Pressure Perturbations and Buoyancy in Convective Clouds

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To treat the pressure field in small-scale atmospheric motions (\(\sim 10 \text{ km}\)) as being horizontally homogeneous with that of the hydrostatic environment, is not satisfactory for the case of convective clouds where updrafts are of the order of 10 m sec\(^{-1}\) or greater (see Ogura and Phillips, 1962; and more recently, Dutton and
Fichtl, 1969). The horizontal accelerations resulting from the convection imply the existence of (horizontal) pressure gradients which cannot be ignored. It should be recognized that the hydrostatic pressure within a cloud is different from that outside, and also that friction affects the nonhydrostatic pressures within the cloud and precipitation region.

The following general form of the Navier-Stokes equation is used to analyze the perturbation pressure and buoyancy terms:

\[
\frac{dv}{dt} = -gk \frac{1}{\rho} \nabla p + d + f
\]

(1)

where \( v \) is the air velocity vector, \( g \) the acceleration of gravity, \( k \) the unit vector in the vertical direction, \( \rho \) air density, \( p \) air pressure, \( d \) the hydrometer drag per unit mass of air, and \( f \) the shearing stress per unit mass, e.g., viscous and Reynolds stresses. It is assumed that \( p, \rho \) and the absolute temperature \( T \) can be described by small perturbations from a reference state (indicated by a subscript \( e \)) which is in hydrostatic equilibrium and horizontally homogeneous. Thus

\[
\begin{align*}
T(z,x,t) &= T_e(z)+\hat{T}(z,x,t), \quad \hat{T} \ll T_e \\
\rho(z,x,t) &= \rho_e(z)+\hat{\rho}(z,x,t), \quad \hat{\rho} \ll \rho_e \\
\rho(z,x,t) &= \rho_e(z)+\delta(z,x,t), \quad \delta \ll \rho_e
\end{align*}
\]

(2)

where \( z \) is the vertical, \( x \) the horizontal coordinate, \( e \) (e.g., \( x,y \) or \( x, \theta \), etc.) and \( T_e, \rho_e, \rho_e \) are prescribed functions of height.

We proceed as follows: 1) substitute for \( \rho \) and \( p \) from (2) and neglect all but first-order terms in the perturbation quantities; 2) split the gradient operator into a horizontal (\( \nabla_h \)) and a vertical (\( \partial/\partial z \)) component; 3) take account of the definition of the environment pressure, \( \nabla_h \rho_e = 0 \) and \( (\partial \rho_e/\partial z) = -\rho_e g \); and 4) write the ideal gas law for the air for small perturbations in the form \( \rho/\rho_e = \rho/\rho_e - \hat{T}/T_e \). After these operations Eq. (1) becomes

\[
\frac{dv}{dt} = -g \nabla_h \hat{\rho} \frac{1}{\rho_e} \frac{\partial \hat{T}}{\partial z} \frac{k}{\hat{\rho}} \hat{T} \nabla_h k + d + f
\]

(3)

i.e., the total acceleration is determined by the horizontal and vertical gradients of the pressure perturbation, the buoyancy (two terms), and the accelerations due to hydrometeor drag and shear forces.

It has been customary in most models of cumulus convection to neglect the term \( (1/\rho_e) (\partial \hat{T}/\partial z) \nabla_h k \) and also to drop the second pressure perturbation term \( \hat{\rho}/\rho_e \nabla k \) on the assumption that \( \hat{\rho}/\rho_e \approx \hat{T}/T_e \). This has happened, particularly in one-dimensional models of clouds, where these simplifications were made quite arbitrarily by the assumption of horizontal pressure surfaces or without even recognizing the “pressure problem” (e.g., Srivastava, 1967). This difficulty has been avoided in two-dimensional numerical modelling where the vorticity equation is used as a basis (Orville, 1968; Arnason et al., 1968), since this equation guarantees proper handling of the perturbation pressure gradients. These models, however, have been restricted explicitly to shallow convection and do not properly account for the \( \hat{p}/\rho_e \nabla k \) term in larger convective clouds.

