A Method of Estimating Vertical Eddy Transport in the Planetary Boundary Layer Using Characteristics of the Vertical Velocity Spectrum

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ABSTRACT

The vertical eddy diffusivity coefficient K is hypothesized to depend upon the parameters that determine the energy spectrum of the vertical velocity fluctuations. Vertical velocity spectra from the lowest 320 m of the atmosphere are used to verify a relation among the rate of dissipation of eddy energy per unit mass, the standard deviation of the vertical velocity fluctuations, and the wavenumber of the peak of the energy spectrum of the vertical velocity fluctuations. Observations at Round Hill, Mass., and Cedar Hill, Tex., are employed to verify that the vertical eddy viscosity K_M is proportional to the product of any two of the above parameters. However, the Richardson number must be included with these parameters in order to estimate the vertical eddy conductivity K_H. In addition, it is shown that the wavenumber maximum of the vertical velocity spectrum and the nondimensional ratio σ_w/u_* may be approximated at heights less than 320 m by empirical formulae.

1. Introduction

In the past, studies of the vertical turbulent transport of various substances in the atmosphere have led to hypotheses concerning the vertical eddy diffusivity coefficient K. A survey of the classical equations for the eddy diffusivity coefficient (see Pasquill, 1962) reveals that much of the previous theory has been based on empirical findings, especially for the region of the planetary boundary layer above a height of about 50 m. In this paper, dimensional reasoning is used to develop an improved method for estimating the diffusivity K in the planetary boundary layer.

2. Theory of vertical eddy diffusivity coefficients

In order to develop valid functional relationships for the vertical eddy diffusivity K, it is first necessary to identify all the physical quantities that are relevant to the problem. The vertical mixing efficiency K of the atmosphere must be dependent upon the general characteristics of the vertical components of the turbulent eddies, i.e., the mean eddy size and the amounts of turbulent energy carried by the eddies of various sizes. However, because these characteristics are described by the energy spectrum of the vertical fluctuations of the wind speed, it is hypothesized that the vertical diffusivity K is dependent upon the gross characteristics of the vertical velocity spectrum.

The important parameters of the vertical velocity spectrum may be identified by investigating the functional forms that have been proposed in the past to simulate observed spectra. Von Kármán (1948) suggested the relation

$$\frac{S_w(k)}{\sigma_w^2}=\left(\frac{M}{\pi}\right)\frac{1.0+(8/3)(1.339kM)^2}{[1.0+(1.339kM)^2]^{11/6}}, \tag{1}$$

where k is the wavenumber in cycles per unit length, $S_w(k)\Delta k$ the amount of vertical turbulent energy per unit mass in the wavenumber interval between k and $k+\Delta k$, M the integral length scale of the longitudinal fluctuations of the wind speed, and σ_w the standard deviation of the vertical fluctuations of the wind speed. Of the many other studies of vertical velocity spectra that have been conducted since von Kármán first proposed (1), the most thorough has been the recent investigation of Busch and Panofsky (1968), who hypothesize the equation

$$\frac{kS_w(k)}{\sigma_w^2}=\frac{0.63k/k_m}{1.0+1.5(k/k_m)^{5/3}}, \tag{2}$$

where k_m is the wavenumber at which the specific energy $kS_w(k)$ is a maximum. Both (1) and (2) suggest that the vertical velocity spectrum may be completely determined by two quantities, the standard deviation σ_w and a scaling length k_m^{-1} or M.

In the inertial subrange of the vertical velocity spectrum, the quantity $S_w(k)$ is theoretically given by

$$S_w(k)=C\epsilon^{2/3}k^{-5/3}, \tag{3}$$

where ϵ is the rate of dissipation of turbulent energy per unit mass. Panofsky and Pasquill (1963) have determined that the constant $C=0.19$. Eqs. (2) and (3)

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may thus be used to derive the identity

$$\epsilon^{\frac{1}{3}}/(\sigma_w k_m^{\frac{1}{3}})=1.5. \quad (4)$$

An identity of this form has been proposed by several authors, including Hinze (1959).

