Atmospheric Gravity Waves from Nuclear Explosions[1]

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ABSTRACT

Atmospheric gravity waves excited by nuclear explosions were recorded on several occasions during the period 1967–68, on a large aperture (250 km×200 km) array of long-period microbarographs (1–60 min period pass-band) in the New York–New Jersey area. The spectrum of these waves peaks near a period of 15 min and their average group velocity ($\sim$600 m sec^{-1}), their dispersion and attenuation conform to theoretical predictions, for the surface mode.

1. Introduction

In 1967–68 a 250 km ×200 km array of 6–12 low-frequency microbarographs (period pass-band 1–60 min) was operated by the authors at Hudson Laboratories Columbia University (Fig. 1). The outputs of all sensors were transmitted by telephone line and recorded on digital tape at the laboratory in Dobbs Ferry. During this period the system recorded three instances of what appear to be surface gravity waves generated by large nuclear explosions. In each case the individual sensors have low signal-to-noise ratios (of the order of unity or less) but the arrivals were clearly seen after filtering and beamforming. The identification of these arrivals as surface waves is based upon the fact that simple model calculations (Tolstoy, 1967; Tolstoy and Englehardt, 1969; Tolstoy and Pan, 1970), as well as calculations based upon realistic stratified models of the atmosphere (Pfeffer and Zarichny, 1962, 1963; Press and Harkrider, 1962; Harkrider, 1964; Harkrider and Wells, 1969), give the observed group velocities ($\sim$600 m sec^{-1}) and dispersion. Furthermore, a simple model for the attenuation gives estimates of the same order as those we observe (Tolstoy and Pan, 1970).

2. The data

Gravity waves were observed for the Chinese explosion of 17 June 1967, and the high energy events of 15 July and 24 August 1968. In all instances the center frequency was in the 12–18 min period band, with a mean near 15 min.

In Figs. 2–7, we exhibit data from the Chinese shot of 16 June 1967. Only six detectors were operating at that time. In all figures the top traces represent the outputs of individual sensors, the lower collection of traces giving the beamforming display; each trace here is the result of delaying and summing the individual sensor outputs, assuming a plane wave traveling from a particular direction with a given velocity. The traces correspond to increments of 10°, starting with 180° true (direction *out* of which the wave is coming). If an arrival traveling with the assumed phase velocity from a specific direction is present, the beamforming display exhibits a group with maximum amplitude corresponding to the correct direction. In addition, a more

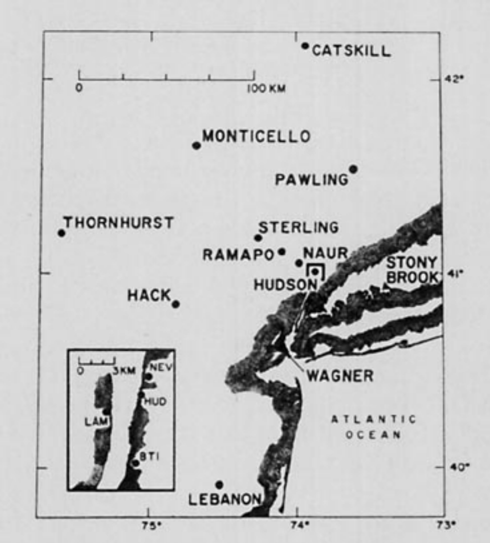

FIG. 1. The Hudson Laboratories long-period microbarograph array.