In order to demonstrate that the magnitude of the two pressure perturbation terms in the \( z \) component of (3) necessitates their inclusion in serious attempts at modelling clouds with heavy precipitation, we refer to Barnes (1969), who measured a perturbation pressure of 3 mb at about 5.5 km (500 mb) in a cumulonimbus. This would result in a contribution to the buoyancy acceleration of \( \hat{p}/\rho_e g \approx 0.06 \text{ m sec}^{-2} \). By comparison, a 2C temperature perturbation in the updraft at the \(-30 \text{C} \) level would result in a buoyancy acceleration of \( \hat{T}/T_e g \approx 0.08 \text{ m sec}^{-2} \). Lozowski and List (1969) showed that the hydrometeor-induced perturbation pressure gradient is of the same order of magnitude as the drag force. Therefore, it may be expected that \( (1/\rho_e) (\partial \hat{T}/\partial z) \) is of the order of 0.03 \text{ m sec}^{-2} for \( d_e = 0.03 \text{ newtons kg}^{-1} \), and hence also of the order of the buoyancy.

Since the perturbation pressure can be represented as the sum of a perturbation hydrostatic pressure \( \hat{p}_h \) and a perturbation nonhydrostatic pressure \( \hat{p}_n \), the former can be linked to the density perturbation through the relation

\[
\frac{\partial \hat{p}_h}{\partial z} = -\hat{\rho} g
\]

(4)

This is equivalent to assuming the following vertical equation of motion for the updraft:

\[
\frac{dw}{dt} = \frac{1}{\rho} - (\partial \hat{p}_n/\partial z) + d_z + f_z
\]

(5)

Buoyancy which appears in the \( z \) component of (3) does not occur in (5); the vertical motion is directly linked to the vertical nonhydrostatic pressure gradient, and the vertical components of drag and friction per unit mass.

For the horizontal motion, where gravity and hydrometeor drag are of no consequence (if the drag is assumed to have no horizontal component), an integration from the distant environment to \( x = \xi \) may be performed. Starting from (3), and making use of the boundary condition \( \hat{p}|_{\xi} = 0 \), one obtains

\[
\hat{p}|_{\xi} = -\rho_e \int_{\xi}^{\xi} \left( \frac{du}{dt} - f_z \right) dx
\]

(6)

The perturbation pressure at the point \( \xi \) and any level \( z \) is proportional to the average net horizontal accelerations.

Differentiating (6) with respect to \( z \) gives

\[
\frac{\partial \hat{p}}{\partial z} = -\frac{\partial}{\partial z} \left[ \rho_e \int_{\xi}^{\xi} \left( \frac{du}{dt} - f_z \right) dx \right]
\]

(7)
indicating that the perturbation pressure gradient which enters the vertical component of the equation of motion (3) is most important where the net horizontal accelerations are strong and vary considerably with height. Such conditions occur particularly in the upper and lower portions of convective clouds, and also in the vicinity of high hydrometeor concentrations. The latter consideration was examined by Lozowski and List (1969), whose results are in agreement with the above conclusion.

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Summary

1) A discussion of the z component of the equation of motion applicable to updrafts, Eq. (3), shows that the two terms containing pressure perturbations can no longer be neglected in the modelling of large convective clouds with precipitation.

2) Dropping the traditional relation of updrafts with buoyancy leads to an unrestricted equation for the vertical, which links the motion to the gradient of the "in-cloud" nonhydrostatic pressure, the drag, and the viscous and turbulent friction forces, Eq. (5).

3) Integration of the equation of motion in the horizontal produces a Bernoulli-type expression, Eq. (6), which relates integrated net horizontal accelerations to total perturbation pressures.

4) A strong vertical gradient of the perturbation pressure is linked with strong net horizontal accelerations which vary considerably with height, Eq. (7).

REFERENCES