If it is assumed that the mixing efficiency of the air is completely determined by the properties of the vertical velocity spectrum, and that the parameters σ_w and k_m are sufficient to describe the vertical velocity spectrum, then it follows from dimensional reasoning that the vertical eddy diffusivity coefficient K may be written in any of the forms

$$K=C_1\sigma_w k_m^{-1}, \quad (5)$$

$$K=C_2\epsilon^{\frac{1}{3}}k_m^{-\frac{4}{3}}, \quad (6)$$

$$K=C_3\sigma_w^4\epsilon^{-1}, \quad (7)$$

where the C's are constants to be determined from observations. Eq. (5) is similar to a functional form suggested by Taylor (1915), although Taylor did not specify any precise form for the length appearing in this equation. For nearly neutral conditions close to the ground, observations have substantiated the relations [see Lumley and Panofsky (1964), and Busch and Panofsky (1968)]

$$K=0.4u_*z, \quad (8a)$$

$$\sigma_w=1.3u_*, \quad (8b)$$

$$k_m=0.3/z, \quad (8c)$$

$$\epsilon=u_*^3/0.4z, \quad (8d)$$

where u_* is the friction velocity. Substitution of these values into (5)–(7) gives the following estimates for the constants:

$$C_1=0.09, \quad C_2=0.06, \quad C_3=0.35.$$

In subsequent sections of this paper, data obtained during extremely diabatic conditions and at heights up to 320 m will be shown to agree with these estimates.

3. Verification of Eq. (4)

In order to justify (5)–(7), it is necessary to first assert the universality of (4). Energy spectra of the vertical component of the turbulent velocity are available from towers at Round Hill, Mass., Cedar Hill, Tex., Hanford, Wash., and Vancouver, British Columbia (over the sea). In addition, the Boeing Company has reported spectra measured by airplanes within the planetary boundary layer over greatly varying terrain. Table 1 summarizes the most important information concerning these locations. The data are too extensive to include here, but have been given by Hanna (1967).

In all cases the quantity $\log[kS_w(k)]$ was plotted as a function of the quantity $\log k$, and the wavenumber k_m of the peak of the spectrum was estimated subjectively. Because of the flatness of the spectral peaks for some of the data, the individual estimates of the peaks of the spectra are accurate only within about ±20%. The standard deviation σ_w was provided with all the original data with the exception of the set of data from Vancouver, where σ_w had to be estimated by calculating the area under the spectral curve. The velocities σ_w and u_* from Round Hill were corrected by Prasad (1967) for the effects of finite averaging and sampling times. The dissipation rate ϵ was calculated for each run by means of (3), for a wavenumber k subject to the condition that the nondimensional number $kz>1$. In several cases, a value of $kz \geqslant 10$ was employed. Although MacCready (1962) states that (3) is valid to values of the nondimensional wavenumber $kz<1$, it is possible that the dissipation rate ϵ may be underestimated slightly if (3) is employed in the lower range of the wavenumber interval suggested by MacCready. However, the spectra utilized in this study suggest that the "minus 5/3" law embodied in (3) extends at least to a value of the number kz of unity.

In Fig. 1 the quantity $\epsilon^{\frac{1}{3}}$ is plotted as a function of the quantity $\sigma_w k_m^{\frac{1}{3}}$ for all runs, yielding a linear regression line which verifies (4). It can be concluded that this relation is universal within the lowest 320 m of the atmosphere, since the data presented in Fig. 1 represent stabilities ranging from free convection conditions to inversion conditions in which turbulence had nearly vanished, heights ranging between 1.6 and 320 m, and terrain ranging from flat plains to high mountains. Thus, provided no other parameters influence the ver-

TABLE 1. Summary of site and vertical spectra characteristics.

