Interrelated Changes of Wind Profile Structure and Richardson Number in Air Flow from Land to Inland Lakes

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(Manuscript received 3 April 1968)

ABSTRACT

A semi-empirical model for wind profile modification, in airflow from land to water on the 2-km scale, is discussed using 1950 Lake Hefner data. Initial (rough-surface) and final (smooth-surface) profiles are derived and an interpolation model is developed and used to calculate intermediate profiles. A distribution of vertical velocity with height is determined with the aid of the assumption that the divergence in the x direction near the surface is compensated by convergence in the y direction aloft. The rate of growth of the internal boundary layer follows as a direct consequence of the model. Theoretical computation is compared to Super's observations of wind profile variations with fetch on Lake Mendota. A diurnal variation of subsidence found over Lake Hefner is related to the variation in thermal stratification and its effect on the wind profile structure. Momentum budgets between land and lake are constructed and give evidence of a diurnal variation in surface stress over Lake Hefner.

1. Introduction

If air has been flowing over a surface of uniform roughness for a certain length of time, a steady state or equilibrium state of a wind profile is developed. For adiabatic surface-layer conditions this equilibrium profile can be satisfactorily described by Prandtl's "log-law," relating wind speed to the logarithm of height. Upon encountering a surface of a new aerodynamic character, be it rougher or smoother than the original, the wind profile gradually begins to adjust to the new roughness, and another equilibrium tends to be established after a sufficiently long fetch downwind. One can also say that at the leading edge of the new surface an internal boundary layer begins to develop. If the new surface is rougher, mechanical turbulence is increased and the internal boundary layer is likely to grow faster than if the new surface were smoother than the original. The thickness of this layer is also in direct proportion to the amount of momentum extracted from the moving air. Along the downwind fetch one finds that within the internal boundary layer the air undergoes readjustment to the new surface, while outside this layer the airflow remains undisturbed. There have been many significant contributions to the study of wind profile modification; for references see Section 12 of Munn (1966). Special consideration will be given here to theoretical work by Elliott (1958) and Panofsky and Townsend (1964).

In addition to a change in roughness as air flows from one surface type to another, there may be a change in temperature or moisture profiles due to changes in the energy budget of the surfaces. Horizontal advection of sensible and latent heat will occur. Surface heating or cooling must affect the curvature of the wind profile, since there will be a change in the stability of the air. Diabatic effects are always involved when air moves from land out across a lake. Super (1964) showed that the wind across Lake Mendota behaved differently under lapse as opposed to inversion conditions. During inversion conditions the wind would accelerate for a horizontal fetch up to about 2 km, then decrease in speed at greater travel distances. If lapse conditions existed, the speed was observed to increase across the entire available length (about 5 km).

Super's experimental work had been carried out for relatively large fetches in comparison with wind profile modification experiments in other studies of the order of centimeters to meters in wind tunnels, and hardly more than a few meters in natural field experiments. In this study, wind profile modification over Lake Hefner will be discussed, for a fetch of approximately 2 km.

Previously, as in Elliott's or Panofsky and Townsend's models, an initial steady-state profile was assumed, and the subsequent modified profiles are extrapolated. In this analysis both the initial and final equilibrium profiles are assumed, and a semi-empirical interpolation model is proposed to calculate intermediate profiles in various stages of development. The growth rate of the internal boundary layer then follows as a direct consequence of the model, and the diurnal variation in Richardson number effects can and will be considered.

2. Lake Hefner Project and analysis preparation

The Lake Hefner Project was initiated in April 1950, as a cooperative effort among the Department of the Navy, the Geological Survey, the Bureau of Reclamation, and the U. S. Weather Bureau; it was concluded in August 1951. Its main objective was the problem of evaporation. Hopefully, the results of the analysis would not only be useful for estimating or predicting evaporation from reservoirs but could also be applied to different types of inland lakes.

Voluminous hydrological and micrometeorological data were accumulated during the 16 months of the Project. For a complete account of the instrumentation, reference is made to Anderson (1954). The evaporative flux was to be derived by three independent methods via 1) the hydrological water budget, 2) the interface heat budget, and 3) the mass transfer or vertical diffusion of water vapor into the lower atmosphere. Dry- and wet-bulb temperatures as well as wind speed and direction at heights of 2, 4, 8, and 16 m were recorded at four micrometeorological stations about the lake; surface water temperature and rainfall were measured at these sites as well.

Most important for profile modification studies appeared to be the data from the south shore, barge, and north intake-tower stations. Since our analysis was to be limited to two-dimensional, but horizontally unidirectional, flow in the lower 16 m, all data were reviewed to isolate those days when there was persistent wind direction from the south. Out of the many days with persistent southerly flow, only 15 had complete instrument records at the three stations. From these 15 it was decided to choose a day with strong winds to insure that all anemometer levels of the upwind station were within the surface layer; 28 September 1950 met this requirement and, in addition, exhibited a significant variation in temperature gradient throughout the diurnal period.

Upon plotting the wind profiles (u vs $\log z$) for several 24-hr periods, it was found that the points for the north (intake-tower) station were erratic. An apparent profile discontinuity between the 4- and 8-m levels suggested a disturbing influence of the dam around the north shore of the lake, especially in strong wind cases. Hence, the wind data from the intake-tower station was considered unusable for the intended detailed analysis, and the study was limited to the profile changes from the south shore to the center of the lake only. In the following discussion, the south shore site will be referred to as the upwind station, and the barge as the downwind station.

3. Data analysis

The presentation of the 3-hr wind and temperature data in the Lake Hefner report make Richardson and Deacon numbers readily computable. First, both the temperature and wind data at the upwind and downwind stations were smoothed for the entire diurnal period using an equally weighted 3-value running time mean. Then Deacon and Richardson numbers were computed at both stations, using the equations of definition

$$\beta = -\partial \log(\partial u/\partial z)/\partial \log z,$$
$$\mathrm{Ri} = (g/\theta_M)(\partial\theta/\partial z)/(\partial u/\partial z)^2,$$

where $\partial u/\partial z$ is the vertical shear, g the acceleration of gravity, θ the potential temperature, and θ_M its layer-mean value. It is conventionally assumed that the Deacon number (or the nondimensional profile curvature β) represents a "shape factor," while the Richardson number serves as a "scaling factor." This distinction suggests that a shape factor can be compared with the role of a dependent variable, the scaling factor with that of an independent variable.

