Characteristics of the Large-Scale Dispersion of Particles in the Southern Hemisphere

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ABSTRACT

An analysis of the Eulerian and Lagrangian velocities at the 200-mb level in the Southern Hemisphere is made. It is found that: 1) the zonal component of the eddy diffusivity in the mid-atmosphere in the Southern Hemisphere is about 50% greater than that in the Northern, whereas the meridional component of the eddy diffusivity in the Southern Hemisphere is about 50% smaller than that in the Northern; 2) the coefficient for the Eulerian-Lagrangian time-scale transformation in the Southern Hemisphere is about 0.6 which is of the same order of magnitude as that in the Northern; 3) the autocorrelation functions and energy spectra of the Eulerian and Lagrangian velocities in the Southern Hemisphere are similar to those in the Northern; and 4) the peak of the energy spectrum of the meridional component of the Lagrangian velocity in the Southern Hemisphere occurs near the frequency 1.8×10^{-2} cycle hr^{-1}, about the same as that in the Northern.

1. Introduction

In recent years, theoretical (Kao, 1962, 1965, 1968) and experimental (Kao and Bullock, 1964; Kao and Taylor, 1964; Kao and Gain, 1968; Murgatroyd, 1969; Kao and Powell, 1969) studies of the dispersion of particles by the large-scale atmospheric motion in the Northern Hemisphere have been made. These investigations have shown that in the mid-atmosphere the eddy diffusivity is of the order of magnitude of 10^{11} cm^2 sec^{-1} for the zonal component and 10^{10} cm^2 sec^{-1} for the meridional component. The ratio of the Lagrangian to Eulerian integral time-scale is found to be 0.5 for large-scale atmospheric motion. In the high-frequency range, the energy spectrum of the zonal component of the Lagrangian velocity is approximately proportional to the minus third power of the frequency, whereas that of the meridional component of the velocity shows an energy peak occurring near the frequency of 10^{-2} cycle hr^{-1}. In view of the difference in the land-sea distribution and the intensity of the general circulation in the Northern and Southern Hemispheres, the question arises as to what are the differences and similarities in the characteristics of the dispersion of particles by the large-scale atmospheric motion in the two hemispheres. The purpose of this paper is to investigate these characteristics.

2. Source of data

The Lagrangian wind data used in this study were extracted from the Global Horizontal Sounding Technique (GHOST) data made available by the National Center for Atmospheric Research (Solot, 1968). The trajectory data that went into the Northern Hemisphere or south of 80S were rejected, since only the Southern Hemisphere data were of interest in this study, and the data south of 80S had large position errors due to the characteristics of the particular navigational system being used. The Lagrangian wind data used in this study were for the period 25 May 1966–23 September 1967. While GHOST balloon data were available at 1-hr intervals, only one observation was taken every 24 hr and the other data were interpolated. The interpolation process was done by two spline functions, one for latitude and one for longitude. To minimize the interpolation error, the data at 12-hr intervals were used in this study.

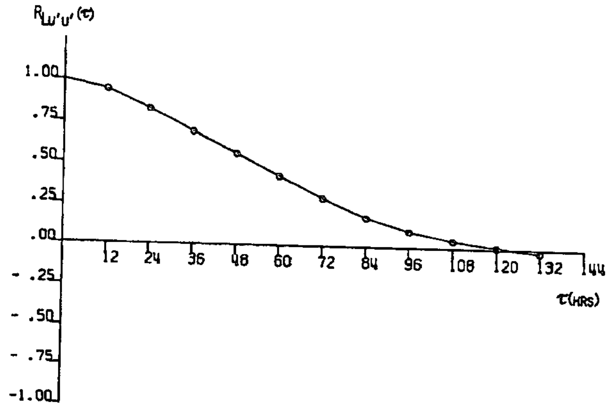

FIG. 1. Lagrangian autocorrelation function of the zonal velocity of individual particles.

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For the Eulerian wind data the station chosen was Hobart, Tasmania. The reasons for choosing this station is because it had the smallest amount of missing data, and it is near the latitude and longitude of Christchurch, New Zealand, at which the constant density balloons were launched. The Eulerian wind data at the 200-mb level were available every 24 hr.