Location	Round Hill, Mass.	Cedar Hill, Tex.	Hanford, Wash.	Vancouver, British Columbia	Vancouver, British Columbia	United States
Observer	Record and Cramer (1966)	Kaimal (1966)	Elderkin (1966)	Smith (1967)	Weiler and Burling (1967)	Boeing Company (1967)
Terrain type	Scattered brush, grassland, and woods	Gently rolling; few trees	Desert sagebrush	Over sea	Over sea	Ranging from plains to high mountains
Number of w spectra	53	81	18	11	8	49
Heights of w spectra (m)	15, 16, 40, 46, 91	46, 137 229, 320	3, 6.1, 12.2, 87	1.6–4.2	1.7–2.7	76, 229

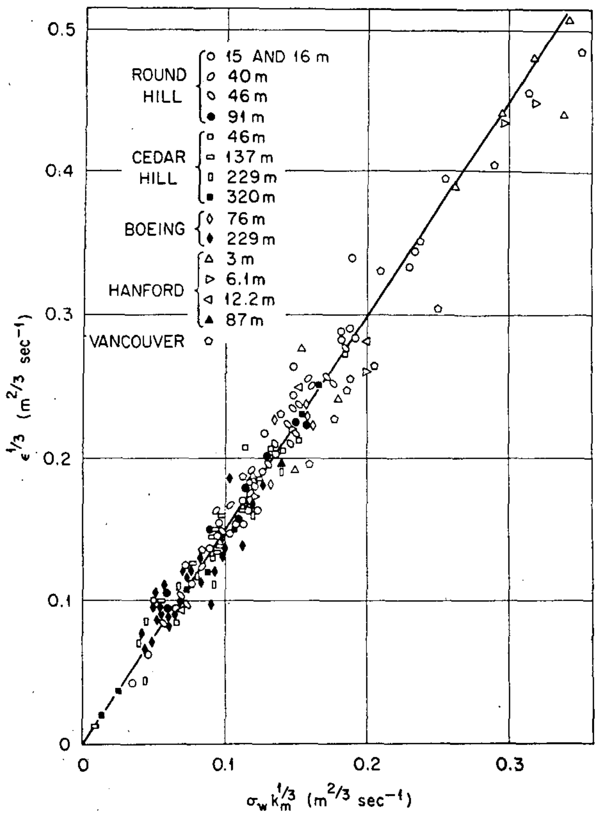

FIG. 1. The quantity $\epsilon^{\frac{1}{3}}$ as a function of the quantity $\sigma_w k_m^{\frac{1}{3}}$ for all data.

tical eddy diffusivity coefficient K, this coefficient may be represented in the form of (5)–(7).

4. Calculations of the vertical eddy diffusivity coefficient K

The eddy viscosity K_M and the eddy conductivity K_H at Round Hill were calculated from the definitions

$$K_M = u_*^2/(dU/dz),$$

$$K_H = -\overline{w'T'}/(d\theta/dz),$$

where U is the mean wind speed, θ the mean potential temperature, and w' and T' are the deviations of the vertical velocity and the temperature from their averages. The overbar denotes a time average over a period of $\sim$1 hr. The upper reference level on each tower at Round Hill was excluded from study because of uncertainties in the vertical gradients of the wind speed and temperature at these levels.

In Fig. 2 the eddy viscosity K_M at Round Hill is plotted as a function of the quantity $\sigma_w k_m^{-1}$, yielding the identity

$$K_M = 0.085\sigma_w k_m^{-1}. \quad (9)$$

The scatter on the figure is greatest for large values of the eddy viscosity K_M, where the wind speed shears are the smallest in magnitude and hence the least accurate. The horizontal nonhomogeneity of the flow at Round Hill, as reported by Panofsky *et al.* (1967), may also contribute to the scatter. The same is true for Fig. 3, in which the eddy viscosity K_M is plotted as a function of the quantity $\epsilon^{\frac{1}{3}} k_m^{-\frac{4}{3}}$, with the result that

$$K_M = 0.055\epsilon^{\frac{1}{3}} k_m^{-\frac{4}{3}}. \quad (10)$$

Eqs. (4) and (9) can be used to calculate the remaining constant of proportionality, i.e.,

$$K_M = 0.30\sigma_w^4 \epsilon^{-1}. \quad (11)$$

Note that these constants agree well with the constants estimated by means of (8a)–(8d).

