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The 4-Day Venus Circulation Driven by Periodic Thermal Forcing

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ABSTRACT

It has been proposed that the observed 4-day retrograde rotation of the Venus atmosphere is a zonal motion of at least the upper atmosphere driven by periodic solar thermal forcing. We have assessed the relative importance of periodic thermal forcing for the atmospheres of Venus, Earth, Mars, Jupiter, Saturn and Uranus. Periodic thermal forcing is likely to play a significant role only in the dynamics of the Venus atmosphere, principally because of the favorable overhead speed of the sun; this is 3 m sec^{-1}, at least two orders of magnitude slower than the overhead solar speeds for the other planets. For a simplified channel flow problem we have examined the detailed mechanisms by which periodic induced circulations transport the momentum necessary to support a mean shear.

1. Introduction

Schubert and Whitehead (1969) have proposed that the observed 4-day retrograde circulation of the Venus atmosphere is the result of thermal forcing by the periodic overhead motion of the sun. The purposes of this paper are to: 1) briefly summarize the evidence supporting the existence of the Venus 4-day circulation; 2) reiterate the proposal of Schubert and Whitehead; 3) discuss the relevant mechanisms of momentum transport induced by periodic thermal forcing; and 4) assess the relative importance of periodic thermal forcing to the dynamics of the atmospheres of the planets Venus, Earth, Mars, Jupiter, Saturn and Uranus.

2. Evidence for 4-day circulation

The principal observations indicating a 4-day zonal retrograde circulation of the Venus atmosphere are ultraviolet (UV) photographs of the upper atmosphere (Boyer and Camichel, 1961, 1965, 1967; Boyer and Guerin, 1966; Dollfus, 1968; Smith, 1967). These photographs indicate the 4-day rotational motion in two ways. First, displacements of individual features on photographs taken a few hours apart reveal motions in the retrograde sense with a period of 4–5 days (Boyer and Guerin, 1966; Smith, 1967). The important point to note is that in photographs including the morning terminator, the observed displacements are toward the subsolar point; on the other hand, in photographs showing the evening terminator, displacements are away from the subsolar point. Therefore, the photographs indicate a zonal retrograde rotation of the upper atmosphere and not a wind blowing always toward or always away from the subsolar point. The second indication of atmospheric rotation given by the UV photographs is the often observed recurrence of similar features in plates separated in time by 4–5 days (Boyer and Camichel; Smith)

Spectroscopic measurements of motions in the Venus atmosphere also indicate a retrograde rotation of the atmosphere (Guinot, 1965; Guinot and Feissel, 1968). Under the assumption that the axis of atmospheric rotation is perpendicular to the orbital plane of Venus, Guinot and Feissel determined a rotation period of 4.3 ± 0.4 days.

There are additional indications of rapid motions in the upper atmosphere of Venus. Data obtained by Mariner 5 indicate a significant ion density in the Venus night-side ionosphere (McElroy and Strobel, 1969). The most plausible mechanism for the maintenance of this ion population is lateral transport of the ions from the day to the night side. Approximate calculations lead to wind speeds of the order of 100 m sec^{-1} (McElroy and Strobel). Also on the basis of Mariner 5 data, Shimizu

(1969) has concluded that there is a deficiency of oxygen atoms in the upper atmosphere. He suggests a dynamical explanation which yields horizontal wind speeds of ~ 100 m sec^{-1}. Temperature measurements of the Venus atmosphere show the temperature to be significantly more latitude than longitude dependent (Murray *et al.*, 1963; Goody, 1965). An efficient lateral transport of heat from the day to the night side of the atmosphere could easily account for this. Thompson (1970) has calculated a heat mixing time scale for the upper Venus troposphere which gives wind speeds of the order of 100 m sec^{-1}. Wind speeds calculated by McElroy and Strobel (1969), Shimizu (1969) and Thompson (1970), although not required to be zonal, are of the same magnitude as the 100 m sec^{-1} speed associated with the 4–5 day retrograde rotation.