3. Lagrangian correlation functions and spectra of single particle velocities

The Lagrangian autocorrelation functions for the zonal and meridional components of single-particle velocities are computed and are shown as $R_{Lu'u'}(\tau)$ and $R_{Lv'v'}(\tau)$, respectively, in Fig. 1 and 2. These correlation functions are similar to those for the Northern Hemisphere (Kao, 1965; Kao and Gain, 1968). The correlation function for the zonal velocity behaves like an exponentially decreasing function, whereas that for the meridional velocity shows a combination of an exponentially decreasing function and a cosine function with a damping amplitude.

The power spectra of the zonal and meridional components of the velocity are shown as $E_{Lu'u'}(n)$ and $F_{Lv'v'}(n)$, respectively, in Figs. 3 and 4. It may be noted that in the high-frequency range the spectrum of the zonal velocity may be approximated by n^{-3}, and that of the meridional velocity shows an energy peak occurring near the frequency 1.8×10^{-2} cycle hr^{-1}, which agrees well with the results obtained in the Northern Hemisphere (Kao, 1965).

The cross-correlation function and cospectrum of the zonal and meridional components of the velocity are shown, respectively, in Figs. 5 and 6. The cross-correlation function shows a negative value at zero time-lag and behaves like a sinusoidal function with a damping amplitude. The cospectrum shows positive values in the low-frequency end, but negative values in the frequency range 3.2×10^{-3}–1.6×10^{-2} cycle hr^{-1}. These results indicate that the low-frequency eddies tend to move

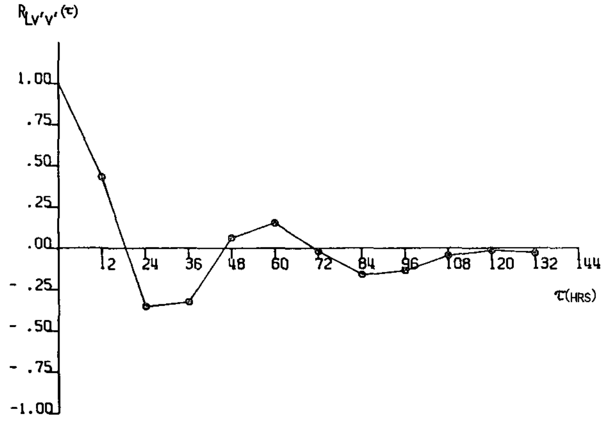

FIG. 2. Lagrangian autocorrelation function of the meridional velocity of individual particles.

FIG. 3. Normalized energy spectrum of the zonal velocity of individual particles.

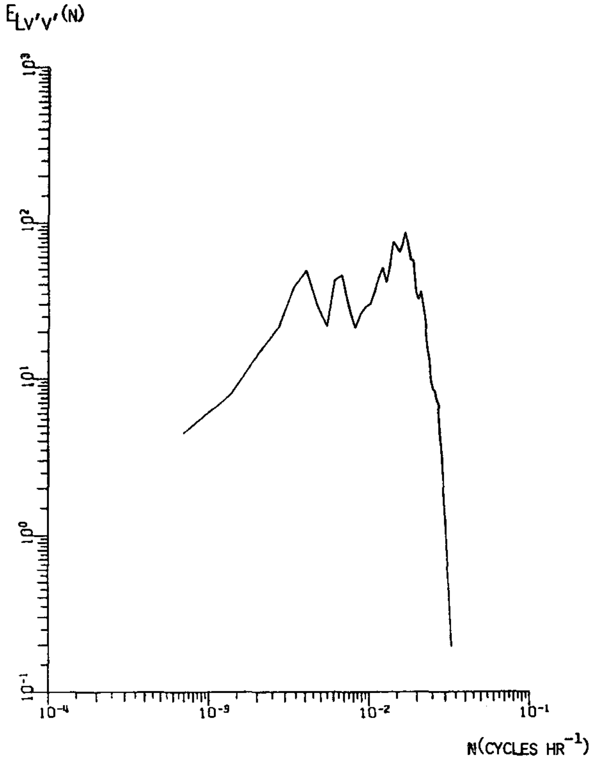

FIG. 4. Normalized energy spectrum of the meridional velocity of individual particles.