A plot of β vs Ri is shown in Fig. 1. The upwind station shows some scatter, but points tend to follow the requirement that $\beta=1$ for $\mathrm{Ri}=0$, with $\beta(\mathrm{RI})$ corresponding tolerably well to the "KEYPS" relation,

$$\beta = (1-18\ \mathrm{Ri})/(1-13.5\ \mathrm{Ri}).$$

Throughout the diurnal period the wind profile at the downwind station, however, shows persistent curvature which obviously does not conform to any direct extension of the logarithmic law. Thus, computed values of the shape factor β of the wind profile range between 1.2 and 1.6 for the entire diurnal period during which the thermal stratification changes back and forth between lapse and inversion conditions. This can be verified in Fig. 1 where it is immediately apparent that the points for the downwind station disagree significantly with the semi-empirical "KEYPS" relation, because β remains larger than unity even for strongly positive Ri.

Such "anomalous" values of β could be attributed to three causes: 1) prevailing lapse conditions, 2) anemometer levels being outside the surface layer, or 3) a wind profile not fully developed. Persistent lapse conditions are ruled out because Ri changes its sign during the diurnal period. Since wind speeds for the period of this analysis were sufficiently high (8–11 m $\sec^{-1}$), most of the anemometer levels can be assumed within the surface layer. This suggests that the anomalous curvature evidenced in Fig. 1 for the lake station is probably due to incomplete profile adjustment. However, even though the Richardson-number dependency can be too weak to alter the overriding influence of incomplete adjustment on the profiles, thermal structure does affect the winds speeds significantly, as will be seen in Section 10. Thus, the first problem will be to construct an adiabatic wind profile which is representative of equilibrium conditions over the water. This is equivalent to an extrapolation to a large fetch downwind, using the observed wind data after the effect of thermal structure has been eliminated.

Tentatively, it will be assumed that surface roughness as well as friction velocity are uniform over the entire lake surface. This assumption is an oversimplification since it is often observed that the surface characteristics of the lake change downwind in connection with currents, upwellings, and/or the development of waves. However, only this assumption makes it possible to say that a neutral profile below the 4-m level at the downwind station will be representative of any other neutral

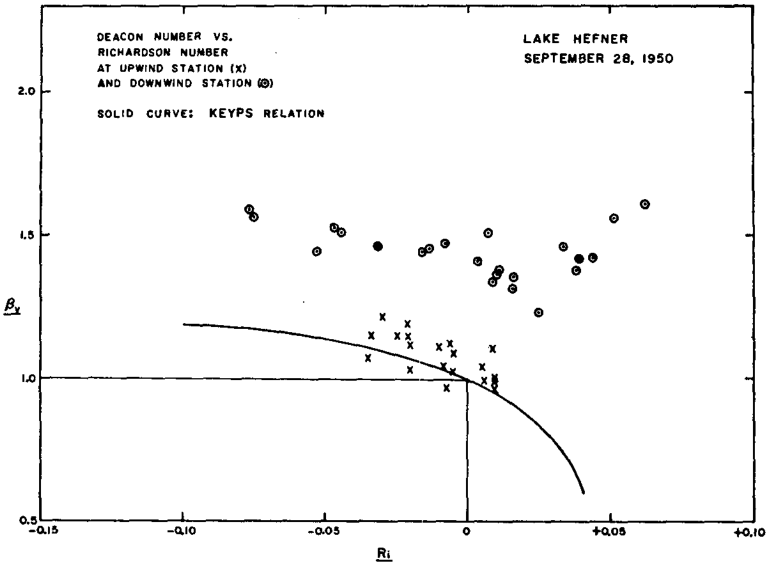

FIG. 1. Deacon number vs Richardson number for upwind and downwind stations on Lake Hefner during a full diurnal cycle. The curve illustrates the theoretical "KEYPS" relation.

profile, in these lowest air layers, at any fetch beyond this station.

4. Model characteristics

The two equilibrium profiles, i.e., for the initial state over rough terrain and the final state over smooth terrain, under adiabatic surface layer conditions are given by two different sets of parameters of the log-law as follows:

$$\left.\begin{aligned} U_I &= (U^*/k)\ \ln(z/Z_0) \\ U_F &= (u^*/k)\ \ln(z/z_0) \end{aligned}\right\}, \tag{1}$$

where the values of U_I, as determined by the set U^* and Z_0, pertain to the initial conditions over land, while U_F, as determined by the set u^* and z_0, refers to the far-downwind conditions over the lake. For the Karman constant k, a value of 0.428 is used after Lettau (1961). Since the lowest anemometer level considered in this analysis was numerically much greater than the roughness heights, z/z_0 and z/Z_0 have been substituted in (1) for the more correct forms $(z+z_0)/z_0$ and $(z+Z_0)/Z_0$. At any point along the fetch between the initial and final states it is assumed the wind profile is in some state of continuous transition. The initial and final wind profiles are functions of z only. The transition profiles $u=u(x,z)$ are a function of both x and z. The problem is to express the function $u(x,z)$.

With the aid of (1), a dimensionless transition function $\psi(x,z)$ is defined by

$$\psi \equiv \frac{\Delta u}{\Delta U} \equiv \frac{u-U_I}{U_F-U_I}, \tag{2}$$

where $u=u(x,z)$ refers to the same height as U_I and U_F. This identity implies that $\psi=0$ at $x=0$ (i.e., at the line of roughness discontinuity). As $u\rightarrow U_F$ at large fetches, $\psi\rightarrow 1$. Following the classical lines of a "similarity" solution, the profile changes with fetch can be described by saying that ψ must be a function of a new independent dimensionless variable $\zeta=z/Z$. Z has dimensions of length and is a monotonically increasing function of x; furthermore, Z is proportional to the height of the internal bounday layer. This last condition suggests that when $\psi=1$, ζ is small in comparison with unity, while for $\psi=0$, ζ is large in comparison with unity.