3. Periodic thermal forcing

The overhead motion of the sun relative to an observer rotating with Venus is from west to east with a speed of 3 m sec^{-1}. If the periodic motion of the sun drives the 4-day circulation, then the thermal forcing must induce mean zonal winds moving in a direction opposite to that of the sun and with speeds more than an order of magnitude larger. Experimental and theoretical studies (Fultz, 1956; Stern, 1959; Davey, 1967; Schubert and Whitehead, 1969) have shown that periodic thermal forcing does indeed induce mean winds opposite to the direction of motion of the thermal source. Schubert and Whitehead have performed a moving flame experiment with liquid mercury as the working fluid and induced mean speeds in the mercury four times greater than that of the flame. In experiments performed at the Meteorological Office, England, with periodic internal heating of a dilute salt solution (Hinch and Schubert, 1970),[1] mean speeds comparable to and larger than the phase speed of the internal heating have been observed.

The experiments with boundary heating of liquid mercury and internal heating of a salt soultion have in common the fact that temperature or density fluctuations exist within the body of the fluid. In the earlier moving flame experiments with water, the thermal fluctuations were confined to relatively thin boundary regions and as a result only small mean speeds, on the order of 1% of the flame speed, were observed. It is clear from the more recent laboratory investigations and from theoretical studies (Schubert, 1969; Hinch and Schubert) that periodic thermal forcing may induce geophysically significant mean velocities. In general, therefore, this phenomenon must be given equal *a priori* significance with other types of circulations when attempting to explain the dynamics of a planetary atmosphere.

We note in concluding this section that Thompson (1970) has suggested a mechanism that could complement periodic thermal forcing; namely, the instability of a Hadley-type circulation between subsolar and antisolar points.

[1] In preparation.

4. Momentum transport with periodic thermal forcing

We seek to understand the mechanisms by which periodic thermal forcing, such as that due to the overhead motion of the sun, can induce mean motion in a planetary atmosphere. This understanding of the physical processes involved in supporting a mean flow by a moving thermal source is best achieved by considering the simplest model that retains the essential features of the phenomenon. Therefore, in the discussion that follows, the model is not chosen to represent a planetary atmosphere, but rather a simple fluid dynamical situation in which the motions induced by periodic thermal forcing can be studied.

Consider a Boussinesq fluid in a two-dimensional channel defined by planes at $z=\pm h$. A travelling wave temperature perturbation, $-T'\cos(kx+\omega t)$, which moves in the negative x direction with speed, $U=\omega/k$, is applied at both walls of the channel. We assume that the fluid is infinitely thermally conducting in a direction normal to the walls so that within the fluid the temperature is the sum of a constant mean temperature and the fluctuating temperature. The periodic thermal forcing produces a fluctuating buoyancy force (the gravitational body force points in the negative z direction) which drives horizontal and vertical velocity fluctuations, u' and w', respectively. The velocity perturbations interact to support a mean horizontal motion $\bar{u}(z)$ in the positive x direction. A linearized analysis of this flow has been given by Stern (1959), while the work of Davey (1967) includes this model as a special case. Schubert (1969) has discussed the flow on the basis of a quasi-linear theory. We shall consider the linear solution to this problem in some detail in order to understand the way in which the velocity perturbations interact to support a mean flow moving in the direction opposite to that of the travelling thermal source.

The magnitude of the thermal forcing is characterized by the dimensionless parameter

$$F=\frac{gh}{U^2}\frac{\rho'}{\rho}, \tag{1}$$

where g is the acceleration of gravity and ρ' the magnitude of the density fluctuations driven by the temperature fluctuations, i.e., $\rho'/\rho=-\alpha T'$, where α is the coefficient of thermal expansion. A second dimensionless parameter which must enter our discussion is