It has been determined by Panofsky *et al.* (1967) that the turbulent Prandtl number K_M/K_H is a function of stability at Round Hill. Consequently, the eddy conductivity K_H cannot be expected to be directly proportional to the quantity $\sigma_w k_m^{-1}$. Pandolfo (1966) has suggested the relation

$$K_M = K_H \phi, \quad (12)$$

where the nondimensional wind shear ϕ is related to the parameters of the wind profile by

$$\phi = \frac{dU}{dz} \frac{0.4z}{u_*}.$$

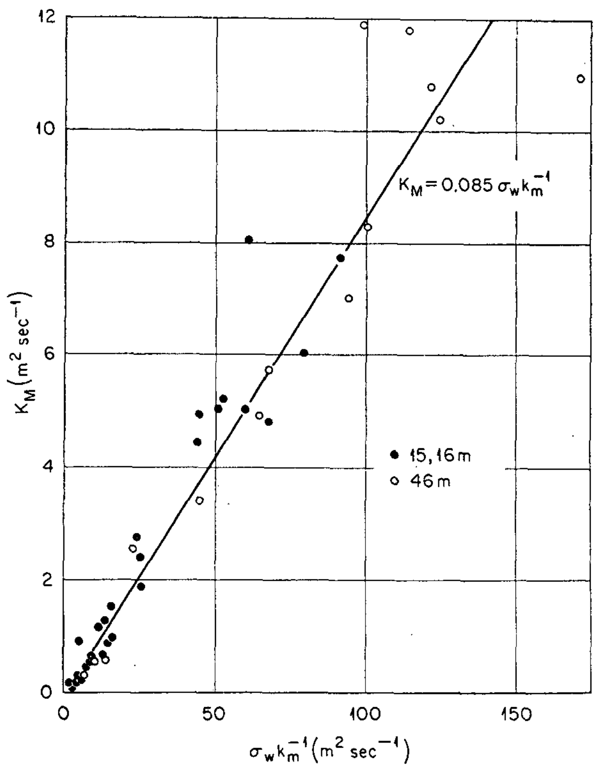

FIG. 2. The eddy viscosity $K_M = u_*^2/(\partial u/\partial z)$ as a function of the quantity $\sigma_w k_m^{-1}$ for the Round Hill data.

According to theory and empirical evidence, ϕ is a function of the gradient Richardson number Ri $= (g/T)(d\theta/dz)/(dU/dz)^2$, where g is the acceleration of gravity. In this study, the functional relationships suggested by Panofsky *et al.* (1960) are assumed to be valid, i.e.,

$$\phi = 1.0 + 5\mathrm{Ri}, \quad \mathrm{Ri} > 0, \tag{13}$$

$$\phi = (1.0 - 18\mathrm{Ri})^{-\frac{1}{4}}, \quad \mathrm{Ri} < 0. \tag{14}$$

Another frequently used stability parameter is the ratio $(z/L) = (-gH/c_p\rho T)/(u_*^3/0.4z)$. In this equation H is the upward flux of enthalpy per unit area, c_p the specific heat of air at constant pressure and ρ the density of air. The stability parameters Ri and z/L are related through

$$z/L = (K_H/K_M)\mathrm{Ri}\phi. \tag{15}$$

Consequently, according to (12) and (15), the stability parameters Ri and z/L are equal. The Richardson number is therefore a linear function of height, for the length L is nearly constant in the lowest 200–300 m of the boundary layer. At Cedar Hill the parameter Ri was calculated using data from heights of 9 and 91 m, from which the values of Ri at other heights on the tower were obtained by linear extrapolation. However, at Round Hill the value of Ri at each height could be calculated from local parameters.