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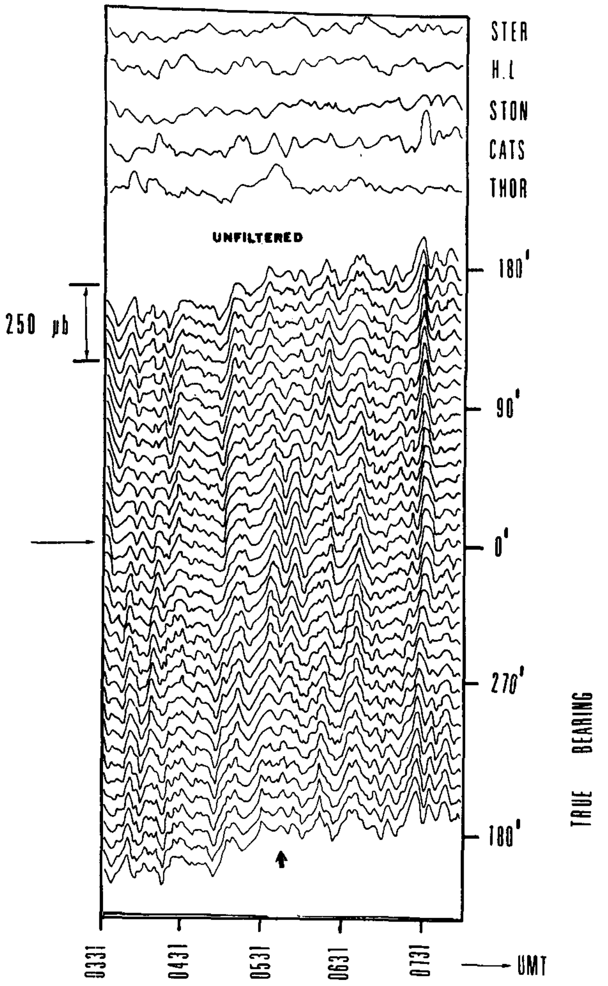

FIG. 2. Unfiltered data of 16 June 1967, 0330–0800 UMT. The top traces are individual sensor outputs, while the lower set corresponds to delaying and summing sensor outputs so as to maximize a plane wave traveling at 600 m $\sec^{-1}$ and coming out of the true bearings shown (10° steps). There is a definite indication of an arrival near 0530 coming out of a 0° bearing, but the signal-to-noise level is marginal. The transit velocity is about 600±60 m $\sec^{-1}$ (deduced from press reported zero time of about 0100).

careful numerical analysis allows one to determine the value of the velocity which gives maximum amplitudes. The displays reproduced in this paper correspond to a velocity of 600 m $\sec^{-1}$, which was in all cases close to the optimum figure (±60 m $\sec^{-1}$).

Fig. 2 shows beamforming with unfiltered data. The individual sensors were quite noisy, due to the passage of a front and the presence of many thunderstorms in the area, with peak-to-trough excursions of several hundred µb over much of the data. Nevertheless, beamforming at 600 m $\sec^{-1}$ suggests the presence of a wave packet arriving from the direction of Lop-Nor (approximately 0° true).

Passband filtering through a 10–25 min digital filter, prior to beamforming, gives a sharply steering, clear arrival from the direction of 0° true (Fig. 3). The maximum peak-to-trough amplitude averaged over all stations is 115 µb. The directionality properties can be determined by taking a given time interval, say 0525–0610 GMT, squaring and integrating each of the traces of a beamforming analysis for an assumed phase velocity of 600 m $\sec^{-1}$, and displaying on a polar plot. The result is shown in Fig. 4 for two slightly different samples. The solid curve in Fig. 4 represents the theoretical array lobe calculated in similar fashion for a 550 km wavelength sine wave arriving from 0° true. It is seen that the observed directionality pattern is very close to what it should be for a 15-min period wave arriving out of the north with 600 m $\sec^{-1}$ phase velocity.

A dispersion analysis, based upon a crude peak, trough and axis crossings counting procedure, of the wave groups recorded by the Catskill and Thornhurst sensors provided the points displayed in Fig. 5. These points are seen to be in fair agreement with theoretical