$$S=\frac{\omega h^2}{\nu}, \tag{2}$$

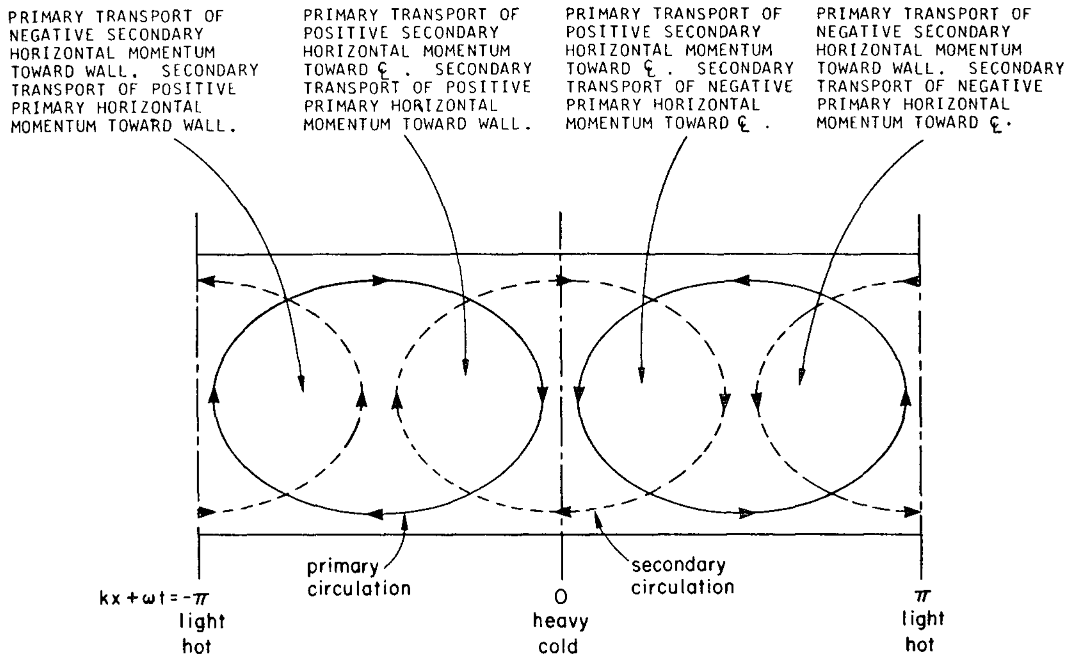

FIG. 1. Momentum transport in the limit $S \ll 1$.

where ν is the kinematic viscosity. The parameter S is the ratio of the viscous diffusion time based on channel half-thickness to the period of the forcing temperature fluctuation. It is instructive to investigate the mechanisms of momentum transport in the limiting cases $S \rightarrow 0$ and $S \rightarrow \infty$.

In the limit $S \rightarrow 0$, a flow dominated by viscous forces must be considered. The velocity fluctuations are

$$u' = UFS\frac{\zeta}{6}(\zeta^2-1)\sin(kx+\omega t) + \frac{UFS^2}{1440}\zeta(\zeta^2-1)(3\zeta^2-7)\cos(kx+\omega t)+\cdots, \quad (3)$$

$$w' = -UFS\frac{(kh)}{24}(\zeta^2-1)^2\cos(kx+\omega t) + \frac{UFS^2}{720}(kh)(\zeta^2-1)^2(\zeta^2-3)\sin(kx+\omega t)+\cdots, \quad (4)$$

where $\zeta = z/h$ and the approximation $kh \ll 1$ has been made to simplify the results. According to (3) and (4) we may consider the velocity fluctuations to consist of a primary flow of $O(S)$ and a secondary flow of $O(S^2)$. The primary vertical velocity fluctuations are in phase with the forcing density fluctuations, while the secondary vertical velocity fluctuations are $\pi/2$ radians out of phase with the periodic forcing. Similarly, the primary horizontal velocity fluctuations are $\pi/2$ radians out of phase with the density perturbations and the secondary horizontal velocity fluctuations are in phase with the density fluctuations. The interaction of these primary and secondary flows provides the mechanism of transporting momentum necessary to support the mean flow, i.e.,

$$\bar{u} = \frac{F^2S^4}{2!5!6!}(1-\zeta^2)^5+\cdots. \quad (5)$$

The vertical shear of the mean flow is related to the momentum transport by the velocity fluctuations according to

$$\nu\frac{d\bar{u}}{dz} = \overline{u'w'}, \quad (6)$$

where the bar denotes an average with respect to the phase, $kx+\omega t$. From (3) and (4) it is clear that the self-interactions of the primary and secondary flows cannot support a mean shear. With the aid of these equations and Fig. 1 we can understand how the primary and secondary flow interactions do transport momentum and support the vertical shear of the mean velocity. The primary [$O(S)$] circulation follows the travelling temperature wave to the left, the motion being downward where the fluid is cold and upward where the fluid is hot (solid-line circulation in Fig. 1). The magnitude of this circulation is independent of U and the flow is precisely the cellular pattern one expects for a standing wave temperature variation. The secondary [$O(S^2)$] circulation (dotted-line circulation in Fig. 1) also moves to the left, but $\pi/2$ radians out of phase with the temperature wave. Since the magnitude of the secondary circulation is directly proportional to U, this flow would vanish for a standing wave temperature variation at the walls.