In a tentative, semi-empirical approach to the interpolation problem, ψ was approximated by negative exponential functions which meet the above criteria, such as

$$\psi=e^{-\zeta} \text{ or } \psi=e^{-\zeta^2}. \tag{3}$$

The first, simple exponential form was discarded because it resulted in an unrealistic rapid rate of increase in Z with distance close to the upwind shore. The second, or Gaussian, form emphasizes a continuous, gradual change of the profile downwind. The model implies an aysmptotic approach to the upwind shearing stress at large heights downwind and also a gradual change in shearing stress with height without a discontinuity at the top of the internal boundary layer.

5. Computation of equilibrium profiles at upwind and downwind stations

For the empirical determination of the transition function over Lake Hefner, it would have been ideal if adiabatic wind profiles were observed simultaneously at both stations. However, none of the measured temperature profiles showed truly adiabatic conditions, and Richardson numbers were normally of opposite sign at the two stations. An estimate of adiabatic profiles with no change in stratification was derived by interpolation in the following way, involving somewhat tedious procedures.

The upwind data were analyzed to obtain the set Z_0 and U^*, with the aid of a scheme for the diabatic surface layer described by Dalrymple *et al.* (1966) and illustrated by Stearns and Lettau (1963). Accordingly, the value of Z_0 was obtained using the equation

$$\log Z_0 = \log(z+D) + 0.43(\Phi - \phi/\alpha), \qquad (4)$$

where ϕ is the diabatic influence function defined as

$$\phi = [k(z+d)\partial v/\partial z]/\sqrt{\tau_0/\rho}.$$

The integral diabatic influence function Φ is defined by

$$\Phi = \int_0^z (z+d)^{-1}(\phi-1)dz,$$

where d is the zero-plane displacement which equals $D-Z_0$, and α is the profile contour number which is a function of height defined by

$$\alpha = \Delta \log V / \Delta \log (z+D). \qquad (5)$$

Since $D \ll z$, a displacement height was considered insignificant in this analysis.

Roughness values were computed using β's at 400 and 800 cm for all eight profiles of the diurnal period. An average for the upwind site was obtained by averaging 16 values of $\log Z_0$ and resulted in $Z_0 = 4.92 \pm 1.27$ cm. This value appears to agree with independent estimates of the aerodynamic roughness of open prairie country (as for the surroundings of Lake Hefner, in Oklahoma).

Using the same diabatic surface-layer model, a value for the friction velocity was also obtained using the expression

$$U^* = k\alpha V/\phi. \qquad (6)$$

As before, an average of 16 values was taken, resulting in $U^* = 69 \pm 8.7$ cm sec^{-1}. The equilibrium profile over land thus obtained is shown as part of Fig. 4.

A plot of Richardson numbers at the upwind vs downwind station shows the inverse relationship between stability over land and lake (see Fig. 2a). The diurnal trend in temperature stratification has a distinct effect on the wind differences between the upwind and downwind stations; namely, a greater or lesser degree of divergence as the air accelerates over the lake. It was with this in mind that another graph (Fig. 2b) was superimposed on Fig. 2a. A base line approximating the slope of the Ri_I vs Ri_{II} points was drawn through the origin. Dashed lines perpendicular to the base line were drawn through each time period on the original graph.

Wind speed differences between the upwind and downwind stations were computed at each level (16, 8, 4, 2 m) for the eight 3-hr averages. These differences $(u-U_i)$ divided by the upwind speed U_i are plotted as the ordinate in the superimposed graph. The abscissa represents time from 0130 to 1330 going left to right and 1330 to 2230 going back to the left again.

Three clusters of points are found. The first, going from left to right, represents inversion conditions over land and lapse over the lake. The second group represents slight lapse conditions over land and slight inver-

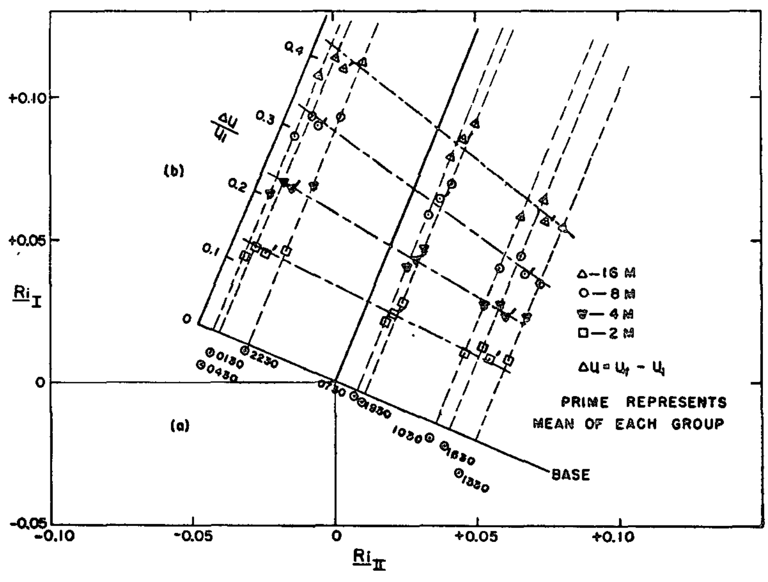

FIG. 2. Stability at upwind station (Ri_I) vs stability at downwind station (Ri_{II}) for eight 3-hr periods during 28 September 1950, a., and superposition of new graph indicating wind differences between downwind and upwind stations during diabatic and adiabatic periods at four levels, b. See text for explanation.

sion over the lake. The third group indicates strong lapse over land and strong inversion over the lake. A straight line was fitted to the means of each group representing the four levels.

