

## Theory of Droplet Growth in Clouds: II. Diffusional Interaction Between Two Growing Droplets<sup>1</sup>

JOHN C. CARSTENS,<sup>2</sup> ALLEN WILLIAMS<sup>2</sup> AND JOSEPH T. ZUNG<sup>3</sup>

*Graduate Center for Cloud Physics Research, University of Missouri-Rolla*

(Manuscript received 7 November 1969, in revised form 26 April 1970)

### ABSTRACT

The interaction of two growing droplets in a supersaturated atmosphere has been examined, and the temperature and vapor density profiles have been determined. It is found that the smaller droplet tends to "catch up" with the larger at a slower rate than predicted by conventional diffusion theory. Consideration of droplet fallspeeds leads to the conclusion that, under atmospheric conditions, growth interaction becomes significant only for droplet "pairs" having equal or nearly equal radii. The number of such pairs is generally small enough so that the effect on the size distribution is quite small. Of a much greater importance is the possibility of a resulting attractive diffusio-phoretic force between two growing drops which, in turn, gives rise to a net velocity of one drop toward the other. If this diffusion force of attraction becomes sufficiently strong to overcome the hydrodynamic and thermo-phoretic forces acting in the opposite direction, both collision efficiencies and coagulation of small droplets could be further enhanced, thus accounting for departures from monodispersity in actual atmospheric clouds.

### 1. Introduction

One of the most difficult tasks in the study of the condensational growth of a system of droplets is to assess quantitatively the effect of droplet interaction upon the rate of growth. So far, this has been done by using either the "cellular" model (Zung, 1967a) or the "continuum" model (Zung, 1967b). Results obtained from these models are only approximate and averaged out over all droplets in the assemblage. In the present work attempts have been made to determine more accurately the influence of a growing droplet upon the growth rate of another droplet located in its vicinity.

Little work has been done on the effect of juxtaposition on diffusional growth. This is partly due to the fact that the average distance between droplets is, for most practical aerosols and especially clouds, quite large compared with the spatial extension of the diffusion profiles around each drop. It is usually assumed, therefore, to a good approximation, that each drop grows in an isolated atmosphere and interacts with its neighbors only by means of its contribution to depletion of bulk vapor concentration and increase of bulk temperature. Competition effects may then be incorporated into the diffusion scheme by the introduction of a "cell" around each drop (Zung, 1967a; Reiss and LaMer, 1950). Should the present calculation of the relative growth rates and diffusional interaction of two juxtaposed drops indicate similar results, it could be

regarded as a quantitative argument for the use of the cellular model; if not, it should indicate at what point the model starts to break down.

It is also a well-known fact (Byers, 1965) that the purely diffusional growth of droplets produces a narrow spectrum of sizes. Such a distribution is, however, not always in agreement with various observations either in clouds or in the more controlled environments of cloud chambers; hence, various mechanisms, such as coagulation or statistical fluctuations (Fuchs, 1964), have been postulated to account for departures from monodispersity. In this paper, we show that when two droplets are juxtaposed, the approach to monodispersity is retarded, but not reversed.

It is found that the effect of growth interaction is to displace a small percentage of the drop population toward radii smaller than that of the remaining drops; thus, a slightly less narrow distribution is predicted than that obtained by conventional diffusion theory. However, this effect is of minor importance under typical atmospheric conditions, though it may be of importance for high populations ( $\sim 10^6$  drops  $\text{cm}^{-3}$ ) under controlled conditions, such as in cloud chambers. Of much greater importance, at least to the atmospheric scientist, is the possibility of an attractive diffusio-phoretic force between drops which we have attempted to suggest by averaging the Stephan flow velocity around each drop. This could explain the coalescence of pairs of drops when they are falling at the same rate. The collision efficiencies of small droplets could be further enhanced if the diffusion forces of attraction were sufficient to overcome the hydrodynamic and thermo-phoretic forces acting in the op-

<sup>1</sup> Research supported by the Atmospheric Sciences Section, Office of Naval Research, under THEMIS Grant N00014-68-A-0497.

<sup>2</sup> Department of Physics, University of Missouri-Rolla.

for the observed departures from monodispersity in actual atmospheric clouds.

**2. Theory of condensational growth of two juxtaposed droplets**

Consider a system consisting of two droplets of radii  $a_1$  and  $a_2$ , respectively, in a supersaturated atmosphere, separated by a distance  $d_0$  between centers. The diffusion profiles of vapor concentration and temperature are calculated for a quasi-steady-state condition by solving the equations

$$\nabla^2 \rho = \nabla^2 T = 0, \tag{1}$$

with the following boundary conditions:

*Thermal continuity*

$$T_e|_{\text{surface}} = T_i|_{\text{surface}}, \tag{2}$$

*Linearized thermal equilibrium*

$$\rho|_{\text{surface}} = c + bT|_{\text{surface}}, \tag{3}$$

*Power balance*

$$\mathbf{n} \cdot [LD\nabla\rho|_{\text{surface}} - K_i\nabla T_i|_{\text{surface}} + K_e\nabla T_e|_{\text{surface}}] = 0. \tag{4}$$

In the above  $T_i$  and  $T_e$  are the temperatures inside and outside the droplets (there would be two equations for each boundary condition),  $K_i$  and  $K_e$  the thermal conductivities characterizing the liquid of the droplet and the gas mixture around it,  $\mathbf{n}$  a unit vector perpendicular to the droplet surface,  $L$  the latent heat of condensation,  $D$  the diffusion coefficient,  $\rho$  and  $T$  the vapor density and temperature fields, and  $c$  and  $b$  equilibrium constants. Under atmospheric conditions the linear approximation is rather good over the region of interest (this, of course, must include the effects of curvature and possibly of soluble impurities on the equilibrium vapor density). In controlled cloud chambers, however, where the supersaturation is usually much higher, the linear fit is not a good approximation. Nevertheless, one can easily guess at the final steady-state temperature, or at least near enough to it so that the linear approximation is valid throughout a region which includes the actual temperature, by calculating this temperature for an isolated drop.