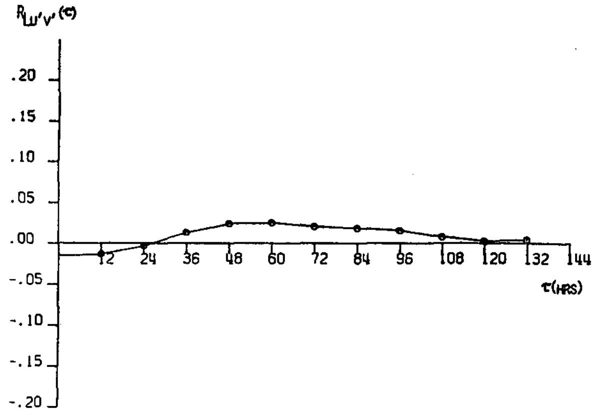

FIG. 5. Lagrangian cross-correlation function of the zonal and meridional velocities of individual particles.

the particles toward the ENE, whereas the eddies of medium frequencies tend to move the particles toward the ESE.

4. Estimate of eddy diffusivity

It has been shown in an earlier paper (Kao, 1962) that the zonal and meridional components of the eddy diffusivity may be estimated by

$$K_{xx}=\frac{\overline{u_L'^2}}{\epsilon_{xx}}, \tag{1}$$

$$K_{yy}=\left(\frac{2\overline{v_f^2}}{\bar{A}^2+2\overline{v_f^2}}\right)\frac{\overline{v_L'^2}}{\epsilon_{yy}}, \tag{2}$$

respectively, where $\overline{u_L'^2}$ and $\overline{v_L'^2}$ are the variance of the zonal and meridional components of the Lagrangian velocities, ϵ_{xx} and ϵ_{yy} are the zonal and meridional components of the frictional coefficient, $\bar{A}$ the mean amplitude of the meridional velocity of the planetary waves, and $\overline{v_f^2}$ the variance of the meridional velocity deviation from the mean planetary wave (Kao, 1962). The values of ϵ_{xx}, ϵ_{yy}, $\bar{A}^2$ and $\overline{v_f^2}$ can be determined from the correlation functions for the planetary wave model (Kao, 1962); i.e.,

$$R_{u'u'}(\tau)=\exp(-\epsilon_{xx}\tau), \tag{3}$$

$$R_{v'v'}(\tau)=\left(\frac{\bar{A}^2}{\bar{A}^2+2\overline{v_f^2}}\right)\left(\frac{\sin\Delta\omega\tau}{\Delta\omega\tau}\right)\cos\bar{\omega}\tau$$
$$+\left(\frac{2\overline{v_f^2}}{\bar{A}^2+2\overline{v_f^2}}\right)\exp(-\epsilon_{yy}\tau), \tag{4}$$

where $\bar{\omega}$ and $\Delta\omega$ are the mean and half-range of the angular frequency of the planetary waves.

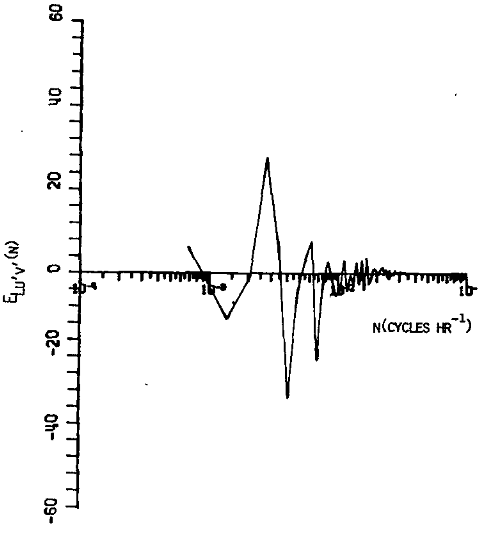

FIG. 6. Normalized cospectrum of the zonal and meridional velocities of individual particles.