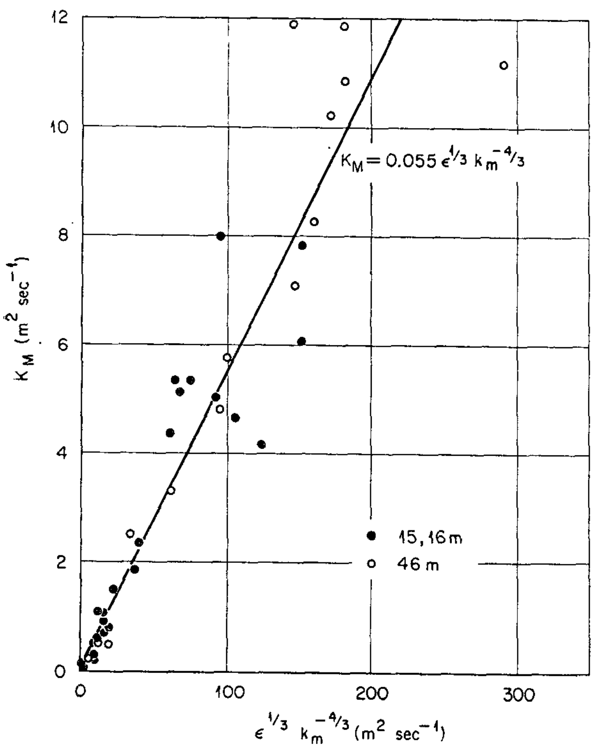

FIG. 3. The eddy viscosity $K_M = u_*^2/(\partial u/\partial z)$ as a function of the quantity $\epsilon^{\frac{1}{3}}k_m^{-\frac{4}{3}}$ for the Round Hill data.

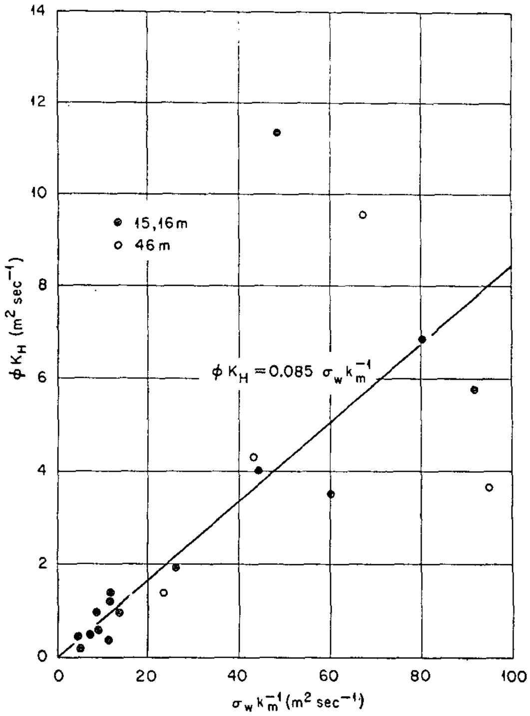

FIG. 4. The quantity ϕK_H as a function of the quantity $\sigma_w k_m^{-1}$ at Round Hill.

Eqs. (9) and (12) imply that it is possible to represent the eddy conductivity K_H by means of

$$K_H = 0.085\sigma_w k_m^{-1}[\phi(\mathrm{Ri})]^{-1}. \tag{16}$$

The quantity ϕK_H is plotted as a function of the quantity $\sigma_w k_m^{-1}$ in Fig. 4 for several runs at Round Hill. The observations, which exhibit considerable scatter, appear to be best represented by (16). Consequently, the eddy diffusivity coefficient proposed in (5) corresponds to the observed eddy viscosity K_M at Round Hill but not to the observed eddy conductivity K_H. While the gross characteristics of the vertical velocity spectrum are sufficient to describe the vertical coefficient of momentum transport, an additional parameter, Ri, is necessary in order to describe the vertical coefficient of enthalpy transport.

Eqs. (9), (10) and (11) were also tested using data from the 433-m tower at Cedar Hill, Tex. Because the quantity $\overline{u'w'}$ was not measured at Cedar Hill, the friction velocity u_* was estimated by integrating a simplified form of the equation of motion, i.e.,

$$\frac{d}{dz}u_*^2 = -fU_g \sin\Omega.$$

Integration of this formula yields

$$u_* = u_{0*}\left(1.0 - \frac{fzU_g \sin\Omega}{u_{0*}^2}\right)^{\frac{1}{2}}, \tag{17}$$