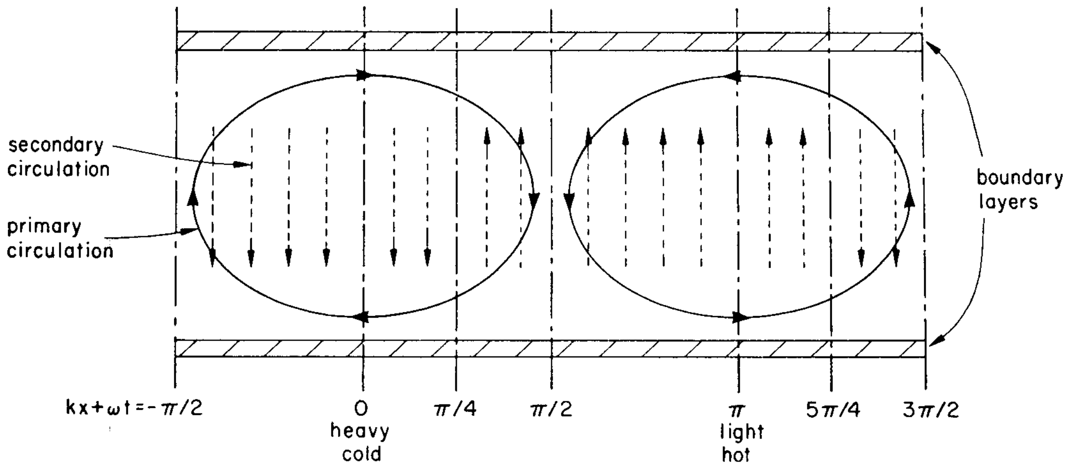

FIG. 2. Momentum transport in the limit $S\gg1$. Vertical secondary flow transports positive primary-flow horizontal momentum toward the channel center for $-\pi/2<(kx+\omega t)<\pi/4$, $\pi/2<(kx+\omega t)<5\pi/4$, but toward the walls for $\pi/4<(kx+\omega t)<\pi/2$, $5\pi/4<(kx+\omega t)<3\pi/2$.

The nonzero contributions to $\overline{u'w'}$ come from the transport of primary-flow horizontal momentum by the vertical motion of the secondary flow,

$$UFS\frac{\zeta}{6}(\zeta^2-1)\frac{UFS^2}{720}(kh)(\zeta^2-1)^2(\zeta^2-3)\,\overline{\sin^2(kx+\omega t)},$$

and the transport of secondary-flow horizontal momentum by the vertical motion of the primary flow,

$$-\frac{UFS^2}{1440}\zeta(\zeta^2-1)(3\zeta^2-7)UFS\frac{(kh)}{24}(\zeta^2-1)^2\,\overline{\cos^2(kx+\omega t)}.$$

As can be seen from Fig. 1, the vertical motion of the secondary flow always transports positive primary-flow horizontal momentum toward the walls, while the vertical motion of the primary flow transports positive secondary-flow horizontal momentum toward the channel center. To support a mean shear flow to the right, the interactions of the velocity fluctuations must transport a net positive horizontal momentum toward the channel centerline. Thus, the transport of secondary-flow horizontal momentum by the vertical motion of the primary flow must be the dominant of the two interactions. The mechanisms discussed here have also been noted by Whitehead (1970).

We now discuss the motion in the limit $S\rightarrow\infty$. As the dimensionless frequency becomes large, the flow tends toward a boundary layer structure. In the inviscid interior the velocity fluctuations are

$$u'=UF\zeta\cos(kx+\omega t)\cdots, \quad (7)$$

$$w'=-UF\frac{kh}{2}(1-\zeta^2)\sin(kx+\omega t)$$
$$-UFS^{-\frac{1}{2}}kh\cos\left(kx+\omega t+\frac{\pi}{4}\right)+\cdots, \quad (8)$$

and the mean flow is

$$\bar{u}=\frac{UF^2S^{\frac{1}{2}}}{4\sqrt{2}}(1-\zeta^2). \quad (9)$$

The primary circulation in the inviscid interior is $\pi/2$ radians out of phase with the temperature fluctuations. The secondary $[O(S^{-\frac{1}{2}})]$ circulation in the inviscid interior, driven by influx and efflux from the boundary layer, is entirely vertical, independent of ζ, and is $\pi/4$ radians out of phase with the thermal wave. The vertical shear of the mean flow can only be supported by the transport toward the channel centerline of positive primary-flow horizontal momentum by the vertically moving secondary flow. The relevant circulations and the direction of horizontal momentum transport are depicted in Fig. 2. In a fluid where the vertical diffusion of heat proceeds at a finite rate, the skewing of the thermal wave as it diffuses inward from the walls will need to be considered in a description of the momentum transport mechanisms.