Using the Lagrangian velocity correlations shown in Figs. 1 and 2, the values of the parameters in Eqs. (3) and (4) are estimated. It is found that

$$\left.\begin{aligned}\epsilon_{xx}&=6.12\times10^{-6}\ \text{sec}^{-1}\\ \epsilon_{yy}&=9.18\times10^{-6}\ \text{sec}^{-1}\\ \bar{A}^2/\overline{v_f^2}&=7.52\\ \bar{\omega}&=3.02\times10^{-5}\ \text{sec}^{-1}\\ \Delta\omega&=1.25\times10^{-5}\ \text{sec}^{-1}\end{aligned}\right\}.$$

Since $\overline{v'}=0$ and $\overline{v'^2}=\frac{1}{2}\bar{A}^2+\overline{v_f^2}$, one finds that

$$\left.\begin{aligned}\bar{A}^2&=\frac{15.04}{9.52}\overline{v'^2}\\ \overline{v_f^2}&=\frac{2}{9.52}\overline{v'^2}\end{aligned}\right\}.$$

The mean and variance of the zonal and meridional components of the velocities determined from the

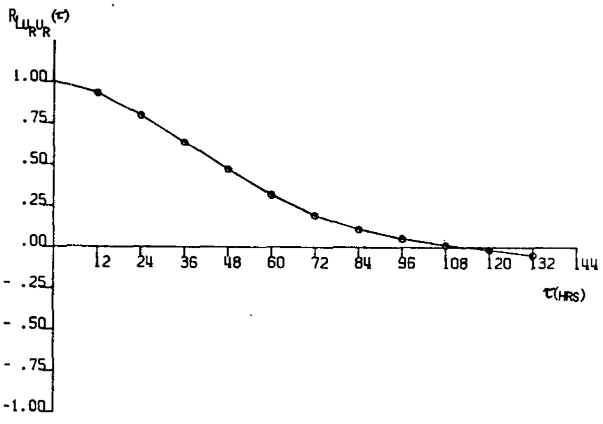

FIG. 7. Lagrangian autocorrelation function of the relative zonal velocitity between particles.

Lagrangian wind data are, respectively,

$$\bar{u}_L = 21.7 \text{ m sec}^{-1}, \quad \overline{u'^2} = 198 \text{ m}^2 \text{ sec}^{-2},$$

$$\overline{v'^2} = 187 \text{ m}^2 \text{ sec}^{-2}.$$

Therefore,

$$\left.\begin{aligned} A^2 &= 296 \text{ m}^2 \text{ sec}^{-2} \\ \overline{v_f^2} &= 39 \text{ m}^2 \text{ sec}^{-2} \end{aligned}\right\},$$

and

$$\left.\begin{aligned} K_{xx} &= 3.29 \times 10^{11} \text{ cm}^2 \text{ sec}^{-1} \\ K_{yy} &= 4.27 \times 10^{10} \text{ cm}^2 \text{ sec}^{-1} \end{aligned}\right\}.$$

Comparing with the zonal and meridional component of the eddy diffusivity in the Northern Hemisphere, one finds that at 200 mb in the Southern Hemisphere the zonal component of the eddy diffusivity is about 50% greater than that in the Northern Hemisphere, whereas the meridional component is about 50% smaller (Kao, 1962; Kao and Gain, 1968; Kao and Powell, 1969).

5. Lagrangian correlation functions and spectra of relative particle velocities

The Lagrangian autocorrelation functions for the zonal and meridional components of the relative particle velocities for the period 15 October 1966–12 January 1967 are computed and shown, respectively, as $R_{Lu_ru_r}(\tau)$ and $R_{Lv_rv_r}(\tau)$ in Figs. 7 and 8. It is seen that the autocorrelation functions for the relative particle velocities in the Southern Hemisphere are again similar to those in the Northern (Kao and Gain, 1968).

The normalized energy spectra of the zonal and meridional components of the relative velocities are shown in Figs. 9 and 10. It may be noted that, in the high-frequency range, the energy spectrum of the zonal component of the relative velocity is approximately proportional to n^{-3}, whereas the energy spectrum of the meridional component of the relative particle velocities shows a peak occurring near the frequency of 1.8×10^{-2} cycle hr^{-1}, which is again similar to that in the Northern Hemisphere.