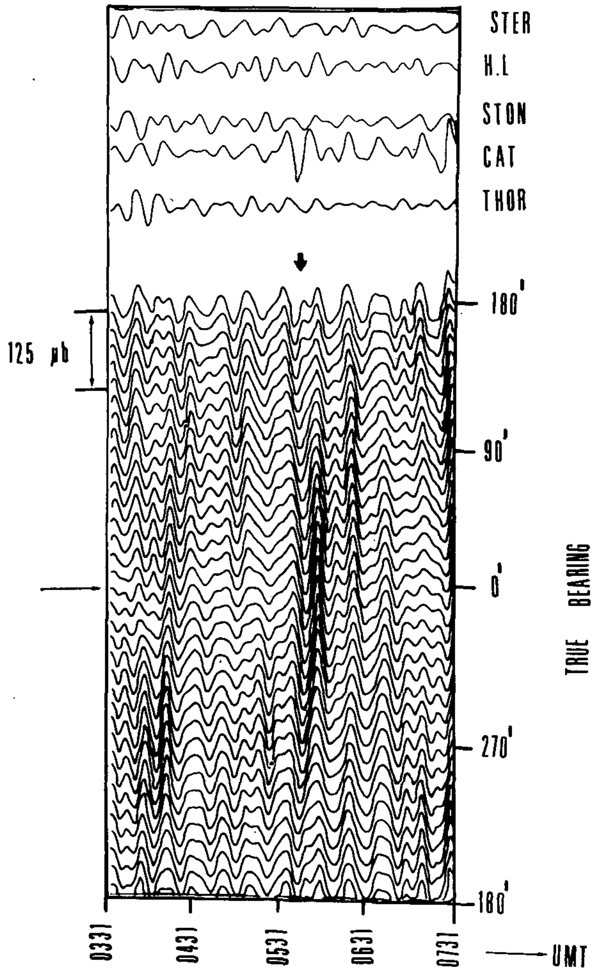

FIG. 3. Delay and sum or beamforming pattern of bandpass (10–25 min) filtered data. A comparison with Fig. 2 shows that the signal-to-noise ratio has been much improved and the beamforming sharpened. The maximum averaged peak-to-trough amplitude is 115 µb.

dispersion curves for the surface gravity mode calculated for the models shown in the inset. These models were selected on the grounds that they provided reasonable two-layer fits to the observed distribution of the Väisälä frequency and the logarithmic derivative of the density in the atmosphere (Tolstoy and Pan, 1970). Curve A took into account compressibility effects by using the mean values of the sound velocity shown in the inset, whereas curve B used the incompressible approximation. Since we estimate the absolute values of the observed group velocity to have a fairly large margin of uncertainty (±10%), the difference between the two fits cannot be considered significant. A check of the internal gravity wave modes both for this model (Tolstoy and Pan, 1970) and for more complex stratified models (Pfeffer and Zarichny, 1962, 1963; Press and Harkrider, 1962; Harkrider, 1964) shows that, *in*

FIG. 4. Observed and calculated array lobes. Squaring and integrating a sample of beamformed data between 0515 and 0600 UMT gives the observed directionality pattern. This procedure gave similar results when applied to the bandpass filtered data of Fig. 3 and the matched filter data of Fig. 6 (dotted lines). The theoretical pattern was calculated for a 550 km wavelength plane wave with a 600 m sec^{-1} phase velocity arriving out of 0° true (solid line). The agreement with the experimental results is seen to be good.

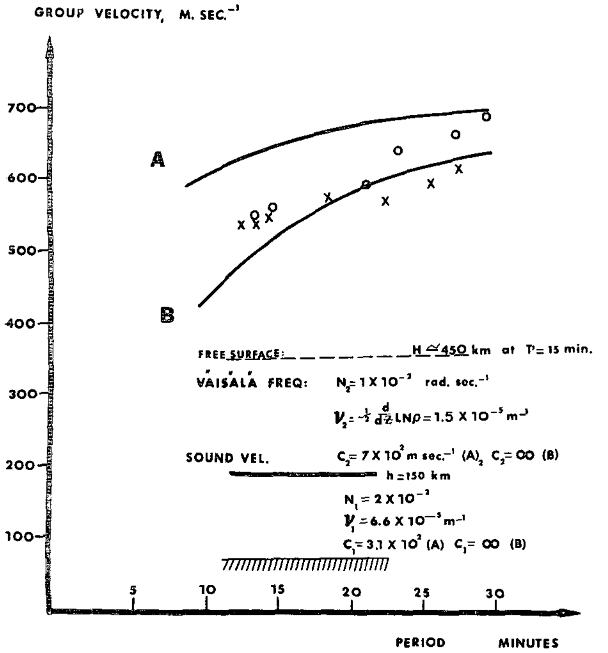

FIG. 5. Dispersion analyses of unfiltered wave groups, recorded at the Catskill and Thornhurst stations, by the peak and trough counting method, and normalized to a nominal value of 600 m sec^{-1} at $T = 20$ min. The result is similar to that of the *surface gravity wave* mode for the two-layer model of the atmosphere shown in the inset (Tolstoy and Pan, 1970). The two curves correspond to the compressible (A) and incompressible (B) models.

