A Model for Atmospheric Pressure Fluctuations in the Mesoscale Range[1]

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ABSTRACT

It is shown, given perturbations of the jet stream wind system similar to those reported from balloon and aircraft studies, that it is possible to calculate ground level pressure fluctuations. Using a density stratified model of the troposphere and a constant gravity field, and assuming the jet stream to act as a traveling disturbance, a simple linear model predicts the correct order of magnitude and power spectra for microbarographic fluctuations in the 5–60 min period range.

1. Introduction

Numerous atmospheric pressure fluctuation power spectra have been published and discussed in the literature during the last 10 years (Gossard, 1960; Lumley and Panofsky, 1964; Golitsyn, 1964; Pinus *et al.*, 1967). The mechanisms supplying the energy depend upon the frequency band. Thus, at periods of the order of days we have large scale meteorological phenomena, and there are prominent peaks at the diurnal and semidiurnal periods. At the short end, i.e., minutes and seconds, one is dealing both with background infrasonic activity and with boundary layer wind and turbulence effects. In between we have a range of periods from a few minutes to a few hours; this is the realm of *mesoscale phenomena.*

The energy input into this part of the spectrum is due to many causes and is quite variable. There is no single mechanism capable of accounting for *all* observed background pressure fluctuations. Nevertheless, it has long been a part of the folklore among students of infrasound that the tropospheric jet stream accounts for a good fraction of the observed pressure fluctuations at periods longer than a few minutes. Recent results confirm the intimate connection between jet stream activity and mesoscale pressure fluctuations. It is quite common to see disturbances traveling with phase speeds of 10–50 m sec^{-1}, i.e., essentially with velocities of the order of jet stream speeds, and whose direction of travel closely follows that of the jet stream core aloft. This remarkable condition is often seen to persist for days or weeks (Herron and Tolstoy, 1969). Thus, there appears to be little doubt that, for long periods of time, most of the energy input into the mesoscale comes from the jet stream.

The purpose of this paper is to show that the energy input from the jet stream into ground level pressure fluctuations may occur through a simple mechanism of internal gravity wave excitation by perturbations of the jet. Similar conclusions have been reached by others (J. Young, private communication; Claerbout, 1967). For example, Claerbout has suggested a relation between the velocity of the ground level pressure disturbances and the wind speed at the height of minimum stability (i.e., lowest Richardson number) in the jet stream cross sections; the best correlations may occur, according to him, with the velocity at a considerable distance upstream from the instrument. In another paper we have indicated a possible correlation of spectrum levels with distance from the region of maximum eddy kinetic energy (Herron *et al.*, 1969). What we will show here, using published jet stream studies (Reiter, 1963; Kao and Woods, 1964), and assuming some reasonable value for the velocity of the radiating disturbance (in the 10–50 m sec^{-1} range), is that the hypothesis of energy radiated into the internal gravity wave spectrum gives acceptable orders of magnitude for some of the observed properties of the mesoscale fluctuation fields.

2. Radiation of jet stream fluctuations into internal gravity waves

Consider the wind velocity power spectra obtained by aircraft measurements along the jet stream axis (Kao and Woods, 1964). We assume that these spectra are stationary, and that they may be interpreted as a frozen-in property of the wind system carried along by the jet core. In order to calculate the spectral distribution of the ground level pressure fluctuations due to the traveling jet stream irregularities, we calculate first of all the effect of a simple traveling harmonic wave moving at jet stream height with a phase velocity equal to some average jet core wind speed. The atmosphere

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below this height is assumed to be stationary. Linear internal gravity wave theory then allows us to calculate the contribution of this spectral component to the ground level pressure power spectrum. Although this type of mechanism is admittedly unrealistic, it does give a surprisingly good estimate of the pressure fluctuation amplitudes. The ground level period T is, of course, simply related to the wavelength λ and the assumed jet core speed c by

$$T=\lambda/c. \tag{1}$$

Note that this method of connecting jet wind fluctuations to ground level pressure spectra does not assume a frozen-in perturbation field, only a frozen-in spectrum.