A new line was then drawn perpendicular to the base line through the origin (or zero-zero of the Richardson number scales); this line can be thought of as representing the adiabatic case common to both sites. Points were read from the graph at the intersection of the new adiabatic line and the four height curves. These values were assumed to represent $\Delta u/U_I=(U-u_I)/U_I$ for the simultaneous adiabatic condition. Since the upwind adiabatic profile has already been computed, it is now possible to obtain Δu at the four levels in question; and, with the aid of Eq. (2), the adiabatic value of the transition function ψ is found.

For an array of assumed Z values, ζ^2 and ψ were calculated for the four levels using Eq. (3). Then ΔU was calculated with the aid of the Δu's obtained in the previous section and Eq. (2). Final U_F values were then computed as $\Delta U+U_I=U_F$. When a profile refers to neutral conditions, velocity differences from level to level must vary linearly with corresponding differences in ln$z$. Consequently, since all the levels in this analysis are double heights, differences in U_F must be constant between all four instrument levels, and this criterion permits us to derive a "best value" of Z through trial and error.

For the fetch of 2 km at the downwind station, the value of Z thus obtained was 35 m. It should be noted that the basis for the estimation of Z hinges critically not only on the choice of relations (3) but also upon the representativeness of Fig. 2b. Unfortunately, there are too many degrees of freedom in fitting the four straight lines to the "cluster means." Thus, the values of $\Delta u/U_1$ ead off the graph for the adiabatic condition could easily vary at least by $\pm 2\%$. Although this amount may seem small, it was found that changing the $\Delta u/U_I$ values, in the same direction, at the 2- and 16-m levels by only 1%, the best estimate of Z changed by 15%, which means 5 m, if not more. However, improvements will be possible only when more detailed and representative observational data are available with an increased number of stations over both land and water surfaces.

Having found a representative downwind adiabatic wind profile, the roughness parameter of the lake was implicitly determined; z_0 turned out to equal 0.235 cm. The values estimated by Harbeck and Marciano (1954) ranged from 0.5–1.2 cm; they used only the lower three anemometer levels, and averages for 98 near-adiabatic cases. Their z_0 values increased directly with increasing wind speed at the 8-m level, suggesting perhaps that the roughness may be related to increasing wave height over the center of Lake Hefner. With a wind speed (at 8 m) of 1000 cm sec^{-1}, Marciano and Harbeck obtained a value of 0.94 cm for z_0, while in this analysis $z_0=0.235$ cm for the same condition. To a certain extent the difference can be explained by the use of $k=0.428$ (Harbeck and Marciano used $k=0.40$). However, it must be mentioned here that in computing z_0, Harbeck and Marciano assumed fully developed flow at the downwind station. It has been shown here that such an assumption is not valid, and may very likely lead to errors.

The computed friction velocity of 52.6 cm sec^{-1} is within 10% of the value 49.9 cm sec^{-1} given by Harbeck and Marciano for a 10 m sec^{-1} wind at 8 m. A plot of the downwind equilibrium profile is included in Fig. 4.

6. Mass continuity and vertical motion

It is assumed on the scale of this analysis that density changes are negligible, since $\Delta T_{\max}$ is never more than 3C (2 km)$^{-1}$ giving a value for the density disturbance of less than 10^{-6} gm cm^{-3}. We write the equation of mass continuity as

$$w_z=-u_x-v_y, \tag{7}$$

where subscripts denote differentiation with respect to the indicated Cartesian coordinate. If ζ is defined as before,

$$w_z=w_\zeta/Z \text{ or } Zw_z=w_\zeta. \tag{8}$$

Remembering also that $Z=Z(x)$,

$$\zeta_x=-Z_x\zeta/Z. \tag{9}$$

Now consider Eq. (2), which upon differentiation with respect to x produces

$$u_x=\psi_x\Delta U \text{ or } Zu_x=Z\psi_x\Delta U. \tag{10}$$

Therefore, Eq. (7) may be rewritten as

$$w_\zeta=-Z\psi_x\Delta U-v_yZ. \tag{11}$$

Differentiation of the Gaussian transition function ψ with respect to x gives

$$\psi_x=-2\zeta_x\zeta\psi, \tag{12}$$

so that

$$Z\psi_x=-2Z\zeta_x\zeta\psi=2Z_x\zeta^2\psi. \tag{13}$$

The continuity equation (11) now transforms into

$$dw=-2Z_x\zeta^2\psi\Delta Ud\zeta-v_yZd\zeta. \tag{14}$$

The negative vertical velocities which result from divergence in the lower layers must somehow be compensated for in the upper layers, or an unrealistic finite vertical velocity at any large height will result. Let us assume that in the atmospheric flow considered the compensation in the upper layers is the result of convergence in the y direction. As a consequence of this assumption it is found that the mean vertical velocity first increases (from the boundary value of $\bar{w}$ equal to zero at the surface) to a maximum of $\bar{w}$ in the middle layers, and then decreases to zero again at some large value of ζ, say in excess of $\zeta=3$.

It is convenient to incorporate the v component by making the special assumption that

$$v_y Z = Z_x \zeta^2 \Delta U \frac{d(\psi\zeta)}{d\zeta}. \qquad (15)$$

Thus, the integral of (14) becomes

$$\bar{w} = -Z_x \zeta^3 \psi \Delta U. \qquad (16)$$

Hence, if (16) is totally differentiated, Eq. (14) is obtained under the above assumption.

With the aid of this special model, the vertical velocity satisfies the boundary conditions of $\bar{w}=0$ at the surface where $\zeta=0$, and $\bar{w}=0$ at large heights ($H\approx 3Z$) where $\psi=0$ since $\zeta \gg 1$. This implies a maximum $\bar{w}$ at about $\zeta=\sqrt{3/2}$. In this analysis, with $Z=35$ m at the downwind station, $\bar{w}_{\max}=-0.830$ cm sec^{-1} at a height of 40 m ($\zeta=1.14$).