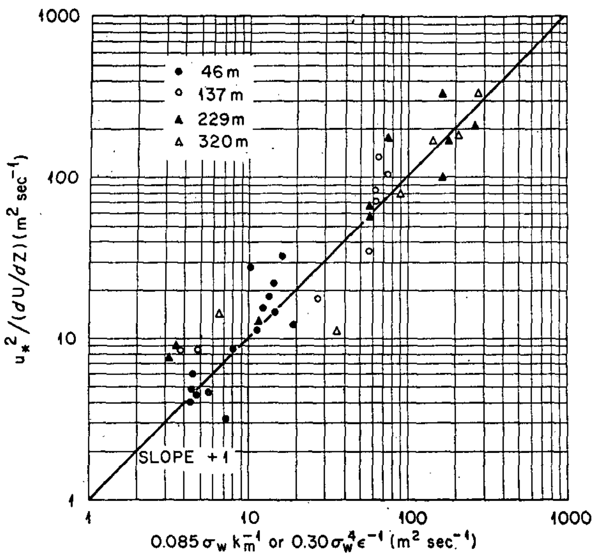

FIG. 5. The eddy viscosity $K_M = u_*^2/(\partial u/\partial z)$ compared with the estimates of (9) and (11) at Cedar Hill.

where u_{0*} is the value of the friction velocity at the surface, f the Coriolis parameter, U_g the geostrophic wind speed, and Ω the angle between the surface wind vector and the geostrophic wind vector (positive if the surface wind blows to the left of the geostrophic wind). The friction velocity u_{0*} at the surface was calculated using the wind speed and the Richardson number at a height of 9 m, i.e.,

$$u_{0*} = 0.4U_9/[3.4 - \psi(\text{Ri})]. \quad (18)$$

A roughness length of 0.3 m has been assumed and the universal function $\psi(\text{Ri})$ may be found from a graph in Lumley and Panofsky (1964). The geostrophic wind speed U_g in (17) was assumed to be approximately equal to the wind speed on the tower at a height of 433 m, and the cross-isobar angle Ω was allowed to vary from 15° for very unstable conditions to 35° for stable conditions (see Blackadar, 1965). Runs were considered only if they seemed to be characterized by steady-state conditions with no vertical irregularities in the wind or temperature profiles.

Thus, whenever the wind speed shear and the friction velocity could be estimated, the eddy viscosity K_M could also be estimated from the definition $K_M = u_*^2/(dU/dz)$. Alternatively, a vertical eddy diffusivity coefficient could be calculated from any of the equations (9)–(11). Fig. 5 compares the eddy viscosity K_M at Cedar Hill with the eddy diffusivity coefficient calculated from (9) or (11). Although there is considerable scatter on the diagram, a linear regression line with a slope of unity appears to best represent the data. Therefore, both the Cedar Hill and Round Hill data support the conclusion that the theoretical eddy diffusivity coefficient developed in this paper corresponds to the eddy viscosity coefficient K_M.

5. Empirical estimates of the standard deviation σ_w and the wavenumber k_m

The variation of the ratio σ_w/u_* with stability could be studied extensively because of the great stability range of the Cedar Hill data. In Fig. 6 the ratio σ_w/u_* has been plotted as a function of the stability parameter Ri for unstable runs, where the value of the friction velocity u_* was obtained by the use of (17) and (18). Runs were excluded if irregularities appeared in the σ_w, wind speed or temperature profiles near the level under consideration. The figure shows that σ_w/u_* increases slowly from a value of about 1.3 during nearly neutral conditions to a value of about 5 at a Richardson number of approximately −700.

For comparison, three theoretical formulas have also been included in Fig. 6. Monin (1959) suggests

$$\sigma_w/u_* = A[1.0 - (z/L)/\phi]^{\frac{1}{3}}, \quad (19)$$

while Panofsky and McCormick (1960) suggest

$$\sigma_w/u_* = A(\phi - 2.4z/L)^{\frac{1}{3}}, \quad (20)$$

where A is a universal constant whose value is most recently estimated to be 1.3 (Busch and Panofsky, 1968). For calculation purposes, the equivalence of z/L and Ri has been assumed. These formulas both correspond to the data for Richardson numbers $\gtrsim -5$. When the Richardson number < -5, the data are best approximated by

$$\sigma_w/u_* = 1.3(1 - 7\text{Ri})^{\frac{1}{6}}. \quad (21)$$

This equation has been obtained by requiring the ratio σ_w/u_* to be equal to 1.3 during neutral conditions and by fitting a power law to the data at large negative values of the parameter Ri. When (21) is combined with (17) and (18), it is possible to estimate the standard deviation σ_w at any height up to 320 m during neutral or unstable conditions without having to measure the vertical velocity fluctuations.