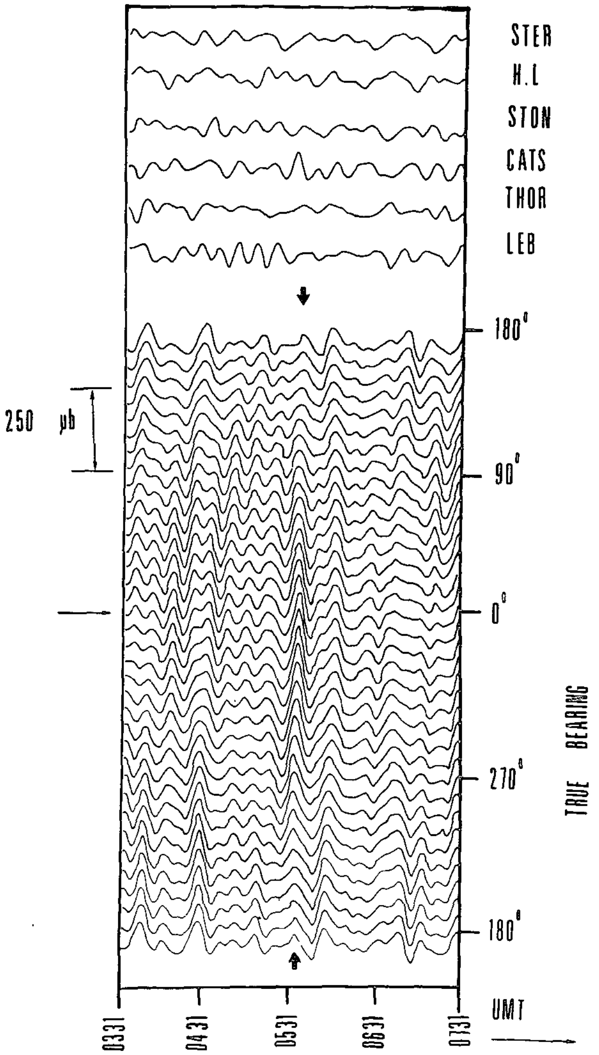

FIG. 6. Result of applying a matched filter to the data of Fig. 2 before beamforming. When the 0°, 0530–0630 UMT data of Fig. 2 are taken as the best representation of the waveform, one may crosscorrelate with individual sensor outputs and then beamform. The result is an improved signal-to-noise ratio and a sharply defined arrival out of 0° true bearing. This corresponds to the *short great-circle path*, with transit velocity of the order of 600 ±60 m sec^{-1}.

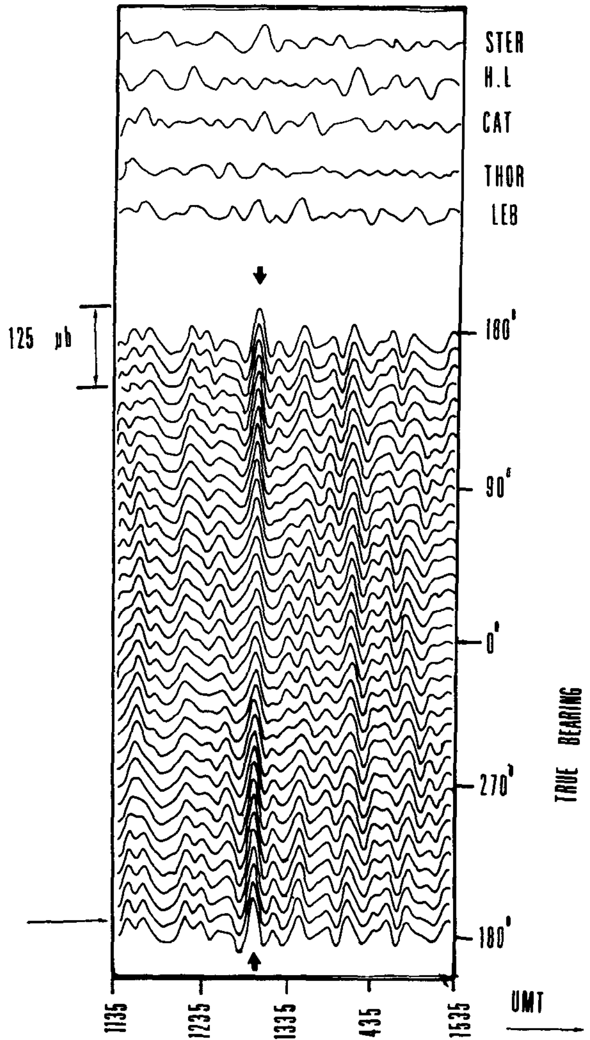

FIG. 7. The *long great-circle path* arrival, after application by the 10–40 min band-pass filter. By the time this arrival was due, the background noise level had dropped considerably and the use of this somewhat broader filter was sufficient to show an arrival beamforming quite sharply out of a 180° true bearing with a 600 m sec^{-1} phase velocity. The pattern is complementary to those of Figs. 6 and 3, as it must be. The transit velocity is close to 600 m sec^{-1} and the maximum peak-to-trough amplitude, averaged over all stations, is 80 μb.

this range of periods, they cannot provide a fit to the observed group velocities.