In order to facilitate a clearer understanding of the foregoing discussion, a three-dimensional schematic diagram of a volume of air between the upwind shoreline and downwind station at Lake Hefner is shown in Fig. 3. The volume is represented by a semicylindrical cutaway which is not drawn to scale for purposes of simplicity and clarity. The upwind and downwind profiles are depicted, with the cross-hatched area on the downwind profile representing the amount of horizontal divergence in the x direction. This divergence gives rise to the increasing vertical velocities indicated in the center of the cutaway. However, instead of increasing further toward the top of the volume, the vertical velocity decreases due to the assumed compensating convergence of air in the y direction.

Eq. (16) indicates that the vertical velocity at a point x along the fetch is directly dependent on Z_x (that is, the derivative of Z with respect to x), a numerical parameter which is obtained through momentum considerations discussed in the following section. Z_x not only permits computation of vertical velocities but, when known at various x, also permits the calculation of Z with fetch by direct numerical integration of Z_x with respect to x. Hence, the gradual change of the wind profile from the upwind shore to the final shape over the lake can be completely described.

7. Momentum continuity and determination of $Z(x)$

Assuming two-dimensional mean flow and no external accelerations or pressure gradient force and $\rho\approx$ constant, then on the vertical scale considered here the continuity equation for x momentum may be written

$$(uw)_z = -(uu)_x. \qquad (17)$$

If the instantaneous velocities are represented as means plus turbulent departures (as indicated by overbars and primes), after cross-multiplication of terms and averaging,

$$(\overline{u'w'})_z = -(\bar{u}\bar{w})_z - (\bar{u}\bar{u})_x - (\overline{u'u'})_x. \qquad (18)$$

The last term can be assumed negligibly small compared to the other terms. Thus, the characteristic balance equation may finally be written

$$(\overline{u'w'})_z = -(\bar{u}\bar{w})_z - 2\bar{u}_x\bar{u}, \qquad (19)$$

where $(\overline{-u'w'})$ is the Reynolds stress which corresponds to U^{*2} over land and u^{*2} over water, respectively. It is postulated that the mean flow is and remains strictly

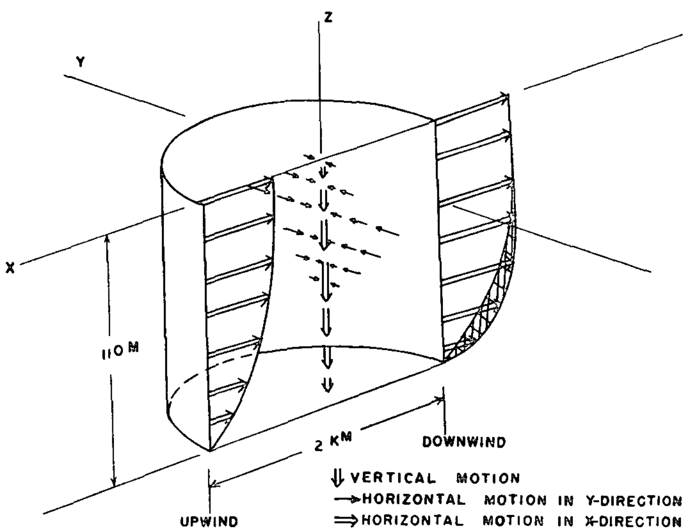

FIG. 3. A generalized schematic diagram (not drawn to scale) of the actual airflow and *assumed compensating airflow* between upwind and downwind station at Lake Hefner.

horizontal and uniform at a certain vertical distance above the interface, i.e., at levels $z \geq H$. This implies that $\bar{w}=0$ at $z \geq H$ since $\psi \approx 0$. Tentatively, it was found sufficient to employ, as a working hypothesis, $H \approx 3Z$. This restriction in height has the obvious advantage that the vertical variation of Reynolds stress can be neglected in the initial wind profile over the land as well as in the final equilibrium profile, but of course, not below $z=H$, during transition.

Thus, with $\bar{u}=\bar{w}=0$ at $z=0$ and $\bar{w}=0$ at $z \geq H$, integrating the momentum balance equation (19) between $z=0$ and $z=H$ gives

$$[\overline{u'w'}]_0^H = -2\int_0^3 Z\bar{u}_x\bar{u}d\zeta. \qquad (20)$$

It is considered that at and beyond the top of the internal boundary layer, the actual wind corresponds to the upwind conditions [i.e., $(\overline{-u'w'})_H = U^{*2}$], while near the surface the actual profile will already be adjusted to the final downwind profile $(\overline{-u'w'})_0 = u^{*2}$. Thus, it follows from (20) that

$$2\int_0^H Z\bar{u}_x\bar{u}d\zeta = U^{*2}-u^{*2} = (U^*-u^*)(U^*+u^*), \quad (21)$$

which implies a value for the integral which is constant for the given overall wind conditions. Upon combining Eqs. (3), (9) and (12) with Eq. (18), we obtain

$$2Z_x\int_0^H \zeta^2\psi\Delta U(U_I+\psi\Delta U)d\zeta = U^{*2}-u^{*2}. \qquad (22)$$

Therefore, for any given Z, the value of Z_x may be obtained by numerical integration of (22). For example, when $Z=35$ m, the integration produces $Z_x=0.0168$. This Z_x value was employed to compute vertical velocities as discussed in the previous section; this Z_x will also be used to estimate the change of the wind profile structure with fetch, i.e., the growth of the internal boundary layer produced by the change in roughness.

8. Growth of the internal boundary layer

Elliott's (1958) mathematical model for the growth of the internal boundary layer under adiabatic conditions would correspond to $Z \sim x^{0.8}$. Specifically, in Elliott's model

$$h = ax^n z_0^{1-n}; \quad n=0.8, \quad a=0.86, \qquad (23)$$

where h denotes the height of the internal boundary layer and z_0 is the roughness parameter of the surface over which the modification takes place. The exponent $n=4/5$ agrees with values obtained experimentally in independent fluid dynamics experiments. Elliott pointed out that the growth of the boundary layer thickness was not dependent on wind speed, which contrasts with results of turbulent boundary layer development studies in wind tunnels.

