indicating that the *perturbation pressure gradient* which enters the vertical component of the equation of motion (3) is most important where the net horizontal accelerations are strong and vary considerably with height. Such conditions occur particularly in the upper and lower portions of convective clouds, and also in the vicinity of high hydrometeor concentrations. The latter consideration was examined by Lozowski and List (1969), whose results are in agreement with the above conclusion.

Summary

1) A discussion of the z component of the equation of motion applicable to updrafts, Eq. (3), shows that the two terms containing *pressure perturbations can no longer be neglected in the modelling of large convective clouds with precipitation.*

2) Dropping the traditional relation of updrafts with buoyancy leads to an unrestricted equation for the vertical, which links the motion to the gradient of the "in-cloud" nonhydrostatic pressure, the drag, and the viscous and turbulent friction forces, Eq. (5).

3) Integration of the equation of motion in the horizontal produces a Bernoulli-type expression, Eq. (6), which relates integrated net horizontal accelerations to total perturbation pressures.

4) A strong vertical gradient of the perturbation pressure is linked with strong net horizontal accelerations which vary considerably with height, Eq. (7).

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The Flashing Behavior of Thunderstorms

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ABSTRACT

The distribution in time of individual lightning flashes recorded visually during 20 storms in New Mexico and on 23-cm radar screens during several storms in the south-central United States has been analyzed. The logarithms of the intervals between flashes within a given storm are normally distributed with a standard deviation σ of approximately one natural-log unit. The available data do not reject the hypothesis that there are no statistically significant variations in σ among storms. The autocorrelation of the intervals between flashes in one storm is very small. The implications of these findings are briefly explored.

1. Introduction

This note is an outcome of a study conducted at Stanford Research Institute (SRI) of the feasibility of observing lightning from satellites (Dennis, 1964). The study differs considerably from others which have been conducted on the same problem (Whiteman and Fryberger, 1961; Hallgren and MacDonald, 1963; Kirkwood, 1965) in that an attempt was made to find the sampling time necessary for the determination of flashing rates. The required sampling time is obviously related to the flashing behavior of thunderstorms. All available data on flashing rates, whether obtained by visual observations, by lightning counters, or through the recording of lightning channels on radar screens, were taken into account. In addition, radar data were analyzed to determine the extent, duration and distribution of complexes of convective cells in thunderstorm situations.

The results of the various analyses can be summarized as follows: Typical thunderstorms are rather small, involving 100–200 km^2 of precipitating cloud. Only one or two cells are electrically active at any one time. The frequency distribution in terms of flashing rates is asymmetric, with the majority of thunderstorms producing less than three or four flashes per minute, but with a few producing over 100 flashes min^{-1}. The storms with high flashing rates do not necesssarily involve a large number of cells; some of the highest rates have been recorded in storms which showed only one cell on a weather radar screen (Mackerras, 1963). In those cases where high flashing rates occur in multicelled storms, the flashes are not uniformly distributed among the cells; rather, one or two cells contribute the majority of the flashes during any given 5-min period.

2. Statistical analysis of flash intervals

A statistical analysis of the times of individual lightning flashes has been made for 20 storms that occurred in New Mexico during 1958, using times of flashes recorded visually to the nearest second at Socorro by personnel of the New Mexico Institute of Mining and Technology. The maximum separation between flashes belonging to the same storm has been set at 15 min. The records for daylight hours include only flashes to ground. The nighttime observations include both ground and cloud flashes. Weather radar data in the SRI Film Library from a site about 160 km from Socorro cover some of the 20 storms. They show, in each instance, only one shower complex likely to produce lightning visible from Socorro.

As normal functions are relatively easy to handle, various parameters related to the flashing rates have been tested for normality. The logarithms of the intervals between successive flashes within a given storm are found to possess this property. (The tests used reveal a tendency for observers to record time to the nearest 5 sec, rather than to the nearest second). The log-normal distribution of intervals between flashes is shown, either implicitly or explicitly, in the data of several authors including Brook and Kitagawa (1960), Norinder and Knudsen (1958) and Lakshminarayan (1962). If t_i is the ith interval between flashes, and if we define

$$x_i = \ln t_i, \qquad (1)$$

then the x_i's are normally distributed. The mean x_i's over individual storms in the sample range from 3.18–5.68, corresponding to median flash intervals between 24 and 290 sec. Let the standard deviation of the x_i's be denoted by σ. The numerical calculations show that σ is approximately one natural-log unit. Although it varies within the range 0.5–1.5 for the various storms, the available data do not reject the hypothesis that there are no statistically significant variations in it. Tests with the available data show no correlation between the value of σ and the flashing rate of a particular storm.