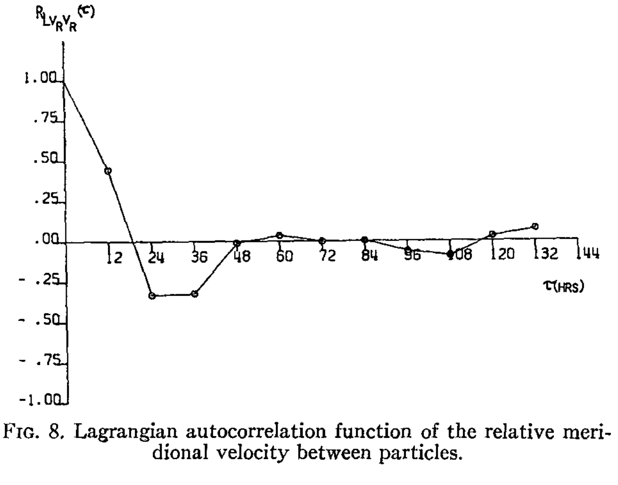

FIG. 8. Lagrangian autocorrelation function of the relative meridional velocity between particles.

FIG. 9. Normalized energy spectrum of the relative zonal velocity between particles.

An estimation of the zonal and meridional components of the eddy diffusivity with the use of the mean squares of the relative displacement of particles gives the same order of magnitude as that obtained in the last section with the use of Eqs. (1) and (2).

The cross-correlation function of the zonal and meridional components of the relative velocities (Fig. 11) shows a negative value at zero time-lag and a sinusoidal function. The cospectrum of the zonal and meridional velocities (Fig. 12) exhibits positive values in the low-frequency end but negative values in the frequency range 3.5×10^{-3}–10^{-2} cycle hr^{-1}.

6. Eulerian correlation functions and spectra of velocities

The autocorrelation functions for the zonal and meridional components of the Eulerian velocities computed with the use of the wind data at Hobart, Tasmania, are shown, respectively, as $R_{Eu'u'}(\tau)$ and

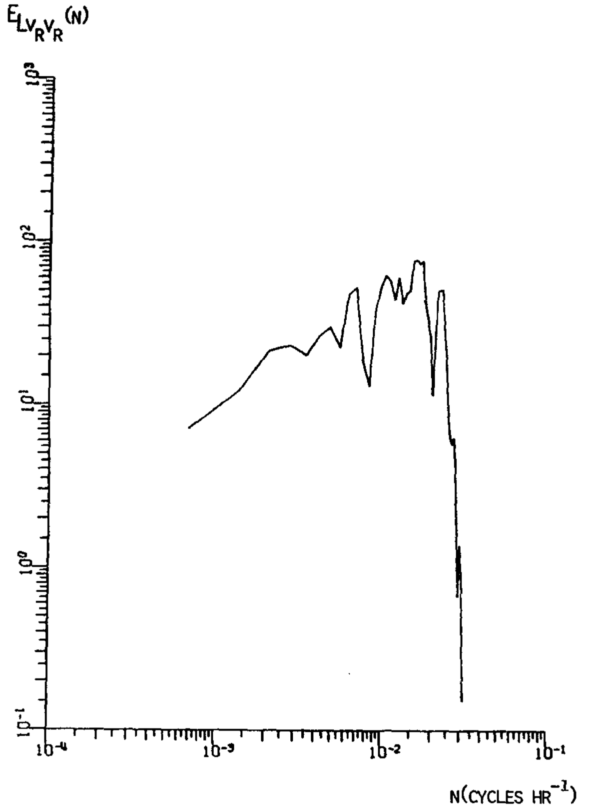

FIG. 10. Normalized energy spectrum of the relative meridional velocity between particles.

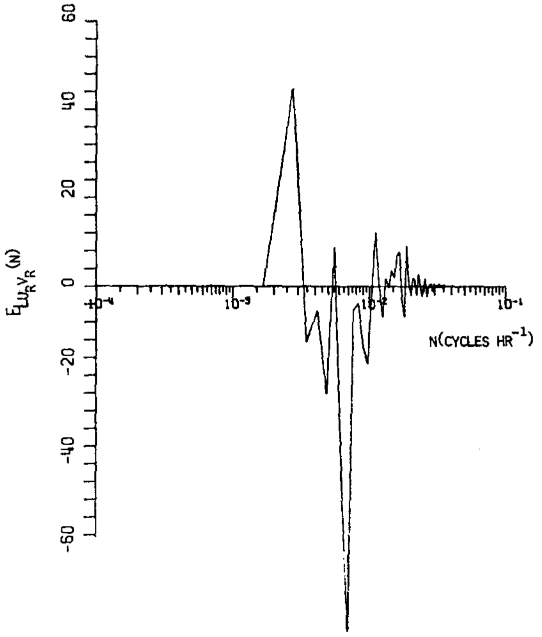

FIG. 12. Normalized cospectrum of the relative zonal and meridional velocities between particles.