The pertinent elements of propagation theory are clear. We assume 1) the longitudinal horizontal and vertical wind fluctuation components u, w are related by the condition of incompressibility, and 2) a harmonic wave train, i.e., proportionality to $e^{i(\alpha x-\omega t)}$ where $\omega/\alpha=c$ is equal to the assumed jet core velocity. The theory of internal gravity waves in a stratified fluid (Tolstoy, 1963) tells us that, if

$$w\propto h\rho^{-\frac{1}{2}}\propto e^{i(\alpha x-\omega t)}, \tag{2}$$

where ρ is the density and h is an auxiliary function, then, using the subscript z to denote differentiation, we have

$$i\alpha u=-w_z, \tag{3}$$

and

$$h_{zz}+\gamma^2h=0, \tag{4}$$

where

$$\gamma^2=\alpha^2\left(\frac{N^2}{\omega^2}-1\right)-\frac{1}{4}\left(\frac{d}{dz}\ln\rho\right)^2-\frac{1}{2}\frac{d^2}{dz^2}\ln\rho,$$

$$\simeq\alpha^2\left(\frac{N^2}{\omega^2}-1\right)=\frac{N^2}{c^2}-\alpha^2. \tag{5}$$

Here N is the Väisälä frequency and c the phase velocity (equal to the assumed average jet core speed). For order of magnitude calculations we assume that

$$N^2=f(z), \tag{6}$$

with $N_1\simeq1\times10^{-2}$ rad sec^{-1} at ground level.

In order to calculate the pressure perturbation p at ground level due to a horizontal wind perturbation u_0 in the jet stream we use, for the harmonic case,

$$p=\rho cu. \tag{7}$$

In general, we may use the WKB approximation, taking care to exclude the vicinity of turning points at $\gamma=0$. The solution to (4) is then

$$\left.\begin{aligned}h&=\gamma^{-\frac{1}{2}}e^{\pm is}\\ s&=\int_{z_0}^{z}\gamma dz,\quad \gamma^2>0\end{aligned}\right\}, \tag{8}$$

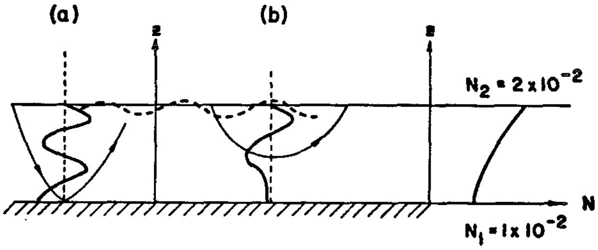

FIG. 1. Model with variable Väisälä frequency at right permitting internal gravity waves to reach ground level (a), or giving total reflection at some intermediate level (b).

or

$$\left.\begin{aligned}h&=|\gamma|^{-\frac{1}{2}}e^{-s}\\ s&=\int_{z_0}^{z}|\gamma|dz,\quad \gamma^2<0\end{aligned}\right\}. \tag{9}$$

Using (2), (3) and (8), we obtain

$$u(z)=u_0\sqrt{\frac{\rho_0\gamma}{\rho\gamma_0}}\begin{cases}\cos s, & \gamma^2>0\\ e^{-s}, & \gamma^2<0\end{cases} \tag{10}$$

and

$$p(z)=c\sqrt{\rho\rho_0}\sqrt{\frac{\gamma}{\gamma_0}}u_0\begin{cases}\cos s, & \gamma^2>0\\ e^{-s}, & \gamma^2<0\end{cases} \tag{11}$$

where the subscript refers to values at jet stream height. These results are approximate, but quite adequate for order of magnitude calculations. A time average gives

$$\langle p^2\rangle=c^2\rho\rho_0\frac{\gamma}{\gamma_0}\langle u_0^2\rangle\begin{cases}\cos^2 s, & \gamma^2>0\\ e^{-2s}, & \gamma^2<0\end{cases} \tag{12}$$

Note the term in $\cos^2 s$, which predicts some *fine structure* for the spectrum, with *maxima* at points

$$s=n\pi. \tag{13}$$

In (10)–(12), we use the e^{-s} or $\cos s$ solutions depending upon whether the corresponding internal gravity waves are ($\gamma^2<0$) or are not ($\gamma^2>0$) totally reflected before reaching ground (Fig. 1). For example, corresponding to (a) in Fig. 1, assuming

$$N^2=N_1^2+az, \tag{14}$$

we have, from (8) and (5)

$$s=\frac{2}{3}\frac{c^2}{a}(\gamma_0^3-\gamma_1^3), \tag{15}$$

where γ_1 is the value of γ at ground level. On the other hand, for case (b) in Fig. 1, for which the wave energy is totally reflected at some height above the earth, we use e^{-2s} in (12) with