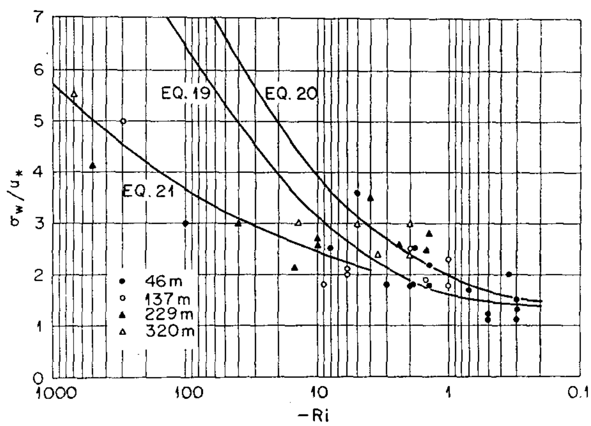

FIG. 6. The ratio σ_w/u_* as a function of the stability parameter Ri for unstable runs at Cedar Hill.

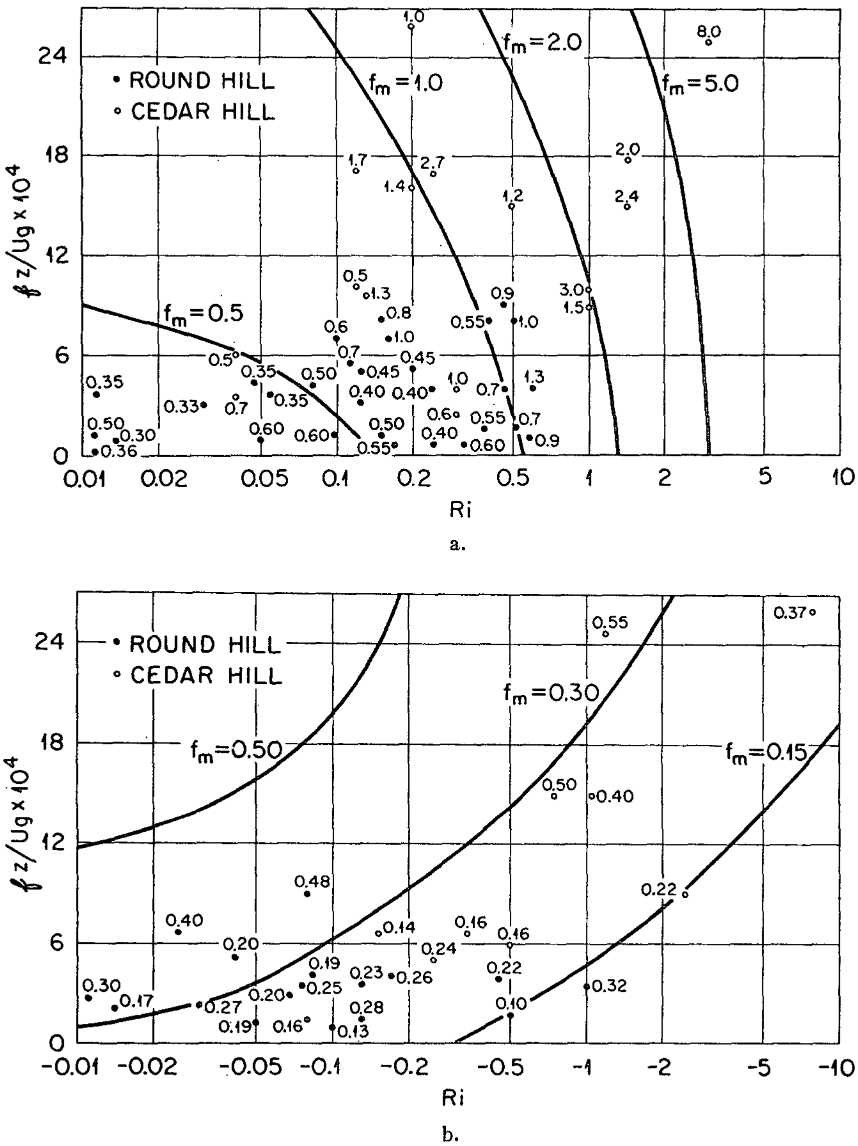

FIG. 7. The nondimensional frequency $f_m = k_m z$ of the peak of the vertical velocity spectrum as a function of Ri and fz/u_g for the Round Hill and Cedar Hill data, for stable conditions, a., and unstable, b. Isopleths of (22) are also shown.