$R_{Ev'v'}(\tau)$ in Figs. 13 and 14. It may be noted that both the Eulerian correlation functions converge slower than the corresponding Lagrangian correlations. This indicates that the Eulerian integral time-scale is greater than the Lagrangian. Therefore, the coefficient for the Eulerian-Lagrangian time-scale transformation is smaller than unity in the Southern Hemisphere.

The normalized energy spectra of the zonal and meridional components of the Eulerian velocities are computed and are shown in Figs. 15 and 16. It may be noted that in the high-frequency range the Eulerian spectra are approximately proportional to n^{-1}, whereas the Lagrangian spectra are proportional to n^{-3}. The same characteristic holds in the Northern Hemisphere (Kao, 1965).

The cross-correlation function and normalized cospectrum of the Eulerian zonal and meridional velocities are shown, respectively, in Figs. 17 and 18. The positive values of the cospectrum in the low-frequency end indicate that the large eddies tend to disperse the particles toward the ENE, whereas the negative values of the spectrum in the high-frequency end indicate that the small eddies tend to disperse the particles toward the ESE.

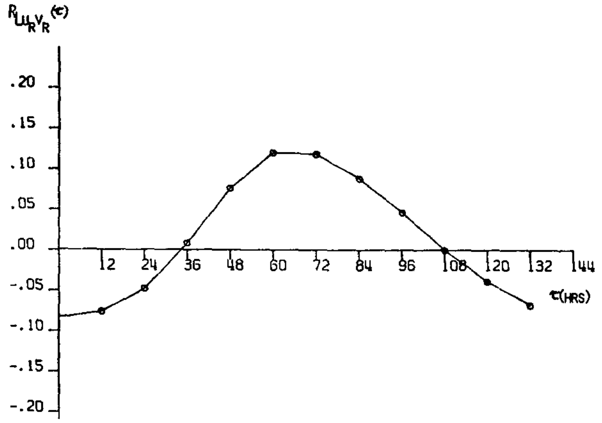

FIG. 11. Lagrangian cross-correlation function of the relative zonal and meridional velocities between particles.

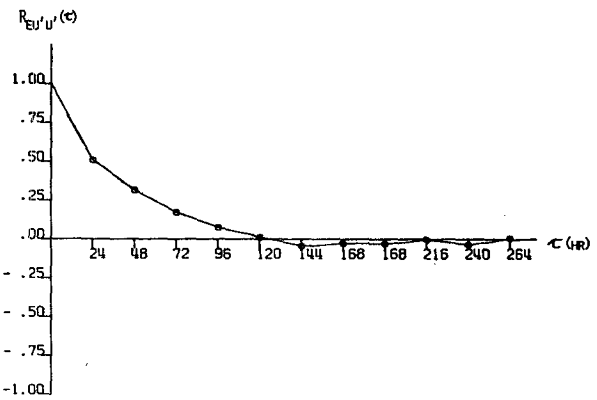

FIG. 13. Eulerian autocorrelation function of the zonal velocity.

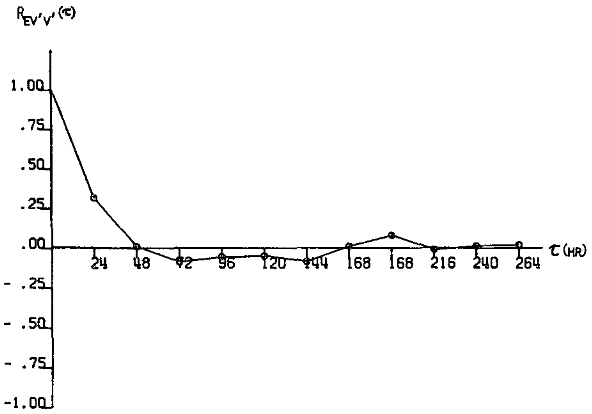

FIG. 14. Eulerian autocorrelation function of the meridional velocity.