In Panofsky and Townsend's (1964) model, the thickness d increases to infinite values with increasing distance, as does Elliott's according to Eq. (23). They considered the expression

$$S = \ln(Z_0/z_0)/[-1+\ln(d/z_0)], \qquad (24)$$

which in the present notation appears to be identical with $(U^*-u^*)/U^*$. Using values estimated for Lake Hefner, a calculation of d was made with $U^*=69$ cm sec^{-1}, $u^*=52.6$ cm sec^{-1}, $Z_0=4.92$ cm and $z_0=0.235$ cm which resulted in $S=0.24$; whereupon (24) yields a thickness $d=800$ m. This appears to be nearly the height of the planetary boundary layer, and obviously is unrealistic. It would be more natural to expect, especially with the relatively long fetches involved in this analysis, that at some finite distance downwind the boundary layer thickness will approach some final value.

It was seen in the preceding section that $Z(x)$ could be attained through momentum continuity considerations. If a sufficient number of pairs of Z_x, Z values are obtained, numerical integration with respect to x will produce $Z(x)$. With Z_x calculated for Z values of 35, 30, 25, 20, 15 and 10 m, such a numerical integration showed that Z varied nearly linearly with x. In other words, an approximate constant value $Z_x=0.015\pm0.001$ was obtained.

9. Intermediate wind profiles

Having established a relation between Z and fetch, intermediate profiles between the upwind and downwind equilibrium profiles were constructed and are illustrated in Fig. 4. This scheme suggests that there is a rather quick adjustment of the wind in the lowest 4 m. Gradually, the intermediate profiles approach the characteristics of the initial (or the undisturbed) flow at the top of the internal boundary layer where the shearing stress becomes equal to the value of the initial flow. At a fetch of 2 km the wind profile curvature resembles that of the profiles plotted from observed Lake Hefner data. This supports the conclusion that the observed profile at the barge station of Lake Hefner expresses a state of significantly incomplete adjustment.

Super (1964) studied air mass modification over Lake Mendota and obtained wind profiles from 0.8–2.8 m above the lake at various fetches between 0.25 and about 5.0 km. The dashed lines in Fig. 5 illustrate Super's results under near-adiabatic conditions with the aid of the ratio of the mean wind speed to the mean wind speed at a height of 2.8 m and fetch of 2.25 km.

In contrast to the prairies around Lake Hefner, the surroundings of Lake Mendota are wooded hillsides, built-up areas, intermingled with a few fields. This makes the estimate of an initial wind profile very difficult.

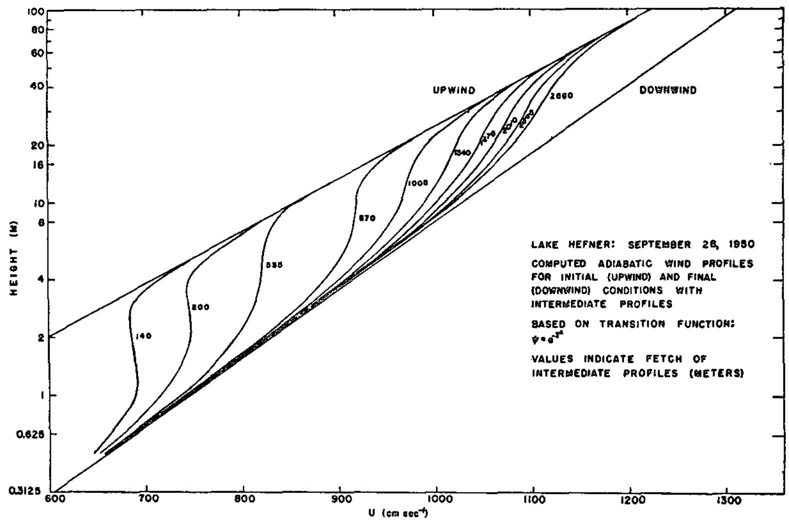

FIG. 4. Calculated upwind and downwind equilibrium wind profiles and intermediate profiles computed from interpolation model $\psi = e^{-\xi^2}$.

For comparison with Super's experimental values, the solid curves in Fig. 5 indicate the variation of wind with fetch as predicted by the interpolation theory for a geostrophic wind of 11.5 m sec^{-1}, $Z_0 = 50$ cm, and $z_0 = 0.235$ cm. Note that this Z_0 is about ten times as large as for the surroundings of Lake Hefner, to account for rougher terrain around Lake Mendota. A noticeable difference between the two sets of curves is the rapid speed increase from 0.25–0.75 km in the observed case. The theory also predicts that with decreasing height the acceleration becomes very small so that there is nearly complete profile adjustment in the lowest meter beyond a fetch of 250 m.

One feature of the theoretical curves, which is not supported by observations, is the relative maximum or reversal of speed increase with height, at fetches < 500 m. This was seen in Fig. 4 where at lower levels the profiles show greater speeds than higher up. Although such S-shaped profiles may occasionally occur, they are not very likely. The theoretical model predicts this to occur at distances < 0.5 km for the given V_g,Z_0 and z_0. However, this is hardly probable, and the "short-fetch" theories of Elliott or Panofsky and Townsend appear to be more realistic for the first 500 m from the shoreline.

Physically, it might be possible to explain the difference in accelerations between the two sets of curves in

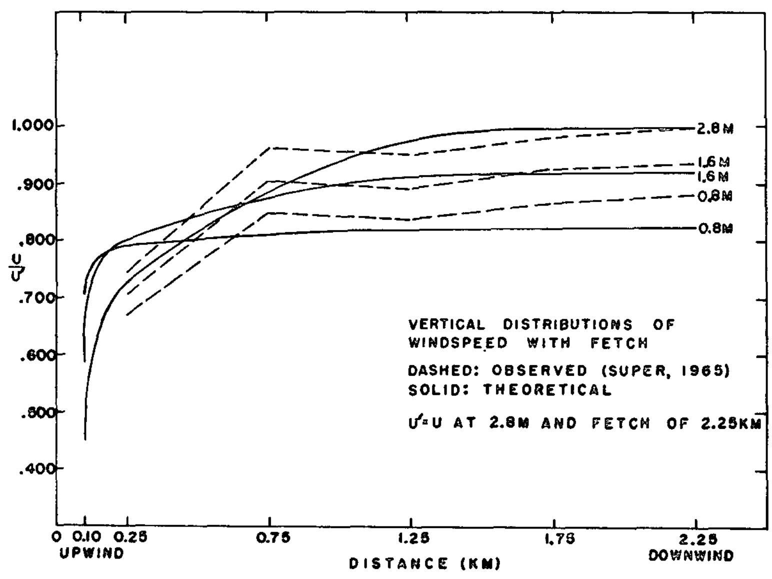

FIG. 5. Observed vertical wind distribution with fetch (Super, 1964) compared with theoretical model curves computed using $Z_0 = 50$ cm, $z_0 = 0.235$ cm, $V_g = 11.5$ m sec^{-1}.

