Comments on "Cloud Droplet Coalescence: Statistical Foundations and a One-Dimensional Sedimentation Model"

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The paper by Warshaw (1967) represents an admirable attempt to measure the validity of the stochastic collection equation as applied to the growth of cloud droplets. It leaves, however, some false impressions and calls for the following comments.

1. The statement made in the first paragraph that Berry (1965) "attempt(s) to simplify the model to a 'continuous-collection' formulation when the droplet has grown sufficiently large" is incorrect. The calculations by Berry were made entirely by the stochastic model. Reduction to the continuous formulation was made unnecessary by the transformation to logarithmic coordinates before the subjection of the problem to numerical computation.

This transformation allows the range in droplet radius from 4 to 400 μ to be covered by 41 storage locations, which is significantly less than the 397 locations that would be required by the method of Warshaw. Had Warshaw used this system his conclusion that stochastic growth calculations are very difficult might have been revised.

2. Section 2 contains the statement that the probability of collection equals Kt only if t is not too small "for when t is very small, the effect of neglecting the volumes of the droplets themselves becomes important." In fact, the probability is meaningful and equal to Kt as t approaches zero, since, from the definition of the collection efficiency E, one is concerned only with the probability that the *center* of the droplet lies in a volume with area $\pi(r+\rho)^2E$ and height $[U(r)-U(\rho)]t$, where r,ρ are the radii of the collector and collected drop, and U is the terminal speed. This is an undistorted volume, well ahead of the collector, that will be swept out by the collector during the interval t.

The only consequence of this point is that it simplifies the derivation of the collection equation by making the appearance of the time derivative, as Warshaw gets in his (2.5), more easily justified.

3. Quite distinct from the above remarks is the following: Warshaw reaches the conclusion that the predictions of the stochastic collection equation contain a moderate to large error due to the statistical fluctuations in the mean collection rate. This conclusion follows from his consideration that the probability that a *single* large droplet collects k of $N(\rho)\Delta\rho$ smaller droplets in a time t is

$$P[\nu=k]=\binom{N(\rho)\Delta\rho}{k}(Kt)^k(1-Kt)^{N(\rho)\Delta\rho-k}, \quad (1)$$

where Kt is the probability that a particular large drop captures a particular small drop. Typical cloud values (below) give $Kt\ll 1$ and the relative dispersion becomes that of Poisson statistics,

$$\sigma/\bar{\nu}=\bar{\nu}^{-\frac{1}{2}}. \quad (2)$$

The "mean number of collections" $\bar{\nu}$, for (1) is

$$\bar{\nu}=N(\rho)\Delta\rho Kt, \quad (3)$$

which, when entered in (2) gives a large relative dispersion. This conclusion is valid when there is only *one* large droplet in the selected volume V inside a cloud.

However, there is not one but $N(r)\Delta r$ large droplets in V and the correct formulation for the number of (r,ρ) captures in V is the same equation from which Warshaw finds the "mean collection rate" $\partial\bar{\nu}/\partial t$, namely,

$$P[\nu=k]=\binom{N(r)\Delta rN(\rho)\Delta\rho}{k}(Kt)^k(1-Kt)^{N(r)\Delta rN(\rho)\Delta\rho-k}. \quad (4)$$

That is, within the volume V and during the time interval t, the test for collection is performed $N(r)\Delta rN(\rho)\Delta\rho$ times. The meaningful fluctuations are those of (4) and not of (1). The relative dispersion is still given by (2), but now

$$\bar{\nu}=N(r)\Delta rN(\rho)\Delta\rho Kt. \quad (5)$$

This produces a very small relative dispersion and significantly changes the conclusions reached by Warshaw.

Let us apply some representative numbers:

a. For *small droplets* we choose a volume $V=1$ m^3 somewhere inside the cloud. Let $r=50\,\mu$, $\rho=16\,\mu$, $N(r)\Delta r=10\cdot10^6$, $N(\rho)\Delta\rho=50\cdot10^6$ and $K=2\cdot10^{-9}$ sec^{-1}. Thus, $Kt\ll1$ for all reasonable t, and by (5) and (2), $\bar{\nu}=10^6t$, and $\sigma/\bar{\nu}=(10^6t)^{-\frac{1}{2}}$.

b. For *large drops* let us take a representative volume of $V=10^3$ m^3, and $r=500\,\mu$, $\rho=200\,\mu$, $N(r)\Delta r=10\cdot10^3$, $N(\rho)\Delta\rho=50\cdot10^3$ and $K=2\cdot10^{-7}$ sec^{-1}. Thus, $Kt\ll1$ and $\bar{\nu}=10^5t$, and $\sigma/\bar{\nu}=(10^5t)^{-\frac{1}{2}}$.

Thus, the relative dispersion in the mean number of collections is very small, even after only 1 sec and for rather small droplet concentrations. Furthermore, it is not necessary to take V to be nearly the whole cloud (as Warshaw concluded) but only a small representative portion of the cloud.

In summary, when only *one* large droplet in V is considered, the relative dispersion is large because the number of independent trials (at collection) is small. But in a real cloud the number of independent trials is proportional to the number of larger droplets in V, and this consideration greatly reduces the relative dispersion.

This reasoning indicates that the stochastic collection equation, contrary to the conclusions reached by Warshaw, is a very good approximation to cloud droplet collection.

REFERENCES

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