7. The Eulerian-Lagrangian time-scale transformation

Based on the similarity in the distribution of the autocorrelation functions of the Eulerian and Lagrangian velocities, Hay and Pasquill (1959) suggested the Eulerian-Lagrangian transformation,

$$R_L(\zeta)=R_E(\tau), \tag{5}$$

with

$$\zeta=B\tau,$$

where R_E and R_L are the Eulerian and Lagrangian velocity correlation functions, respectively, and B a coefficient for the Eulerian-Lagrangian time-scale transformation. Experiments show that the value of B is about 0.6 for the large-scale atmospheric motion (Kao and Bullock, 1964; Murgatroyd, 1969).

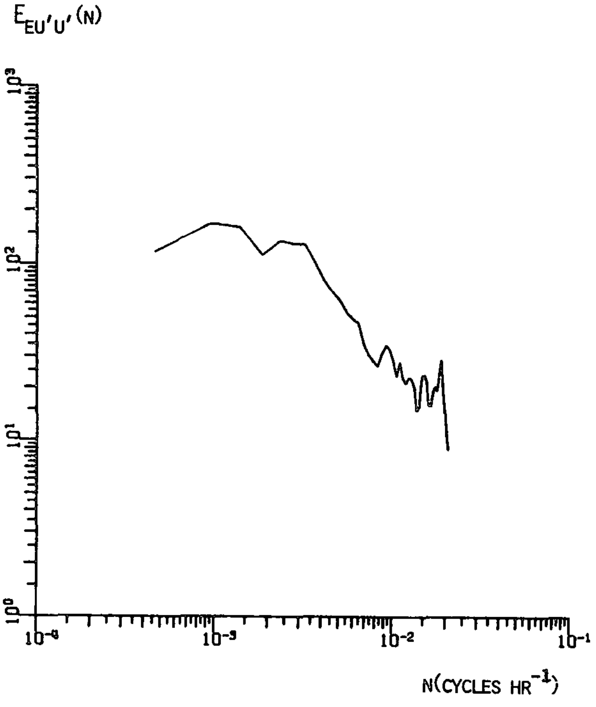

FIG. 15. Normalized Eulerian energy spectrum of the zonal velocity.

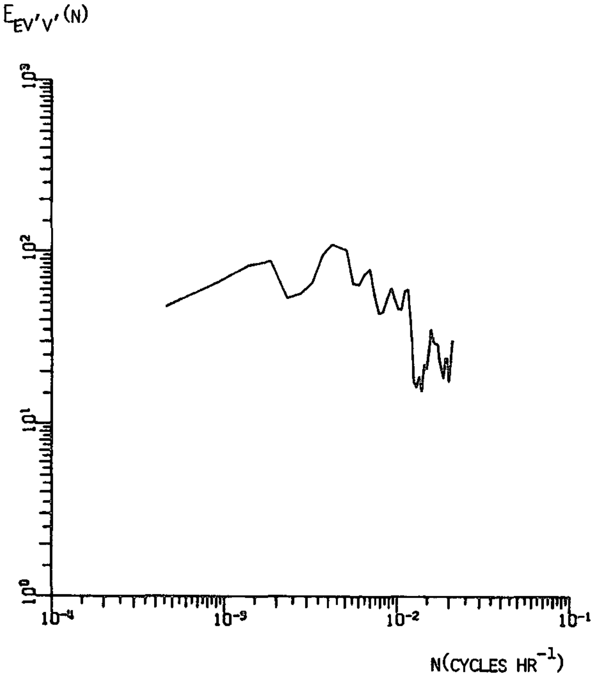

FIG. 16. Normalized Eulerian energy spectrum of the meridional velocity.

It can be shown, by applying the Fourier transform,

$$E(n)=4\int_0^\infty R(\tau)\cos(2\pi n\tau)d\tau, \tag{6}$$

to Eq. (5) that

$$E_L(n)=BE_E(Bn), \tag{7}$$

where E_E and E_L are the normalized Eulerian and Lagrangian energy spectra, respectively.

