Transmission of Submillimeter Waves Through Water Clouds and Fogs

C. M. R. Platt

Division of Meteorological Physics, CSIRO, Aspendale, Australia

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ABSTRACT

Some computations are made of submillimeter wave extinction in clouds and fogs using recent spectrometric results of the optical properties of water. At wavelengths $>1000\,\mu$, an approximate formula is adequate in which extinction is proportional to cloud water content. At 2000 and 1000 μ the extinction is 6.5 and 15.2 dB km^{-1} per gm m^{-3}, respectively. Between 200 and 1000 μ additional extinction occurs due to large droplets of diameters $>20\,\mu$. Extinction for a typical fog distribution is computed and is found to be 41.1 dB km^{-1} per gm m^{-3} at 337 μ and 92.8 dB km^{-1} per gm m^{-3} at 200 μ. Comparisons with experimental data at 1200 and 337 μ shows qualitative agreement, but insufficient data on the composition of the clouds and fogs investigated precludes accurate comparison.

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1. Introduction

The absorption and scattering of electromagnetic radiation by water spheres and fogs has been extensively studied. The full Mie theory has been discussed in detail by van de Hulst (1957). Deirmendjian (1964) and Herman (1962) have discussed extinction and scattering by water droplets in the visible and infrared regions of the spectrum, respectively, and Gunn and East (1954) have discussed the absorption and scattering of microwaves by clouds and rain.

The purpose of this article is to present some computations of the attenuation by water clouds and fog in the submillimeter wave region and to compare them with the only two experimental investigations of which the author is aware, at 1200 and 337 μ, respectively. The possibility of using submillimeter wave techniques to investigate cloud properties, as well as the development of extreme infrared astronomy (Bastin *et al.*, 1964), seem to make such a study worthwhile.

2. The Mie theory of droplet extinction

The following definitions are needed in a discussion of Mie theory: 1) the size parameter $\alpha=2\pi a/\lambda$, a being the droplet radius and λ the wavelength, and 2) the extinction efficiency Q_{ext}, which is defined by $C_{\text{ext}}/(\pi a^2)$, where C_{ext} is the total extinction cross section. Similarly, the scattering efficiency is defined by $Q_{\text{sca}}=C_{\text{sca}}/(\pi a^2)$, and the absorption efficiency by $Q_{\text{abs}}=C_{\text{abs}}/(\pi a^2)$. When the size parameter $\alpha\leqslant 0.7$ (see Section 4a), Q_{ext} is given by (van de Hulst, 1957, p. 274)

$$Q_{\text{ext}}=-\text{Im}\left\{4\alpha\kappa+(4/15)\alpha^3\left[\kappa^2\left(\frac{m^4+27m^2+38}{2m^2+3}\right)\right]\right\} +(8/3)\alpha^4\,\text{Re}(\kappa^2)+\cdots, \tag{1}$$

where m is the complex refractive index, $n-ik$, $\kappa=(m^2-1)/(m^2+2)$, and Im and Re stand for the imaginary and real parts, respectively. Also,

$$Q_{\text{sca}}=(8/3)\alpha^4|\kappa|^2,\quad (\alpha\leqslant 0.7), \tag{2}$$

$$Q_{\text{abs}}=Q_{\text{ext}}-Q_{\text{sca}}. \tag{3}$$

When $\alpha\leqslant 0.2$ then only the first term of (1) is significant and

$$Q_{\text{ext}}=-4\alpha\,\text{Im}(\kappa). \tag{4}$$

Furthermore, Q_{sca} is then negligible and $Q_{\text{abs}}=Q_{\text{ext}}$.

Now the intensity of plane-parallel radiation is reduced from I to I_0 in travelling a distance dx through a cloud of water droplets such that

$$\log_e(I/I_0)=\sum_{a=1}^{n} N_a C_{\text{ext}}(a)dx, \tag{5}$$

where N_a is the number of droplets of radius a, C_{ext} (a) their extinction efficiency, and a must be summed over all droplet radii in the cloud.

If $\alpha\leqslant 0.2$, (4) can be written as

$$C_{\text{ext}}=-\frac{8\pi^2a^3}{\lambda}[\text{Im}(\kappa)]$$

$$=-\frac{6\pi V_a}{\lambda}[\text{Im}(\kappa)], \tag{6}$$

where V_a is the droplet volume. From Eqs. (5) and (6)

$$\log_e(I/I_0)=-\frac{6\pi M}{\rho\lambda}[\text{Im}(\kappa)], \tag{7}$$

where M is the mass of water (density ρ) per unit volume of cloud. In this case, attenuation is proportional to M.

