Grid Interval Effects on a Numerical Model of Upslope Winds and Mountain-Induced Cumulus

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1. Introduction

Previous studies of upslope winds and cloud initiation (Orville, 1964, 1965, 1968) have used grid intervals of 100 m. An experiment has been run in which 50-m grid intervals are used. The results are pertinent because 1) the choice of 100 m to resolve upslope winds that have been depicted in theory and observations as reaching maximum speeds at 30 m and decreasing thereafter (Defant, 1949) may be insufficient, 2) the growth of clouds normally proceeds by a succession of hydrothermals of varying sizes—from a few hundred meters to a kilometer or more (Todd, 1964)—and thus raises questions about the suitability of 100-m grid point spacings, and 3) the upstream differencing technique used in the model has a low order truncation error and introduces diffusion into the problem that is dependent upon the grid interval and velocity (Molenkamp, 1968).

2. Method

The basic numerical model has been described in Orville (1965). Case 5 of that paper (herein identified as A100) is rerun here but with the grid interval reduced to 50 m from the 100 m of the original. The time step was also reduced, from 30 to 15 sec at first, but when linear computational instability developed early, the model was started again with a time step of 7.5 sec. The reduction from 30 to 15 sec satisfied the Courant-Friedrichs-Lewy linear stability condition (1928) but not the combined criterion of advection plus diffusion (Deardorff, 1967). The initial time step of 7.5 sec was again cut in half when the developing motion required it and before any further numerical instabilities occurred. The value of the eddy coefficient $K$ in the Fickian diffusion term was held constant at 40 m$^2$ sec$^{-1}$ in the 50-m model (referred to as A50), the same as in A100. Case 2 of Orville (1965), B100, used eddy coefficients of 4 m$^2$ sec$^{-1}$ and some of those results are also compared with the 50-m model.

3. Results

The principle results are depicted in Figs. 1 and 2. Fig. 1 shows the trajectory of the stream function centers in the two 100-m cases and the 50-m case and the cloud outline for two of the cases, at 154 min for Case A50 and 165 min for Case A100. The time difference is caused by the earlier cloud initiation in Case A50 (by
10 min). Reasons for this are discussed below. It is seen in the figure that the cloud outlines are similar. Also, the trajectory of the stream function centers are alike, although the center in Case A50 has risen about 50 m nearer to the slope and central axis than in Case A100. The center in Case B100, with \( K = 4 \) \( \text{m}^2 \text{sec}^{-1} \), has risen even closer to the slope and symmetry axis.

The fields of potential temperature, stream function and water vapor are also nearly identical in their evolution.

The earlier cloud initiations might conceivably mean that the cloud development is quite different in the two cases. However, Fig. 2, a graph of the maximum liquid water content vs time in the two cases, shows this not to be the case. The times are adjusted to make the cloud initiations coincident. The exponential growth of the clouds is evident.

4. Discussion

The earlier cloud initiation in A50 compared to A100 can be attributed to the numerical technique. Upstream differencing is used in the advection terms, centered differences in the other space derivatives, and non-centered time differences throughout the numerical integration. The time difference of cloud initiation may be explained by the fact that upstream differencing introduces mixing characterized by eddy coefficients pro-

**Fig. 1.** The stream function trajectories and the cloud outlines for the 100- and 50-m grid interval models. The trajectories for Case A100 are indicated by the broken line and crosses, Case A50 by the broken line and open circles, and Case B100 by the broken line and solid circles. The numbers indicate the time in minutes since the beginning of integration and the cloud outlines are indicated for Case A100 at 165 min and A50 at 154 min.
Fig. 2. A graph of the maximum liquid water content vs time in Cases A100 and A50. The double time scale on the abscissa refers to Case A50 on the top and A100 on the bottom. The liquid water contents are in units of gm kg$^{-1}$.

Fig. 3. An example of small turret growth in a cumulus cloud developing in an ambient wind numerical model. The numbers to the right of the cloud outline indicate time in minutes since the beginning of the numerical integration. Heating at the mountain slopes has interacted with an initial ambient flow from the left to create an updraft and the resulting cloud.

portional to the grid spacing and velocity (Molenkamp, 1968). Table 1 summarizes the results with respect to the times of cloud initiation and their dependence on the diffusion in the various cases.

The implicit $K$-value is attributed to the upstream differencing technique. This "implicit" diffusion, or numerical damping, arises because upstream differencing applied to a pure advection equation is consistent not with a pure advection equation but with an equation that treats both advection and diffusion. The "eddy coefficient" of the implicit diffusion term amounts to approximately 35 m$^3$ sec$^{-1}$ in the 100-m model and 20 m$^2$ sec$^{-1}$ in the 50-m model for characteristic wind speeds of 1 m sec$^{-1}$. Cases 2 (B100) and 5 (A100) reported in Orville (1965) indicate that a decrease in the explicit diffusion term, by decreasing the $K$-value from 40 to 4 m$^2$ sec$^{-1}$ in the Fickian diffusion term, has the effect of decreasing the time of cloud initiation (from
141 to 112 min). The reduction of the total diffusion (explicit plus implicit), by decreasing the grid interval, has resulted in the cloud initiation at 131 min in A50, in agreement with theory and the numerical results.

These results plus those of the stream function trajectories and development support the result of smaller diffusive effects in Case A50 compared with A100 (but greater diffusive effects than in B100). I do not believe that the truncation error is serious or that higher order finite difference schemes are necessary for these local scale models, at least at the present level of sophistication and with Fickian diffusion terms used to model turbulence. More elaborate techniques are needed if turbulent mixing is modeled differently (e.g., Lilly, 1967) or if certain fields, such as precipitation, are to be excluded from turbulent mixing.

The implicit diffusion does not act normal to the flow (Molenkamp, 1968) and so explicit diffusion is necessary to transfer the heat and the vapor from the lower surfaces. In fact, one other case (with a different initial wind condition, however) integrated with $K=0$ exhibited much different characteristics than those with finite $K$'s because of the lack of diffusion away from the heated lower boundary.

Recent work by Crowley (1968) shows that upstream differencing applied to a pure advection equation is not accurate (because of the implicit diffusion). The question of how appropriate the upstream differencing technique is to problems with both advection and diffusion is still not answered. A test of the higher order schemes described by Crowley on a diffusion-advection problem is needed.

5. Conclusions

The simulation of upslope winds, and cloud initiation and development, with grid intervals of 100 m is not materially changed when a grid interval of 50 m is used. The principal differences can be attributed to the smaller diffusive effects in the 50-m grid interval model, caused by the finite differencing technique. The optimum grid size is not known. Ackerman's (1966) results on the size of convective elements in cumulus convection indicate that not much more than twice the 100-m grid interval should be used if the grid is to resolve the thermally produced motions in a cloud. Fig. 3, depicting the cloud growth in one of our ambient wind models with a 100-m grid interval, shows that cloud turrets of 200–300 m width can develop in the model.

The details of the slope wind next to the earth's surface are not resolvable with 100- or 50-m grid intervals. However, the motions farther away from the slope, particularly the reversal of the slope winds and the wind speeds, are adequately represented by the models utilizing 50- or 100-m grid intervals.

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REFERENCES