The signal-to-noise ratio of the beamforming display for this arrival can be further improved with the use of a matched filter procedure as follows. One assumes first of all that the 0° trace of Fig. 2, sampled between 0530 and 0630, gives the best possible estimate of the true signal waveform. One may then use it as a matched filter by cross correlating it with each individual sensor output; this is equivalent to using a matched filter, i.e., the best filter in the Wiener sense. Having performed this filtering, one now applies the beamforming procedure. The result given in Fig. 6 shows considerable improvement over Fig. 3 which had been merely bandpass filtered before beamforming.

After passage of the weather front, noise levels dropped appreciably and quite suddenly and the individual trace variances in the 10–40 min period bandpass became smaller by a factor of ~3. As a result we obtained a clear cut identification of the long great-circle path arrival by beamforming 10–40 min *bandpass filtered* sensor outputs, assuming once again a 600 m sec^{-1} velocity. The pattern is displayed in Fig. 7 and is seen to be *complementary to that of Figs. 6 and 3*, giving maximum amplitudes for 180° true where the short path arrival was minimized and vice versa. The arrival time corresponds again to a group velocity of 600 m sec^{-1}, and the average maximum double trace amplitude is 80 μb. It is interesting to gauge the quality of these newly discovered events in terms of the known 300 m sec^{-1} signals. A glance at Fig. 8, which displays the result of filtering and beamforming at 300 m sec^{-1} for the expected time of arrival of these waves, reveals that the quality of the detection of these two events is comparable insofar as the signal-to-noise levels are concerned.

Similar identifications were made in the case of the high energy events of 25 August and 16 July, with the use of bandpass filtering and beamforming. Peak-to-trough amplitudes were 50 and 30 μb for the short and long paths, respectively, on 25 August, and 30 and 20 μb on 15 July. (See Figs. 9 and 10.) The ratios of long path to short path amplitude for the Chinese and 1968 cases were therefore 0.7, 0.6, 0.7 for a mean of 0.7, enabling us to estimate the attenuation with range.

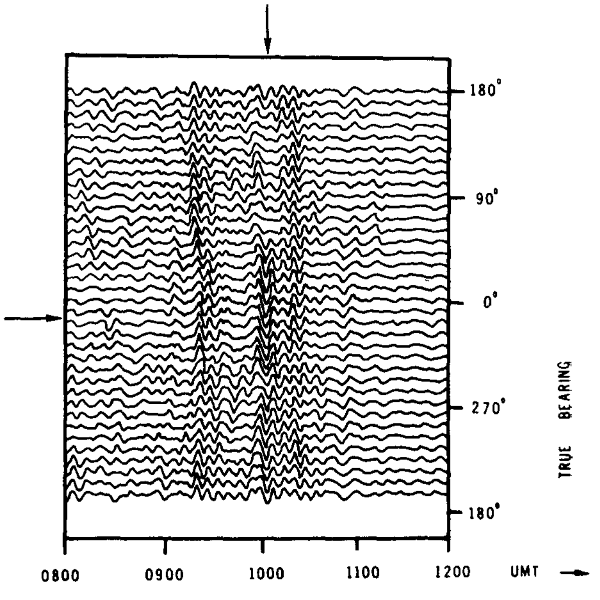

FIG. 8. Beamformed 300 m sec^{-1} arrival, following application of 8 min half-octave filter. The signal-to-noise ratio is hardly more impressive than for 600 m sec^{-1} arrivals shown in Figs. 6 and 7. Use of shorter period filters was not possible due to 1 min data averaging and sampling rate. The transit velocity is ~320 m sec^{-1}.