Fig. 5 by irregularities in the surface discontinuity which could greatly alter wind accelerations over the lake. Here it was assumed that z_0 was uniform over the lake surface, while, in reality, z_0 probably increases with fetch.

10. Influence of thermal stratification on wind profile modification

Fig. 2a illustrated the inverse relationship of stability between the upwind and downwind stations. Typically, overland lapse conditions prevail during the day and inversion at night, in contrast to daytime stability and lapse at night over the lake. This illustrates the known moderation of microclimates near lake shores, since the water stores heat at daytime as a result of high thermal admittance and releases it at night.

During the entire diurnal period under study the air was accelerating as it encountered the smoother surface of Lake Hefner. Following continuity requirements, negative vertical velocities (subsidence) resulted from this divergence of air flow. An attempt was made to investigate the diurnal trend in the magnitude of subsidence over the lake.

Smoothed values of the vertical and horizontal wind speeds at 16 m were used to compute the ratio $(\bar{w}/\bar{u})_{16}$ where $\bar{u}$ is an average between the upwind and downwind stations. These ratios are listed in Table 1.

TABLE 1. Diurnal variation of vertical velocity at the 16-m level above Lake Hefner, 28 September 1950, as evidenced by the ratio of $(\bar{w}/\bar{u})_{16}$.

Time (CST)	$\bar{w}_{raw}$ (cm sec^{-1})	$\bar{w}_{smooth}$ (cm sec^{-1})	$\bar{u}_{smooth}$ (cm sec^{-1})	$(\bar{w}/\bar{u})_{16}$ ($\times 10^{-3}$)
0130	−1.486	−1.273	865	−1.47
0430	−1.038	−1.218	905	−1.34
0730	−1.131	−1.134	928	−1.22
1030	−1.233	−1.126	966	−1.16
1330	−1.014	−1.199	991	−1.20
1630	−1.349	−1.177	936	−1.25
1930	−1.169	−1.271	875	−1.45
2230	−1.294	−1.316	860	−1.53

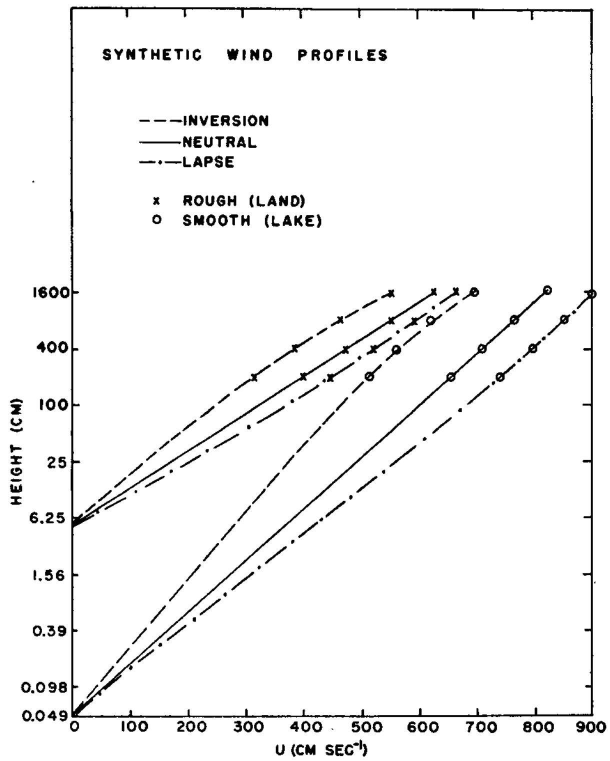

FIG. 6. Synthetic adiabatic and diabatic wind profiles for upwind land surface ($Z_0=5$ cm) and downwind lake surface ($z_0=0.05$ cm).

A diurnal variation, although small, appears to be significant in the values of $(\bar{w}/\bar{u})_{16}$. A minimum occurs at 1030 hours and a maximum twelve hours later at 2230. The higher values of $(\bar{w}/\bar{u})_{16}$ are associated with the period (from 2000–0500) when advection was from a stable to unstable regime. Correspondingly, lower values of $(\bar{w}/\bar{u})_{16}$ coincide with the period of transition from instability over land to a stable stratification over the lake (between 1000–1700).

Fig. 6 illustrates a possible explanation for this $\bar{w}$ cycle as suggested by diabatic surface layer theory (Lettau, 1962). Fully developed synthetic wind profiles for a given geostrophic speed $V_g=15$ m sec^{-1} for the adiabatic cases are shown by solid lines for upwind station ($Z_0=5$ cm) and downwind station ($z_0=0.05$ cm). The two friction velocities were computed with the aid of geostrophic drag coefficients, using the surface Rossby number as a scaling factor for predicting ground drag from the horizontal pressure gradient field. Inversion and lapse profiles are shown for both stations as dashed and dot-dashed curves, respectively. During the night the upwind station is under the inversion regime and wind speeds are less than during the neutral period, while the downwind station is under lapse conditions with speeds greater than those of the downwind neutral profile. Therefore, the divergence is strongest at night, thus producing a greater vertical velocity. Correspondingly, during the day, the divergence has a minimum value. Thus, it can be shown quantitatively how the thermal stratification affects the horizontal wind speed which in turn alters the amount of divergence and consequently the subsidence above the water. However, it is re-emphasized here that the thermal structure is not solely responsible for the transient shape of the wind profiles over the lake.