Autocorrelation coefficients for the x_i's have been computed for each storm over various lags from 1–20. The autocorrelation for lag 1 averaged over the 20 storms is 0.12; that for lag 2 is 0.11.

In addition, a mean autocorrelation coefficient over all possible lags has been computed for each storm according to

$$\bar{\rho}_k = \frac{2}{k(k-1)} \sum_{\delta=1}^{k-1} (k-\delta)\rho_\delta, \qquad (2)$$

where ρ_δ is the autocorrelation of the intervals with lag δ and k the number of intervals between flashes for the storm. No statistical significance can be attached to the variations in $\bar{\rho}_k$, which ranges from 0–0.2 for the different storms. The very low values of $\bar{\rho}_k$ indicate that intervals between flashes throughout a given storm are essentially independent of one another.

The data needed to examine thoroughly the statistics of lightning flashes in areas containing several simultaneously active storms are not available. A limited check has been made, using the times of occurrence of lightning echoes observed on radar in three thunderstorm complexes occurring simultaneously in the south-central United States, the lightning echoes being recorded on 23-cm radar. Consideration of beam width and rates of antenna rotation indicate that approximately one flash in 20 was recorded (Ligda, 1956). The logarithms of the intervals between echo occurrences in each individual complex satisfy the tests for normality and show a standard deviation near one natural-log unit. The intervals between the occurrence of successive echoes anywhere within the three complexes also follow the log-normal distribution with a standard deviation near one natural-log unit, that is, they are statistically indistinguishable from those of a single storm.

3. Implications of the statistical analysis

The most reasonable assumptions concerning any sample of lightning observations for a given area, regardless of whether all flashes are recorded or not, are that the flash intervals are drawn from a log-normal distribution with a standard deviation near one natural-log unit and that the observed intervals are statistically independent. Assuming that the central limit theorem applies, it is necessary to observe about 20 flashes in order to specify a flashing rate to within a factor of 2 at the 5% confidence level. This requirement can be significant in cases where sferics equipment is used to plot thunderstorm activity as a function of azimuth from the receiving station (Kohl and Miller, 1963). It also limits the spatial resolution attainable from a low-orbit lightning-detecting satellite, regardless of the type of sensor employed. Examination of typical flashing rates per unit area in thunderstorm regions shows that it would generally be unrealistic to specify flashing rates for areas <150 km on a side on the basis of low-orbit satellite observations (Dennis, 1964).

The low autocorrelations among flash intervals in single-cell storms show that the occurrence of a flash, or of several flashes close together, has little bearing upon the time of the next flash in the same cell. This is, of course, in line with the results of Kuettner (1950), who identified localized regions of 1 km or so diameter within thunderstorms as the basic discharge units. The local charge concentrations in thunderstorms are undoubtedly related to the small-scale variations in the precipitation rate, but it is probably an oversimplification to expect a one-to-one correspondence between the electrical and precipitation sub-cells. [See, however, Imyanitov and Shifrin (1962).]

In the absence of detailed information on the small-scale distribution of charge within a thunderstorm, the occurrence of an individual flash must be considered as a random phenomenon. It is generally agreed that the initiation of a lightning flash requires a local intensification of the electric field which leads to a dielectric breakdown of the air (e.g., Hagenguth, 1951). As charges are carried about by cloud particles, by precipitation, and by the air itself, it can readily be seen that the electrical conditions inside a cloud, particularly as they relate to the probability of a dielectric breakdown, can change on a time scale of the order of seconds. Conditions outside the cloud are also of some importance; the changing location of a thundercloud with respect to hills, tall buildings, regions of high ground conductivity, and nearby clouds undoubtedly plays a part in the timing of its lightning discharges. For these reasons, little meteorological significance can be attached to details of the time distribution of the flashes in a particular storm.

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