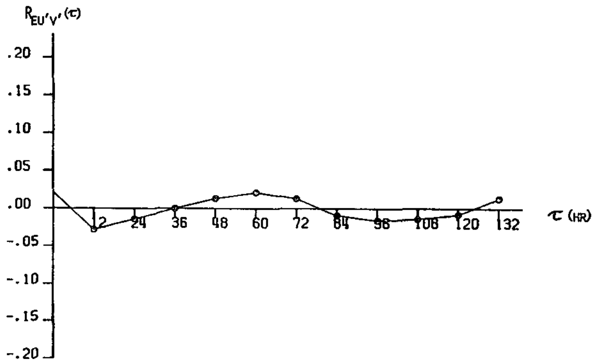

FIG. 17. Eulerian cross-correlation function of the zonal and meridional velocities.

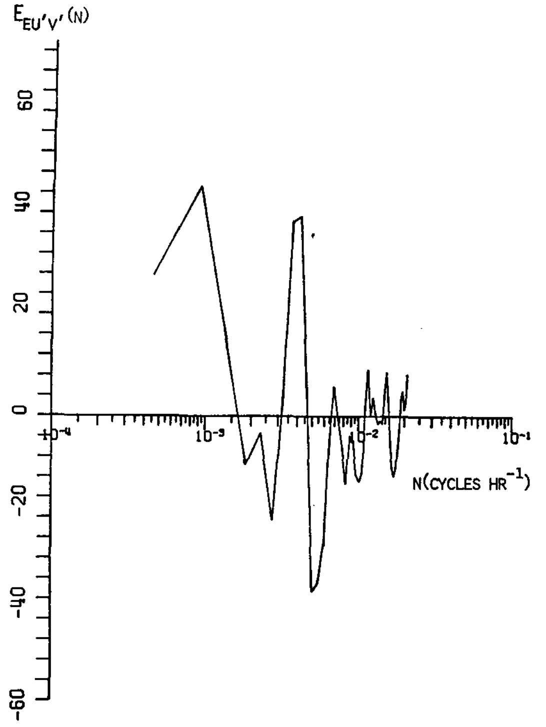

FIG. 18. Normalized cospectrum of the Eulerian zonal and meridional velocities.

In a planetary wave model, Kao (1965) has shown that coefficient B may be approximated by

$$B \approx \left| 1 - \frac{U\bar{N}^2}{2\Omega r \cos^3\bar{\phi}} \right|, \tag{8}$$

where U is the mean zonal velocity, $\bar{N}$ the mean angular wavenumber of the planetary waves, Ω the angular velocity of the earth, r the radius of the earth, and $\bar{\phi}$ the mean latitude of the trajectories. It has been shown that in the Southern Hemisphere the large-scale atmospheric motion is primarily dominated by waves of wavenumbers 1–7 (Anderssen, 1965; van Loon, 1965, 1967). Therefore, the mean wavenumber $\bar{N}=4$. At $\bar{\phi}=50$S and a pressure of 200 mb, $U=25$ m sec^{-1}. The value of B computed with the use of (8) is 0.60, which is about the same as that in the Northern Hemisphere.

To check the value of B determined above, we shall use Eq. (7), in which the maximum value of the normalized spectrum of the Eulerian velocity must correspond to that of the Lagrangian. Therefore, B may be determined from the ratio of the maximum value of $E_{Lv'v'}$ to that of $E_{Ev'v'}$. At 200 mb in the Southern Hemisphere, the maximum value of $E_{Lv'v'}$ ≈ 70 hr cycle^{-1}, and that of $E_{Ev'v'} \approx 110$. Therefore, $B=0.64$, which agrees well with that determined with the use of (8).

The value of B may also be determined from the ratio of the Lagrangian to Eulerian integral time-scale. Because of the large time-interval (24 hr) between observations, the integral time-scales and, therefore, the value of B cannot accurately be determined.

8. Summary

An analysis of the Eulerian and Lagrangian velocities at 200 mb in the Southern Hemisphere has been made. It is found that: 1) the zonal component of the eddy diffusivity in the mid-atmosphere in the Southern Hemisphere is about 50% greater than that in the Northern, whereas the meridional component of the eddy diffusivity is about 50% smaller than that in the Northern; 2) the coefficient for the Eulerian-Lagrangian time-scale transformation in the Southern Hemisphere is about 0.6 which is about the same as that in the Northern; and 3) the autocorrelation functions and energy spectra of the Eulerian and Lagrangian velocities in the Southern Hemisphere are similar to those in the Northern.

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