TABLE 1a. The optical constants of water at 20C.

λ (μ)	n	γ_λ (cm^{-1})	k
200	2.09±0.1	325±5	0.52±0.03
337	2.13±0.05	200±5	0.54±0.03
500	2.20±0.1	190±5	0.76±0.04
1000	2.50±0.05	120±5	0.95±0.07
2000	3.10±0.1	92±5	1.46±0.07

TABLE 1b. Temperature variation of n and k at three wavelengths.

	T (°C)	Wavelength 100 μ[a]	337 μ[b]	6.2 mm[c]
n	20	—	2.15	4.44
	10	—	2.14	3.94
	0	—	2.13	3.44
k	20	0.53	0.54±0.03	2.59
	10	0.48	0.42±0.05	2.37
	0	—	0.32±0.1	2.04

[a] Draegert *et al.* (1966).
[b] Chamberlain *et al.* (1966).
[c] Gunn and East (1954).

3. The optical constants of water (200–2000 μ)

Values of n and k for this region are very few in number. Recently, Chamberlain *et al.* (1966) have reported measurements of n at 337 μ and 118 μ and these values, together with some previous millimeter wave measurements which are reviewed by Chamberlain *et al.*, enable the interpolation of values at intermediate wavelengths to a fair degree of accuracy.

Chamberlain *et al.* have measured the volume absorption coefficient of water (γ_λ) from 100–1000 μ using an interferometer. Values of γ_λ and $k[=\gamma_\lambda\lambda/(4\pi)]$ taken from their curves are shown in Table 1a. The value for 2000 μ was taken from Stanevich and Yaroslavskii (1961).

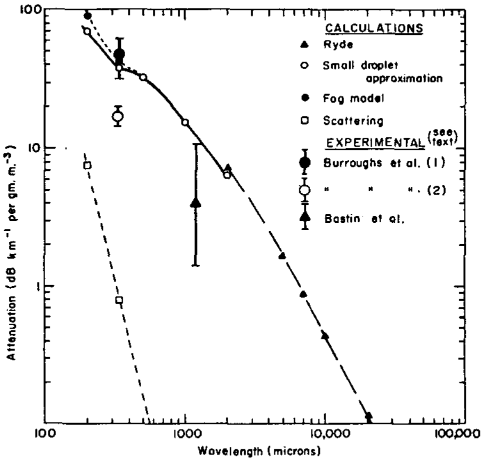

FIG. 1. Submillimeter-wave and millimeter-wave attenuation by clouds and fog. Ryde's values are quoted by Gunn and East (1954). Burroughs *et al.* (1) refers to Eldridge's water content results and Burrough's *et al.* (2) to Houghton and Radford's results.

TABLE 2. Calculated attenuation for $\alpha \leqslant 0.2$.

λ (μ)	κ	Attenuation (dB km^{-1} per gm m^{-3})
200	0.666–0.17i	69.6
337	0.576–0.155i	37.5
500	0.626–0.198i	32.4
1000	0.712–0.186i	15.2
2000	0.834–0.158i	6.5

The temperature at which the above measurements were made was 20C. The temperature dependence of n and k is obviously important. What seems to be the only existing information on this topic is summarized in Table 1b. The effect of the variations of n and k on attenuation is described in Section 4a.

4. Calculations

a. Extreme infrared attenuation by water clouds and fog

The calculated attenuation for $\alpha \leqslant 0.2$ is shown as the full line in Fig. 1, the expression for the attenuation being given by

$$\frac{8.18}{\lambda}\,\mathrm{Im}(\kappa)\ [\mathrm{dB\ km^{-1}\ per\ gm\ m^{-3}}] \qquad (8)$$

[from Eq. (7)]; values are also shown in Table 2. The validity of (8) will depend on the droplet distributions of the clouds in question and the wavelength. From values of maximum droplet diameter, $d_{\max}$, (e.g., Mason, 1962) for various types of cloud, the minimum values of λ for which either $\alpha \leqslant 0.2$ or $\alpha \leqslant 0.7$ is valid can be determined. Typical values are shown in Table 3. Values of $d_{\max}$ for fogs[1] of two different types are taken from Houghton and Radford (1938) and Eldridge (1966).

Except for heavy cumulonimbus clouds, Eq. (8) is apparently adequate for wavelengths $>1000\,\mu$. For shorter wavelengths it is necessary to take account of additional attenuation by large drops by the use of Eq. (1), and $\alpha \leqslant 0.7$ is satisfied down to 200 μ in most cases[2].