For order-of-magnitude calculations, we may assume the short great-circle paths to Dobbs Ferry to be about the same in all cases, 10^4 km or $\frac{1}{4}$ of the earth's circumference. In the absence of dispersion, the wave energy refocuses at the antipodes so that with this geometry the effective cylindrical spreading factor is the same for the short and long great-circle paths. Since the above are maximum amplitudes after filtering through narrow filters and since the frequency of arrivals remains effectively the same, we may consider the amplitude ratio obtained above as reflecting the behavior of harmonic waves with a period of 15 min. This is tantamount to neglecting the role of dispersion in diminishing the wave amplitude, which is legitimate for the narrow filters used; however, we may expect this analysis to give figures for the attenuation which will be slightly in excess of the true values. If we use the relation

$$e^{-\beta \Delta r} \approx 0.7,$$

where $\Delta r = 2 \times 10^7$m is the difference between long and short paths, we find that

$$\beta \approx 2 \times 10^{-8}\,\mathrm{m}^{-1}.$$

It has been shown elsewhere (Tolstoy and Pan, 1970) that one may estimate β from simplified models similar to those used in the calculation of group velocities, if one assumes that most of the surface wave dissipation takes place in a surface layer about 75 km thick. A reasonable set of upper atmospheric parameters gives β in the range 3×10^{-8} to $6 \times 10^{-8}\mathrm{m}^{-1}$, which is of the observed order of magnitude. However, these rough estimates of the attenuation are, at most, a plausibility test for our model; conditions in the atmosphere are just too uncertain for anything more. There are reasons to think that β, when it can be defined, may vary widely depending upon solar conditions, time of day, and direction of propagation.

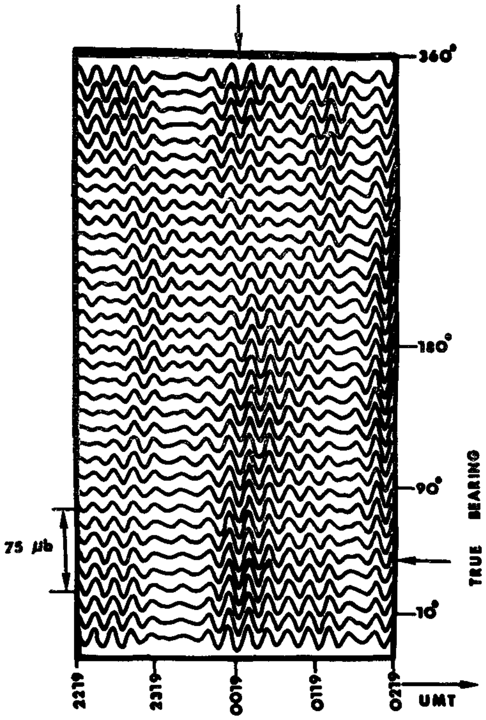

FIG. 9. The short path 600 m sec^{-1} beamformed arrival from the event of 15 July 1968. Maximum peak-to-trough amplitude is 30 μb, the reported zero time (N. Y. Times) being 1900 UMT.

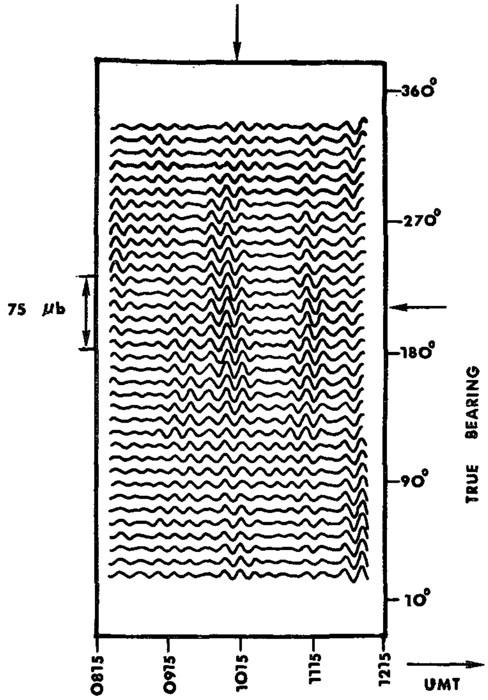

FIG. 10. The long path 600 m sec^{-1} beamformed arrival from the event of 15 July 1968, with a maximum peak-to-trough amplitude of 20 μb.

3. Conclusions

The 600 m sec^{-1}, 15 min period arrivals recorded on the Hudson Laboratories microbarograph system and described above are, in all probability, *surface gravity waves* traveling along what is effectively the top of the atmosphere. The height H of this "effective free surface" depends on the wavelength L, since it corresponds approximately to the region where the mean free path l of the air molecules is of the order of L (indeed, the condition explicitly applied in making calculations is $2\pi l/L = 1$). At the wavelengths of interest in this problem, this surface corresponds approximately to the base of the exosphere ($H \approx 500$ km).