11. Momentum budget

Momentum budgets between the upwind and downwind stations for the eight time periods were constructed using the x-momentum continuity equation (19) rewritten as

$$\rho_0[2\bar{u}\bar{u}_x+(\bar{u}\bar{w})_z]=\tau_z, \qquad (26)$$

where $\tau=-\overline{\rho u'w'}$. Upon integration with respect to height,

$$\tau_0 = \tau_{(z=h)} - \rho_0 (\bar{u}\bar{w})_{(z=h)} - \rho_0 \int_0^h 2\bar{u}_x \bar{u} dz. \quad (27)$$

The vertical transport of x momentum ($\rho_0 \bar{u}_h \bar{w}_h$) and the horizontal divergence of x momentum are described by mean motion components. The eddy-flux contributions were determined knowing U^*, with τ_0 becoming the residue in Eq. (27). It will be assumed that the shearing stress at 16 m between the two stations is equal to the shearing stress of the upwind, undisturbed flow. The U^* values used in the budgeting problem were obtained by averaging the two calculated U^* values for each time period (see Section 5).

Four budgets are shown in Fig. 7. Significant is the fact that τ_0 over the lake shows a diurnal variation, with a maximum of 6.5 dyn cm^{-2} at 1330, and a nighttime minimum of 3.6 dyn cm^{-2} at 0130. The wind accelerates over the lake both day and night, even though lapse conditions during the day produce relatively strong winds on land. The daytime downwind acceleration may also produce an increasing z_0 with fetch. Thus, the diurnal variation of τ_0 illustrates the strong influence of Richardson number on the process of air mass modification.

Although the τ_0 variation is apparently real, the actual values associated with the eddy flux in Fig. 7 are somewhat questionable chiefly because there is some doubt as to whether the shearing stress at 16 m is, in fact, the

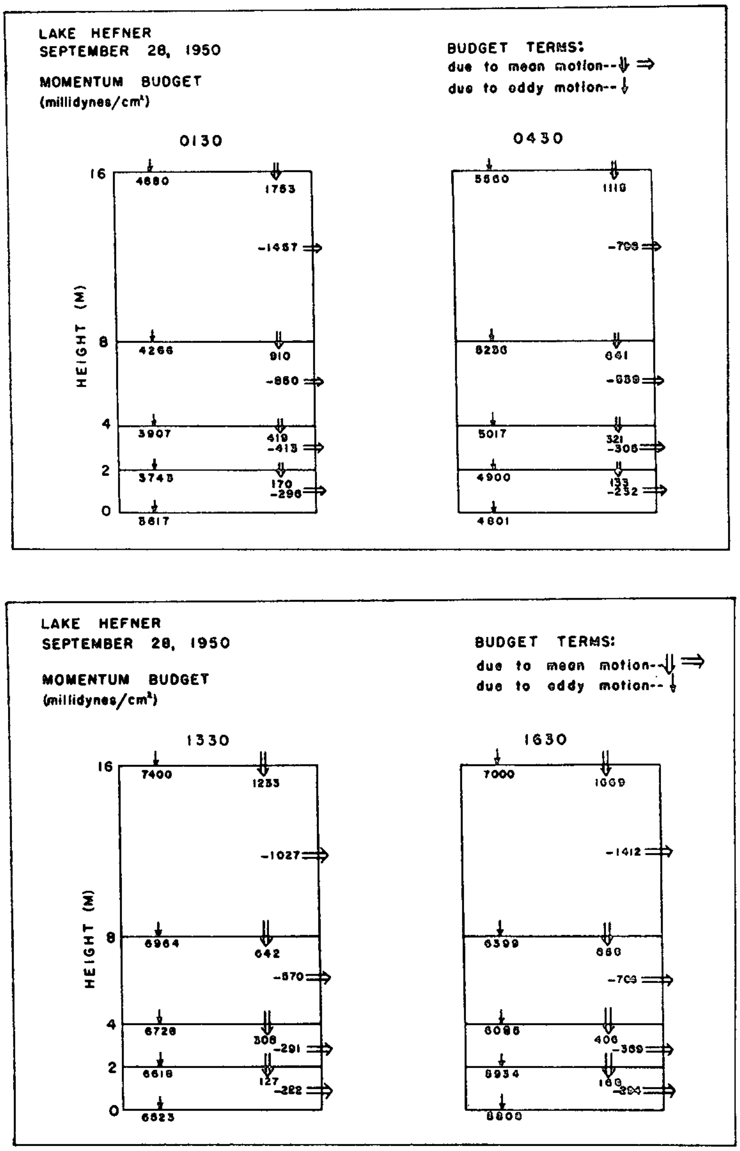

FIG. 7. Momentum budgets between upwind and downwind stations centered at four 3-hr periods during 28 September 1950.

same as the upwind shearing stress. More likely it is less, since the flow at 16 m shows some adjustment to the water surface, at least theoretically (see Fig. 4). In view of this, the surface stress values should be lower.

12. Conclusion

Assuming that both upwind and downwind equilibrium wind profiles can be estimated, it has been shown that an interpolation model offers a possible approach to the problem of wind profile modification. Admittedly, the possible choices of transition functions are numerous, and the Gaussian form selected for this analysis was rather arbitrary. Improved observations of simultaneous upwind and downwind neutral profiles over land and lake are highly desirable. The thermal effect on wind profile modification had been already clearly demonstrated in Super's work on Lake Mendota; however, a complete description of profile modification, including mechanical influences, was not possible because upwind (land) data was lacking due to the fact that the surroundings of Lake Mendota are aerodynamically too complicated. This study took into account the mechanical effects on profile modification and primarily dealt with the adiabatic case. It should be quite apparent that the entire scope of modification, discussed here, cannot be fully described without detailed information from both land and lake surfaces under all thermal conditions. Further studies should be conducted with this in mind.

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