The second and third terms of (1) are nonlinear in the droplet volume so that a droplet size distribution has to be specified. A typical fog distribution based on the measurements of Arnulf *et al.* (1957) is given in Table 4. [There would be, of course, many drops of diameter $<20\,\mu$, but their extinction properties are covered by Eq. (4)].

The submillimeter wave attenuation at 200 and 337 μ for this droplet distribution has been calculated from (1), the contributions from each term being shown in Table 5. The total water content is assumed to be, quite

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[1] A fog is defined as a cloud which has formed at ground level and which restricts the visibility of an observer to 1000 m or less.
[2] A recent discussion covering computation for any value of α is given by Kattawar and Plass (1967).

TABLE 3. Minimum wavelengths at which the given approximations hold.

Cloud type	$d_{\max}$ (μ)	$\lambda_{\min}(\alpha \leqslant 0.2)$ (μ)	$\lambda_{\min}(\alpha \leqslant 0.7)$ (μ)
Stratus	90	1440	410
Stratocumulus	24	375	116
Cumulus	60	950	270
Cumulonimbus	200	3140	1003
Altostratus	30	470	135
Fog (natural)	200	3140	1003
Fog (small particle)	50	780	225

arbitrarily, 0.33 gm m^{-3}, to which the large droplets contribute 0.24 gm m^{-3}. The broken line in Fig. 1 illustrates the attenuation. Attenuation due to scattering for the distribution of Table 4 has also been calculated and is illustrated as the dotted line in the same figure.

Finally, the change in attenuation with temperature can be estimated for the three wavelengths in Table 1b. For a decrease in temperature from 20 to 10C, the attenuation decreases by 5% at 100 μ, decreases by about 25% at 337 μ, but *increases* by 25% at 6.2 mm. At 1000 μ it is thus probable that the change is less than at either 337 μ or 6.2 mm.

b. The effective precipitable water absorption coefficient, γ_λ'

A cloud of thickness 1km and water content 1 gm m^{-3} is equivalent to approximately 0.1 cm depth of precipitable water. Let γ_λ' be defined by

$$\log_e(I/I_0) = -\gamma_\lambda' dx',$$

where dx' is the depth of precipitable water. Then, when $\alpha < 0.2$, γ_λ' (cm^{-1}) can be written as

$$\gamma_\lambda' = -10(6\pi/\lambda)[\mathrm{Im}(\kappa)]$$

[from (7)].

Values of γ_λ' and γ_λ are compared in Table 6, γ_λ' being consistently lower than γ_λ for small droplets. This has also been found to be so in the near infrared (Herman, 1962) when γ_λ is appreciable, even when there is also appreciable scattering.

5. Experimental investigations

a. Measurements at 1200 μ (Bastin *et al.*, 1964)

TABLE 4. Part of a typical fog distribution for $d > 20\,\mu$ (after Arnulf *et al.*, 1957).

Droplet diameter range (μ)	d (μ)	Number (cm^{-3})	Mass (gm m^{-3})	α (337 μ)	α (200 μ)
20–24	22	6	0.0335	0.205	0.345
24–30	27	9	0.093	0.252	0.422
30–40	35	5	0.110	0.314	0.550

TABLE 5. Attenuation from Table 4, illustrating the contributions from each term of Eq. (1). Units, dB km^{-1} per gm m^{-3}.

λ (μ)	1st term	2nd term	3rd term	Total
200	69.6	15.5	7.7	92.8
337	37.5	2.8	0.8	41.1

Bastin *et al.* measured the attenuation of solar radiation by clouds with a ground-based telescope, the radiation bandwidth of the detector being restricted to $\sim\pm 400\,\mu$. A comparison solar signal through clear sky was taken both before and after the cloud transmission measurement. The fractional transmissions selected from their Table 2 through three grades of cumulus, together with the estimated attenuation using reasonable values of water content and cloud thickness, are shown in Table 7. The upper and lower limits are also estimated from available cloud data. The low mean value may reflect the fact that the original cloud "types" were only rough assessments by ground-based observers.

Cloud *emission* would tend to give a low value for the attenuation; however, as the telescope beamwidth was less than the solar disc, a calculation shows that the error varies from only 5% for 0.8 transmission to 25% for 0.05 transmission. The extra *water vapor* attenuation through a saturated cloud compared to unsaturated clear sky would tend to give an attenuation value too high. [The water vapor attenuation at 1200 μ is ~ 2 db km^{-1} for saturated air, 8C and 1 atm (Bastin, 1966).]

b. Measurements at 337 μ (Burroughs *et al.*, 1966)

These authors studied the transmission of CN laser radiation through fog over a horizontal path near the ground. A correction of 54 dB was necessary for water vapor attenuation.