The Round Hill and Cedar Hill data were also used to develop an empirical relation between the wavenumber maximum k_m of the vertical velocity spectrum and the height, the stability, and the nondimensional ratio fz/U_g. If the wavenumber maximum k_m is multiplied by the height z, the resulting nondimensional number is equal to a constant for vertical velocity spectra near the ground during nearly neutral conditions. Therefore, observed values of $k_m z$ are plotted in Fig. 7 as a function of the stability parameter Ri and the number fz/U_g, showing that the observations can be approximated by the isopleths of the empirical function

$$k_m z = 0.3[\phi(\mathrm{Ri})](1.0+500 fz/U_g), \qquad (22)$$

where $\phi(\mathrm{Ri})$ is given by (13) and (14). The standard deviation of the percentage difference of the observed points from the predictions of the nomogram is 39%. Many factors contribute to this deviation, including the uncertainty of the values of U_g and Ri and the subjectivity of the observed values of k_m.

Eqs. (9), (17), (18), (21) and (22) yield the empirical relation

$$K_M = \frac{0.4 u_{0*} z}{\phi}\left(1-\frac{fzU_g \sin\Omega}{u_{0*}^2}\right)^{\frac{1}{2}} \times\left(1+500\frac{fz}{U_g}\right)(1-7\,\mathrm{Ri})^{1/6}, \qquad (23)$$

which can be used during nearly neutral or unstable conditions to estimate the diffusivity K_M in the lowest 300 m of the boundary layer using data that can be obtained near the ground.

6. Conclusions

Energy spectra of the vertical fluctuations of the wind speed within the lowest 320 m of the atmosphere have been shown to conform to the universal law

$$\epsilon^{\frac{1}{3}}/(\sigma_w k_m^{\frac{1}{3}}) = 1.5.$$

The hypothesis was made that the standard deviation σ_w and the wavenumber peak k_m completely determine the vertical eddy diffusivity coefficient K. This hypothesis was tested upon data from Round Hill, Mass., and Cedar Hill, Tex., leading to the expressions

$$K_M = 0.085\sigma_w k_m^{-1},$$
$$K_M = 0.055\epsilon^{\frac{1}{3}} k_m^{-\frac{4}{3}},$$
$$K_M = 0.30\sigma_w^4 \epsilon^{-1},$$

where K_M is the eddy viscosity coefficient. In addition, it was shown that the data from Round Hill may be represented by

$$K_H = 0.085\sigma_w k_m^{-1}\phi^{-1},$$

where K_H is the eddy conductivity coefficient. These formulas allow the coefficients K_M and K_H to be estimated using relatively slow-response turbulence instrumentation. If an estimate of the vertical wind shear is also available, then the stress $[\tau = \rho K_M(dU/dz)]$ and the mechanical eddy energy production $[P = \rho K_M(dU/dz)^2]$ may also be estimated from these equations in the lowest 320 m of the boundary layer.

The vertical eddy diffusivity coefficient K_Q for moisture has been shown to be equal to the eddy conductivity K_H by Dyer (1967). If the assumption is made that the coefficient K_Q for any conservative substance other than momentum is equal to the eddy conductivity K_H, then the upward flux F_Q per unit area of this substance can be estimated from

$$F_Q = -\rho K_Q \frac{dQ}{dz} = -0.085\rho\sigma_w k_m^{-1}\phi^{-1}\frac{dQ}{dz}, \qquad (24)$$

where Q is the amount of the substance per unit mass of air. This formula, which clearly must be tested with more observations, has wide applications in the fields of air pollution and micrometeorology.

The relations proposed in this paper have been tested upon data from the lowest 320 m of the atmosphere. In the future it would be interesting to test the theory in other regions of the atmosphere, where there also may be a direct relation between vertical eddy transport and the energy spectrum of the fluctuations of the vertical wind speed.

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