of various types of basic wind shear and turning as well as the effects of density stratification on the oscillations in the upper levels.

5. Summary

In this paper, a mathematical model for the diurnal oscillations in the tropospheric wind field that are associated with the boundary layer jet phenomenon has been presented. This theoretical model has demonstrated that the basic perturbation equations of motion and continuity, when solved even for a reasonably simple model of the troposphere, will admit solutions that show many of the important kinematical features of the tropospheric wind field as observed by Hering and Borden. While there are many conceivable models, several presently under study by the authors, it is significant that the results presented here yield generally correct relationships between observation and theory. A plausible dynamical explanation of the existence of diurnal oscillations above the boundary layer has been given. This dynamical argument with the successes of the mathematical model strongly implies that the diurnal oscillations that have been observed in the middle troposphere are responses of the atmospheric fluid to the horizontal convergence patterns associated with the boundary layer oscillation.

REFERENCES

Blackadar, A. K., 1955: The low level jet phenomenon. Institute of the Aeronautical Sciences Reprint #519.

——, 1957: Boundary layer wind maxima and their significance for the growth of nocturnal inversions. *Bull. Amer. Meteor. Soc.*, **38**, 283–290.

Bonner, W., 1965: Statistical and kinematical properties of the low-level jet stream. Mesometeorology Research Paper 38, University of Chicago, 54 pp.

Buajitti, K., and A. K. Blackadar, 1957: Theoretical studies of diurnal wind structure variations in the planetary boundary layer. *Quart. J. Roy. Meteor. Soc.*, **83**, 486–500.

Hering, W. S., and T. R. Borden, 1962: Diurnal variations in the summer wind field over the Central United States. *J. Atmos. Sci.*, **19**, 81–86.

Hoecker, W. H., 1965: Comparative physical behavior of southerly boundary layer wind jets. *Mon. Wea. Rev.*, **93**, 133–144.

Kaimal, H. C., and Y. Izumi, 1965: Vertical velocity fluctuations in a nocturnal low level jet. *J. Appl. Meteor.*, **4**, 576–584.

Lettau, H., 1954: Graphs and illustrations of diverse atmospheric states and processes observed during the Great Plains turbulence field program. Occasional Rept. No. 1, Atmospheric Analysis Laboratory, Air Force Cambridge Research Center.

——, 1964: Preliminary note on the effects of terrain slope on low-level jets and thermal winds in the planetary boundary layer. *Studies of the Effects of Variation in Boundary Conditions on the Atmospheric Boundary Layer*, Dept. of Meteorology, University of Wisconsin, Annual Rept., Contract DA-36-039-AMC-00878, USAERDA, Fort Huachuca, Ariz., 122 pp.

Wexler, H., 1961: A boundary layer interpretation of the low level jet. *Tellus*, **13**, 369–378.