The visibility was measured simultaneously. The results of their measurements are shown in Table 8. The third column gives the deduced fog water content assuming the droplet distribution of Table 4 and the resulting attenuation from Mie theory of 41 dB km^{-1} per gm m^{-3}.

These results can be compared with the visibility–water content relationships obtained by other workers. Houghton and Radford (1938) measured water content directly. Eldridge (1966) computed fog droplet distributions from the visible and near infrared spectral transmission measurements of Arnulf *et al.* (1957). The computed values agree well with the direct measure-

TABLE 6. Comparison of γ_λ and γ_λ' (Section 4b).

λ (μ)	γ_λ (cm^{-1})	γ_λ' (cm^{-1})
200	325	160
337	200	86
500	190	74
1000	120	35
2000	92	15

TABLE 7. Measurements of Bastin *et al.* (1964) and estimated attenuation.

Cloud type	Fractional transmission	Attenuation (dB km^{-1}	Outer limits per gm m^{-3})
Light cumulus	0.90	1.82	15 0.5
Medium cumulus	0.70	2.5	6.8 1.2
Heavy cumulus	0.35	4.6	10.4 2.3
Mean		3	11 1

ments of Arnulf *et al.*, who used the spider web technique (Table 4 is based on these values), except for the large droplets, to which the computation method is apparently not very sensitive. Hence, the computed values of water content would tend to be low. However, there still would appear to be an appreciable difference between the water content–visibility relationships of Houghton and Radford and Eldridge, which are shown[3] plotted in Fig. 2, together with Burroughs *et al.*'s results. Now George (1951) has pointed out that "dry," low humidity, industrial-type fogs have many more small droplets and a low visibility compared to "wet" natural fogs of the same water content, but containing larger and fewer droplets. Houghton and Radford state that their measurements were made on either frontal or advection fogs and droplet diameters ranged up to 100 μ, consistent with "wet" type fogs. Arnulf *et al.* made measurements near Paris, but did not state the meteorological conditions.

The measurements of Burroughs *et al.* are fairly consistent with Eldridge's results but not with Houghton and Radford's. The attenuations inferred from both the above authors' curves, assuming their water content–visibility relationships to be correct, are plotted in Fig. 1. Due to the presence of droplets in the 50–100 μ range, the theoretical attenuation would be >41 dB km^{-1}, so that the discrepancy in the Houghton and Radford value is actually greater than that shown.

Thus, assuming that Burroughs *et al.*'s measurements were made in fog with droplet distributions not too unlike those of Arnulf *et al.*, the results agree quite reasonably with the Mie theory.

TABLE 8. Measurements of Burroughs *et al.* with water content deduced from theory.

Estimated visibility (m)	Measured attenuation (dB km^{-1})	Inferred water content (gm m^{-3})
35	25±8	0.61±0.2
70	14±5	0.34±0.1

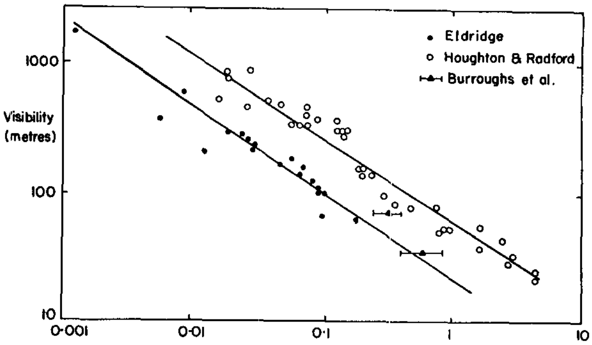

FIG. 2. Water content in fogs vs horizontal visibility.

6. Conclusions

The extreme infrared attenuation by clouds and fog increases markedly from wavelengths 2000 through 200 μ. The region is one of transition between cloud transparency in the microwave region to near opacity in the infrared. As such, attenuation, particularly at 1000–2000 μ, varies widely with both cloud thickness and water content. The dependence of transmission only on water content may thus afford a method of measuring this quantity. In the "window" at 337 μ, it should be possible to differentiate between the various types of fogs, although the high water vapor attenuation would make this method attractive only in the denser fogs.

If astronomical measurements are to be made, due account must be taken of the fact that clouds are rather opaque at 337 μ but semi-transparent at 1200 μ. Eq. (8) affords a method of readily estimating the lower limit of attenuation for any given meteorological conditions.

The experimental measurements to date give reasonable agreement with theory, but further measurements are needed.

It is obvious that before reliable measurements of water content can be made, additional data on the variation of n and k with temperature are also needed.

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[3] Some of the points represent other workers' results.

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