$$s=\frac{2}{3}\frac{c^2}{a}|\gamma_1|^3. \tag{16}$$

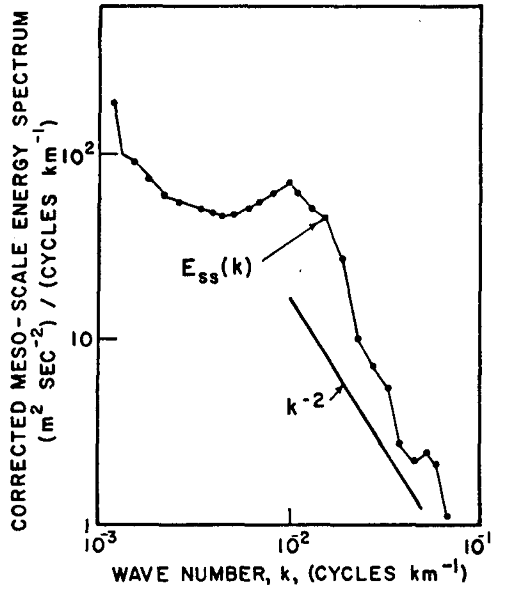

FIG. 2. The longitudinal power spectrum for longitudinal wind velocity fluctuations near the jet stream core, as determined by Kao and Woods (1964).

However, in calculated smoothed power spectra (without fine structure), the $\cos^2 s$ terms in (10)–(12) are averaged and the result is not sensitive to the precise form of (14).

Assuming a density at $z=10$ km of about 0.4 kg m^{-3} and smoothing out the fine structure in (12) by taking $\overline{\cos^2 s}=\frac{1}{2}$ gives (for $\gamma^2>0$)

$$\Pi\simeq 0.2c^2\frac{\gamma}{\gamma_0}P(u_0) \tag{17}$$

in mks units, where Π is the power spectrum for the ground level pressure perturbations and $P(u_0)$ is the wind velocity power spectrum in the jet stream. For the portion of the spectrum $\gamma^2<0$, i.e., $\alpha>N/C$, corresponding to energy reflected before reaching ground, we multiply (17) by e^{-2s}. In actual fact, the atmospheric density stratification is such that N is more nearly constant. The case of a static atmosphere with constant N corresponds to Eq. (17) with $\gamma=\gamma_0=$constant, and calculations for $N=1\times10^{-2}$ have been performed also.

We have applied this conversion to the power spectrum $P(u_0)$ given by Kao and Woods. Thus, their $E_{ss}(k)$ (Fig. 2) is related to $P(u_0)$ by

$$P(u_0)=kE_{ss}, \tag{18}$$

with k in cycles km^{-1}. Converting to $(\mu\text{b})^2\times$(cycles hr^{-1}) and taking logarithms, we obtain orders of magnitude comparable to those observed by Herron and Tolstoy, as shown in Fig. 3. In this illustration the curves have been computed for two assumed jet stream velocities of 20 and 40 m sec^{-1}. Dashed curves correspond to the model of Eq. (14) with $N_1=1\times10^{-2}$ rad sec^{-1} and $N(z)=2\times10^{-2}$ at $z=10$ km. The dotted curves correspond to a constant $N\simeq1\times10^{-2}$.

The steep fall-off of the calculated spectrum for short periods corresponds in all cases to periods less than the ground level Väisälä period. It is seen that the variable $N(z)$ model gives better results. This may well be due to the effects of wind shear, since classic hydrodynamic approximations (Yih, 1965) indicate, for phase velocities higher than the local wind velocity U_0, that we may sometimes replace the constant N^2/c^2 in Eq. (5) by $N^2/(c-U_0)^2$. This is equivalent to replacing a model with shear flow and constant N by a static model with variable N, for given c.

3. Results and conclusions

Fig. 3 shows that:

1) The assumption that ground level pressure perturbations in the mesoscale region are primarily due to internal gravity waves excited by the jet stream core provides a reasonable order of magnitude estimate of the observed fluctuation amplitudes.

2) This assumption is also in accord with the existence of observed changes in slope and humps in the power spectrum for the mesoscale range.