5. Importance of periodic thermal forcing in planetary atmospheres

In the simple model discussed in the previous section, vertical transport of heat was assumed to be infinitely rapid, the fluid was confined between rigid walls, and the thermal boundary condition was given in terms of applied temperature fluctuations at the walls. However, in a realistic model of a planetary atmosphere, the thermal diffusivity is finite, the upper boundary is a free surface, and the thermal boundary condition at the upper surface is given in terms of heat flux. Whatever model one uses to describe the atmosphere, the response to periodic thermal forcing can be discussed in terms of the dimensionless parameters F, S, $\mathrm{P}=\nu/\kappa$, and kh, where κ is the thermal diffusivity and P the Prandtl number. If the incident heat flux is specified at the free surface, it may be convenient to rewrite F to explicitly show the magnitude of the incident heat flux. We first proceed to estimate the orders of magnitude of the above parameters for Venus, Earth, Mars, Jupiter, Saturn and Uranus.

The parameter kh is much less than unity for all the planets since k is of the order of the reciprocal of a planetary radius and the scale height h for each planet is of the order of 10 km [$h=O(\Theta/mg)$, where Θ is the product of Boltzmann's constant and the absolute temperature of the upper atmosphere and m is the mean mass per molecule]. For the inner planets kh is $O(10^{-4})$ and for the outer planets $kh=O(10^{-5})$. Calculations which have been carried out so far, both linear and nonlinear (Davey 1967, Hinch and Schubert) in the limit $kh \ll 1$, show the induced mean velocity and the horizontal velocity fluctuation to be independent of kh, while the vertical velocity perturbation is directly proportional to kh. In describing the motions of an atmosphere driven by periodic thermal forcing, we can use the approximation $kh \ll 1$ and further assume, when comparing the magnitudes of induced mean velocities in the atmospheres of the different planets, that these mean velocities will not explicitly depend on kh.

The Prandtl number P is the most uncertain of the parameters, due principally to difficulties in estimating κ. The dynamic viscosity ν is probably of the order of 10^4 cm^2 sec^{-1} or less (Goody and Robinson, 1966), whereas κ can vary widely depending on whether the transport of heat is convective or radiative. If convective heat transport is dominant, P is of order unity, and if radiative heat transfer is the important mechanism, P may be small compared with unity. Later in this section we will return to a discussion of the effect of P and estimate P for the upper Venus atmosphere.

The values of the heat flux q absorbed at the subsolar point {$q=1.37\times10^6\,(1-A)/r^2$[ergs cm^{-2}sec^{-1}], where A is the albedo and r the distance from the sun in astronomical units}, the equatorial overhead velocity U of the sun, and F and S are given in Table 1. In the Boussinesq approximation, which is valid for atmospheric motions whose vertical length scale is smaller than the atmospheric scale height, $\Delta\rho/\rho=-\alpha\Delta T$, and for a perfect gas, $\alpha=1/T$. For Venus, longitudinal temperature variations are on the order of a few degrees Kelvin (Murray *et al.*, 1963) and in the region of the tropopause, T is of the order of several hundred degrees Kelvin. Thus, $\Delta\rho/\rho$ for Venus is $O(10^{-2})$ and for the other planets we scale $\Delta\rho/\rho$ with the absorbed heat flux at the subsolar point. We have assumed that all the absorbed solar radiation is absorbed in the atmosphere of each planet and none at its surface. This is probably a good assumption for Venus, but for planets like Earth and Mars we have overestimated F.

The forcing parameter for Venus is 10^4-10^9 larger than that of the other planets listed! This is a consequence of two facts. First, Venus sees the slowest overhead speed of the sun by 2–4 orders of magnitude. Since g and h are comparable for every planet, the U^{-2} dependence of F is the major factor in distinguishing the forcing parameter for Venus. Second, q for Venus is as large as or larger than the absorbed heat flux of the other planets by as much as 3 orders of magnitude.

TABLE 1. The periodic thermal forcing parameters for various planetary atmospheres.