Our identification is based on the following features:

1) Calculations on simple two- and three-layer models of the atmosphere for periods of the order of 15 min tell us that group and phase velocities of the order of

600 m sec^{-1} must correspond to the surface gravity mode. Internal gravity modes do not yield these group velocities at these periods. This result is entirely in accord with calculations made on more complex atmospheric models by numerous investigators for a series of *arbitrarily* selected values of H (Pfeffer and Zarichny, 1962, 1963; Press and Harkrider, 1962; Harkrider, 1964; Harkrider and Wells, 1969).

2) Rough group velocity vs period curves obtained from the Chinese test are also in agreement with our surface gravity mode calculations.

3) An order-of-magnitude theoretical analysis of the attenuation, based upon an approximate surface wave model, gives results consistent with those obtained from a comparison of amplitudes for long and short great-circle paths (Tolstoy and Pan, 1970).

The equivalent free surface concept we have used assumes $L \gg d$, where d is the thickness of the near surface transition layer ($\sim$75 km) in which, strictly speaking, one may not use the equations of continuum mechanics but which we have effectively replaced by a high kinematic viscosity surface layer whose principal effect is to produce attenuation of the gravity mode; the attenuation becomes prohibitively large for $L < 2\pi d$. Thus, it is probable that the periods observed do not correspond to a peak in the source spectrum or in the medium response, but rather to the shortest periods of the surface gravity mode that can propagate over long distances without drastic attenuation. One may imagine, for instance, a spectral curve rising toward a peak at shorter periods being counteracted by a sharp rise in attenuation. However, in view of the extreme variability of atmospheric conditions at heights of the order of 500 km, one may expect the effective values of this period and of the attenuation to be variable. Although the mechanism of excitation of this mode of propagation is still somewhat obscure, our identification is based upon firmly established propagation properties. It seems likely that a number of published ionospheric observations of fast traveling disturbances generated by U. S. and U.S.S.R. thermonuclear tests in the early 1960's can be explained in a similar way, although the nature of the available data is not precise enough to establish this with any degree of certainty (Obayashi, 1963; Breitling *et al.*, 1967). Our results lead one to suspect that reported ionospheric disturbances with horizontal group velocities $\gtrsim$ 500 m sec^{-1} would be surface gravity waves, whereas lower velocities correspond to internal gravity waves (Hines, 1967). Acoustic modes of propagation (Wickersham, 1966) are also a possibility for spectral components with periods shorter than 10 min.

A surface wave model is clearly capable of explaining the *propagation properties* of our arrivals. However, there still remain some difficulties in reconciling the *amplitudes* of the observed pressure perturbations with probable and plausible values of ionospheric and upper atmospheric displacements. A set of ionospheric measurements made on 24 August 1968, has been discussed elsewhere (Herron and Montes, 1970). The displacements observed at that time at a height of about 220 km correlate with microbarograph signals of 50 μb peak-to-trough amplitude and correspond to neutral gas vertical displacements of the order of 10 km or more. This observation agrees in order of magnitude with some reports of 400–800 m sec^{-1} ionospheric perturbations created by thermonuclear explosions set off by the U. S. and the U.S.S.R. in the early 1960's, which gave vertical displacements of 10–50 km (Breitling *et al.*, 1967). A much oversimplified one-layer model of the atmosphere (Tolstoy, 1967) can be used to give peak-to-trough pressure perturbations at ground level of 2 μb km^{-1} displacement at these heights, a result which would be quite adequate were it not for the fact that a more realistic two-layer model decreases this estimate to 0.2 μb or less. Proper inclusion of the effects of a layer between altitudes of 110 and 150 km, in which the acoustic cut-off frequency of the medium is lower than the Väisälä frequency, might eliminate the discrepancy; the problem is discussed in more detail elsewhere (Tolstoy and Pan, 1970).