These very crude calculations are therefore at least consistent with observation. They also appear to explain

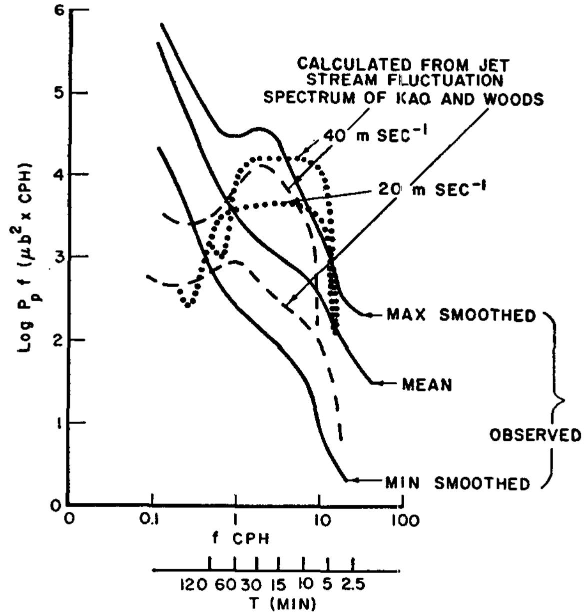

FIG. 3. Comparison of mean pressure power spectra at ground level measured by Herron and Tolstoy with calculated spectra for model of Eq. (6) (dashed lines) and for model with constant N (dotted line) obtained by applying the internal gravity wave radiation hypothesis to the Kao and Woods spectrum of Fig. 2. Maximum observed power spectra are related to extreme weather conditions and should not be representative of jet stream activity.

the frequent appearance of fine structure in the spectrum (Herron and Tolstoy, 1969) since, by means of (13), it is easy to calculate the spacing and position of peaks in the spectrum. The results are sketched qualitatively in Fig. 4. The location and spacing of these spectral lines are consistent with many observations of unsmoothed spectra (Herron and Tolstoy, 1969). A more quantitative elaboration of this point has not been attempted so far; in view of the numerous uncertainties and approximations besetting our model, detailed calculations of this kind would be premature.

This approach is also consistent with the peculiarly rapid decorrelation properties of the traveling disturbances observed in the Dobbs Ferry area; the mechanism proposed here implies that the correlation properties of pressure perturbations at ground level simply mirror the statistics of the jet stream fluctuations. In other words, our pressure measurements at ground level are simply defining the correlation distances of the wind field in or near the jet stream core.

Finally, wind speed fluctuations produced at ground level by these internal gravity waves turn out to be very small. A simple calculation based upon (10) and the Kao and Woods spectrum gives a fluctuation level somewhat smaller than what one would deduce from published ground level wind speed spectra (van der Hoven, 1957). Thus, here again, the hypothesis is in accord with existing data.

Note, however, that it is often possible to identify the appearance of slowly traveling wave trains having different origins, e.g., one observes similar disturbances originating near weather fronts, thunderstorms, and other violent perturbations of the atmosphere. It appears that these are frequently coherent over longer ranges than the typical jet stream induced wave fields described above. Thus, the calculations proposed here do not purport to be relevant at all times, but only when no interference from severe meteorological conditions is likely. Our studies suggest that such conditions pertain more than 50% of the time in our area.

Finally, we should emphasize that it is actually surprising that the crude model we have used provides the correct orders of magnitude for ground level pressure fluctuations. Certainly the image of each spectral component of the wind fluctuations in the jet stream acting as a traveling corrugation is highly questionable. The problem really is to explain why this model should work at all. Perhaps the wind fluctuations reported by Kao and Woods are due to an already *fully developed field of internal gravity waves*. In this case, our approximate model could be expected to yield the correct order of magnitude for the pressure fluctuations at ground level. The possibility also remains that stability waves in the shear flow below the jet stream may constitute a significant (but perhaps less efficient) mechanism for the transmission of these fluctuations.

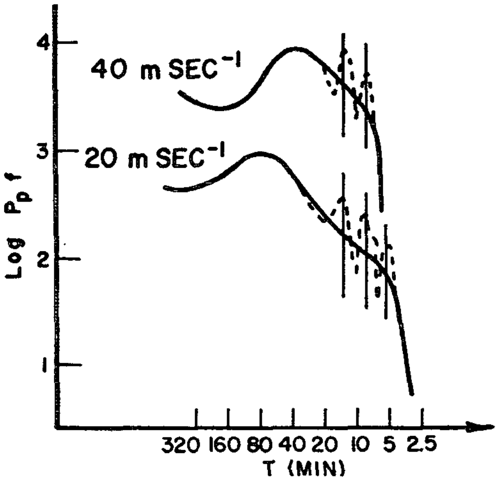

FIG. 4. Theoretical fine structure of power spectrum of ground level pressure perturbations.

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