Radiative Time Constants in the Atmosphere of Jupiter

PETER J. GIERASCH AND RICHARD M. GOODY

Division of Engineering & Applied Physics, Harvard University, Cambridge, Mass.
(Manuscript received 12 May 1969)

ABSTRACT

We have computed radiative decay times for thermal disturbances near the cloud tops of Jupiter and conclude that they are much larger than probable dynamical time constants. Under these circumstances radiative equilibrium calculations are of little significance.

1. Introduction

The object of this paper is to investigate whether radiative equilibrium calculations for Jupiter are likely to be valid in view of the established state of motion near to the cloud tops. We wish to know whether or not the longwave thermal radiation is an important mode of heat transfer at all levels as compared with heat transfer by planetary motions. We may reach tentative conclusions, without a complete knowledge of the circulation, by comparing characteristic time constants \( T(l) = 2\pi / N(l) \) for the dissipation of thermal disturbances of scale \( l \).

2. Hydrogen absorption

For radiative dissipation of a harmonic thermal disturbance [Goody, 1964, Eq. (9.15)], we have

\[
N(l) = \frac{4\pi}{\rho c_p} \int_0^\infty dk_{sr} \left( \frac{dP_s}{dk_{sr}} \right) \left( 1 - k_{sr} \tan^{-1} \left( \frac{1}{lk_{sr}} \right) \right),
\]

where \( \rho \) is the gas density, \( c_p \) the specific heat, \( v \) the frequency, \( k_{sr} \) the volume absorption coefficient, \( P_s \) the Planck function, and \( \theta \) the temperature.

According to Trafton (1967), absorption near the top of the Jupiter clouds is mainly by hydrogen, and the absorption coefficient is

\[
k_{sr} = \frac{n^2}{c} (A_s + qB_s),
\]

where \( n \) is the number density of hydrogen atoms, \( q \) the helium-to-hydrogen number density ratio, and \( c \) the velocity of light. Calculations of \( A_s \) and \( B_s \) for a temperature of 160K are given by Trafton. We shall follow him by assuming that the NH\(_3\) absorption is of secondary importance in the transfer of thermal radiation.

Since \( \rho \propto n \), it is appropriate to compute \( N(n_0/n) \) as a function of \( l^{-1}(n_0/n)^2 \), where \( n_0 \) is the number of hydrogen molecules at 160K and 1 atm; the results of this computation are shown in Fig. 1, for \( q=0, 0.5 \) and 1.0. The differences between these curves are small, and for the rest of this paper we shall use the data for \( q=0.5 \) only.

We may regard these data in many different ways, but the following probably include all matters of importance.

The characteristic scale length for response of the whole atmosphere is a scale height \( (H=19 \text{ km for } q=0.5 \) and 160K). Provided \( (n_0/n) \) is not \( \gg 1 \) (i.e., near or below the "cloud tops"), we find

\[
N(19 \text{ km}) \approx 4 \times 10^{-6} \left( \frac{n_0}{n} \right)^2 \left[ \text{sec}^{-1} \right].
\]

Alternatively, if we take \( l \) to be constant and ask under what conditions \( N \) has its maximum value, we find

\[
N(\text{max}) \approx 10^{-4} \text{sec}^{-1},
\]

when

\[
\frac{n_0}{n} \approx 4.5 \times 10^{-3} \text{sec}^{-1}.
\]

3. Comparison of results

Dynamical time constants, with which we might compare these figures, depend upon the physical balance of terms in the heat and dynamical equations, which we do not know a priori. Goody and Belton (1967), in a study of Martian dynamics, selected two time constants which were very likely to be relevant. For the advection of heat from one side of the planet to the other by a wind of velocity \( v \), they found

\[
\frac{R}{v} \sim \tau_H,
\]

where \( R \) is the radius of Jupiter. If we choose \( v=10^4 \text{ cm} \) sec\(^{-1} \) (the equatorial acceleration) we have \( \tau_H = 9 \times 10^{-8} \text{ sec}^{-1} \).
If vertical motions are important in the heat equation, Goody and Belton\(^1\) give

\[ \tau_Y \sim \left( \frac{R}{H} \right)^2 \frac{f \theta}{g \Gamma}, \]  

where \( f \) is the Coriolis parameter \((1.8 \times 10^{-4} \text{ sec}^{-1})\) and \( \Gamma \) the adiabatic lapse rate \((\sim 1.9 \times 10^{-5} \text{ K cm}^{-1})\). Hence, \( N_Y \approx 8 \times 10^{-7} \text{ sec}^{-1} \).

Provided that \((n_0/n)\) is not \( \gg 1 \), we see that \( N(19 \text{ km}) \ll N_H \) and \( N_Y \), and we may conclude that longwave radiative transfer is probably of negligible importance compared to heat transfer by fluid motions. Alternatively, if we take \( N(\text{max}) = (N_Y N_H)^4 \), we find \( I \sim 10^5 \) cm. Perhaps for very small length scales, at great depths, radiative transfer may be important, but not for planetary scale motions. Of course, the emission of radiation to space is the ultimate sink of all planetary heat, and thermal radiation must be included in the outer boundary condition, but probably not otherwise.

If these statements are correct, then a balance between thermal emission and solar absorption at each level (radiative equilibrium) has almost no significance, even as a rough first approximation, and we can have no real understanding of the thermal structure of the planet before we understand the planetary dynamics. Here we are faced with a problem of immense complexity, about which little is understood (see, e.g., Goody, 1969). Radiative equilibrium calculations such as Trafton's may not be entirely inaccurate, however, for the general level of cloud temperatures is constrained by the thermal emission to space, and the interior of the cloud may be adiabatic on account of large scale motions (see, e.g., Goody and Robinson, 1966). The result cannot differ markedly from a radiative convective profile for the average insolation.

Acknowledgments. Our work is supported by the Atmospheric Sciences section of the National Science Foundation under Grant No. GP4293 to Harvard University.

REFERENCES


\(^{1}\) Goody and Belton include a factor \( \pi \) in this and the subsequent equation. After careful consideration we prefer the present estimates.