Finally, *one* reason for which these arrivals have not been reported in previous barographic measurements of nuclear explosions is clear, since under average conditions the signal-to-noise levels of individual sensor outputs are of the order of unity. In order to observe these arrivals it is essential to use noise cancellation techniques, e.g., appropriate filtering and array beamforming. Under average noise conditions these techniques work quite well; this is due, at least in part, to simple properties of jet stream generated noise (in moderate latitudes) such as its fairly rapid decorrelation with range (Herron and Tolstoy, 1969; Tolstoy and Herron, 1969; Herron *et al.*, 1969). However, this is certainly not the full story. Other factors, the roles of which are hard to assess at the present state of the art, will also play a role. Thus, the *mechanism* of excitation of this mode is still obscure and sources of the kind commonly used in the literature (Harkrider, 1964; Harkrider and Wells, 1969) may not be able to account for the size of the events we have observed. The blocking or amplifying effects of upper atmospheric wind shears are also difficult to evaluate. Simple arguments (Tolstoy and Pan, 1970) suggest, however, that they may critically affect the ground-level pressure signal associated with waves ducted at those altitudes.

The attenuation of the surface gravity mode depends upon the kinematic viscosity near the "equivalent free surface" at the base of the exosphere (Tolstoy and Pan, 1970); the height and effect of the attenuating layer are quite variable and depend, among other things, upon solar activity. Large-scale shear flows and atmospheric turbulence can also be expected to affect the attenuation; these quantities are, at best, somewhat unpredictable. Thus, there are theoretical grounds for expecting some variability in signal to noise even under given noise conditions. Further work along the lines reported in this paper is needed before quantitative predictions can be made with any degree of certainty.

To conclude, we must emphasize that the identification of this arrival as a *surface* mode must still be considered as somewhat tentative. It is conceivable that the inclusion of acceptable models for the anomalous layer, with or without wind shears, may force everyone to revise *all* gravity mode group velocities in the upward direction. In such an eventuality, it may become necessary to identify the arrivals described above with the first *internal* gravity wave mode. However, currently available numerical models of the earth's atmosphere favor our present interpretation.

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REFERENCES

Breitling, W. J., R. A. Kupferman and G. J. Gassmann, 1967: Traveling ionospheric disturbances associated with nuclear detonations. *J. Geophys. Res.*, **72**, 307–315.

Harkrider, D. G., 1964: Theoretical and observed acoustic-gravity waves from explosive sources in the atmosphere. *J. Geophys. Res.*, **69**, 5295–5321.

——, and F. J. Wells, 1969: The excitation and dispersion of the atmospheric surface wave. *Proc. ESSA-ARPA Acoustic-Gravity Wave Symp.*, Boulder, Colo., U. S. Gov't. Printing Office, 299–313.

Herron, T. J., and I. Tolstoy, 1969: Tracking jet-stream winds from ground level pressure signals. *J. Atmos. Sci.*, **26**, 266–299.

——, —— and D. W. Kraft, 1969: Atmospheric background pressure fluctuations in the mesoscale range. *J. Geophys. Res.*, **74** (in press).

——, and H. Montes, 1970: Correlation of atmospheric pressure waves with ionospheric Doppler signals. *J. Atmos. Sci.*, **27**, 51–54.

Hines, C. O., 1967: On the nature of traveling ionospheric disturbances launched by low altitude nuclear explosions. *J. Geophys. Res.*, **72**, 1877–1882.

Obayshi, T., 1963: Upper atmospheric disturbances due to high altitude nuclear explosions. *Planetary Space Sci.*, **10**, 47–63.

Pfeffer, R. L., and J. Zarichny, 1962: Acoustic-gravity wave propagation from nuclear explosions in the earth's atmosphere. *J. Atmos. Sci.*, **19**, 256–263.

——, and ——, 1963: Acoustic-gravity wave propagation in an atmosphere with two sound channels. *Geofis. Pura Appl.*, **55**, 175–199.

Press, F., and D. Harkrider, 1962: Propagation of acoustic-gravity waves in the atmosphere. *J. Geophys. Res.*, **67**, 3889–3908.

Tolstoy, I., 1967: Long period gravity waves in the atmosphere. *J. Geophys. Res.*, **72**, 4605–4622.

——, and J. Engelhardt, 1969: Note on long gravity waves in layered atmospheres. *J. Geophys. Res.*, **74**, 3436–3439.

——, and P. Pan, 1970: Simplified atmospheric models and the properties of long-period internal and surface gravity waves. *J. Atmos. Sci.*, **27**, 31–50.

——, and T. J. Herron, 1969: A model for atmospheric pressure fluctuations in the mesoscale range. *J. Atmos. Sci.*, **26**, 270–273.

Wickersham, A. F., Jr., 1966: Identification of acoustic-gravity wave modes from ionospheric range-time observations. *J. Geophys. Res.*, **71**, 4551–4555.