Planet	q (ergs cm^{-2} sec^{-1})	U (m sec^{-1})	F	S
Venus	6.3×10^5	3.0	10^2	10^2
Earth	8.4×10^5	4.7×10^2	10^{-2}	10^4
Mars	5.0×10^5	2.8×10^2	10^{-3}	10^4
Jupiter	2.5×10^4	1.2×10^4	10^{-6}	10^4
Saturn	7.6×10^3	1.0×10^4	10^{-7}	10^4
Uranus	1.2×10^3	3.7×10^3	10^{-7}	10^4

The full significance of Venus having a forcing parameter at least 4 orders of magnitude greater than that of the other planets is realized when the dependence of $\bar{u}/U$ on F is considered. For the linear case discussed earlier in this paper and for the linear cases discussed by Davey (1967), $\bar{u}/U \propto F^2$. This dependence is unchanged for the linear cases when heat flux conditions instead of temperature boundary conditions are specified at the boundaries. In fact, nonlinear calculations also show an F^2 dependence (Hinch and Schubert). If for planetary atmospheres the induced mean velocity is proportional to F^2, then based on the forcing parameter Venus is more than 8 orders of magnitude more likely to have a significant response to periodic thermal forcing than any of the other planets considered.

The above discussion leads one to suspect that the Venus atmosphere is the prime candidate for exhibiting significant motions driven by periodic thermal forcing. We must, however, be careful to assess the importance of the dependence of $\bar{u}/U$ on S. From Table 1 it can again be seen that Venus is unique; it has an S about 2 orders of magnitude less than that of any other planet listed. This statement would have to be modified if ν changes by orders of magnitude between Venus and some other planet. Since S is at least $O(10^2)$, we can look at the dependence of $\bar{u}/U$ on S in the limit $S\rightarrow\infty$. In the channel flow case considered before, $(\bar{u}/U)\propto S^{\frac{1}{2}}$. This same dependence also holds for nonlinear channel flow (Hinch and Schubert). When the boundary conditions are changed, however, the dependence of $\bar{u}/U$ on S changes. When heat flux conditions are specified at an upper free surface and at a lower rigid boundary, $\bar{u}/U\propto S^{-2}$. This may be seen by taking the free surface problem considered by Davey (1967), and simply replacing the temperature boundary conditions by conditions on the temperature gradients. On the basis of the nonlinear results for channel flow, it might be expected that $\bar{u}/U$ should still be proportional to S^{-2} in the nonlinear region of this problem. Thus, if $\bar{u}/U \propto S^{-2}$ for planetary atmospheres, then this parameter dependence further enhances the importance of periodic thermal forcing for the Venus atmosphere as compared to the other planets.

Finally, let us return to a consideration of the dependence of $\bar{u}/U$ on P. The linear problems using temperature boundary conditions and a nonzero Prandtl number show $\bar{u}/U \propto$ $P^{-2}(P+1)^{-1}$ for $S \gg 1$. This dependence changes to $P^{-3}(P+1)^{-1}$ if heat flux is specified at the boundaries. For planetary atmospheres in which heat and momentum are transported convectively, P is O(1), and the dependence of $\bar{u}/U$ on P is not likely to introduce significant variations among such planets. However, if radiative heat transport is important, the Prandtl number may be significantly less than unity, and the induced mean motions in such atmospheres will be accordingly strengthened. It is possible that such is the case for the Venus atmosphere. In the atmosphere of Venus, a near-infrared band of CO_2 absorbs a significant amount of the incident solar radiation. At altitudes of tens of kilometers, a kilometer of CO_2 absorbs several percent of the incident radiation. Thus, the radiative transfer would be characterized by an effective diffusion coefficient

$$\kappa = \frac{16}{3}\sigma T^3 l/(\rho c_p),$$

where σ is the Stefan-Boltzmann constant, l the mean free path of radiation, and c_p the specific heat at constant pressure. If l is at least of the order of a kilometer, then κ is of order 10^5 cm^2 sec^{-1} and P is $O(10^{-1})$ or smaller.

There is presently no reliable way of estimating the magnitude of the induced mean velocity on Venus. However, we have shown that for Venus the values of the parameters characterizing the response of its atmosphere to periodic thermal forcing are such as to make it likely that large mean zonal motions are indeed induced by this mechanism. In addition, there is now experimental (Schubert and Whitehead, 1969) as well as theoretical (Schubert, 1969) evidence that induced mean velocities many times greater than the velocity of the thermal source can be achieved.

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