In addition to the above conditions, another relationship is needed in order to specify completely the density and temperature profiles. This is obtained by requiring the two-drop solution to reduce to that for isolated drops as the ratio of their radii to distance between centers becomes small. This condition may be obtained by letting the radius of the droplets become very small, i.e.,

$$\lim_{a_0 \rightarrow 0} T_i = T_0,$$

where  $T_0$  is the internal temperature of the isolated drop.

The validity of the thermal equilibrium condition depends upon the requirement that  $(\lambda/a_0\delta) \gg 1$ , which is usually satisfied for droplets of radius  $\geq 1 \mu$  (Carstens and Kassner, 1968). Here  $a_0$  is the radius of the droplet,  $\lambda$  the mean free path of a vapor molecule and  $\delta$  the sticking coefficient. Thermal continuity is valid under the same circumstances as thermal equilibrium. The power balance is obtained by equating to zero the net energy (heat) flux into an elemental pill-box-like volume on the drop surface. [For an example of this technique, see Panofsky and Phillips (1955).]

The nature of the problem suggests the use of bi-spherical coordinates which are described in standard texts (Morse and Feshbach, 1953; Happel and Brenner, 1965). In what follows we have adopted the notation of Morse and Feshbach, along with various other calculations relevant to the solution of the Laplace equation in bi-spherical coordinates.

*a. Temperature and density profiles*

Following Morse and Feshbach, the temperature and vapor density profiles can be obtained directly from the solution to the Laplace equation, which, in terms of the independent variables  $\mu$  and  $\eta$ , may be written as

$$T_e = \sqrt{f} \sum_{n=0}^{\infty} [(A_n \cosh \mu_n + B_n \sinh \mu_n) P_n(w)] + T_{\infty}, \tag{5}$$

$$T_i^{(1)} = \sqrt{f} \sum_{n=0}^{\infty} [F_n^{(1)} \exp(-\mu_n) P_n(w)], \tag{6}$$

$$T_i^{(2)} = \sqrt{f} \sum_{n=0}^{\infty} [F_n^{(2)} \exp(\mu_n) P_n(w)], \tag{7}$$

$$\rho = \sqrt{f} \sum_{n=0}^{\infty} [(C_n \cosh \mu_n + D_n \sinh \mu_n) P_n(w)] + \rho_{\infty}, \tag{8}$$

where  $f = \sinh \mu - \cos \eta$ ,  $\mu_n = (n + \frac{1}{2})\mu$ ,  $w = \cos \eta$ , and  $\rho_{\infty}$  and  $T_{\infty}$  are the bulk values of vapor density and temperature far away from either droplet;  $T_i^{(1)}$  and  $T_i^{(2)}$  are the internal temperatures of droplet 1 (in the region  $\mu > 0$ ) and droplet 2 (in the region  $\mu < 0$ ). The coordinate parameter describing droplet 1 is  $\mu_1$ , and that for droplet 2 is  $-\mu_2$ .

Direct substitution into the boundary conditions [Eqs. (2)-(4)] now yields a sufficient number of equations to specify  $A_n$ ,  $B_n$ ,  $C_n$ ,  $D_n$ ,  $F_n^{(1)}$  and  $F_n^{(2)}$ . For example, for drop 1, thermal continuity leads to

$$A_n + B_n \tanh \mu_{n1} - [F_n^{(1)} - \sqrt{2}T_{\infty}]g(\mu_{n1}) = 0, \tag{9}$$

with a similar equation for drop 2. Here  $g(\mu_{n1}) = \exp(-\mu_{n1})/\cosh \mu_{n1}$ . The power balance results in a

sum over Legendre polynomials, and for drop 1 is

$$\sum_{n=0}^{\infty} \alpha_n P_n(w) = \alpha'_0 P_0(w) + \sum_{n=1}^{\infty} \alpha'_n [nP_{n-1}(w) + (n+1)P_{n+1}(w)], \quad (10)$$

where

$$\begin{aligned} \alpha'_n &= Q_1 \sinh \mu_{n1} + Q_2 \cosh \mu_{n1} + (K_i/LD) F_n^{(1)} \exp(-\mu_{n1}), \\ \alpha_n &= Q_1 G_{11} + Q_2 G_{12} - (K_i/LD) F_n^{(1)} \exp(-\mu_{n1}) H_1, \\ Q_1 &= C_n + (K_e/LD) A_n, \\ Q_2 &= D_n + (K_e/LD) B_n, \\ G_{11} &= \cosh \mu_{n1} \sinh \mu_1 + (2n+1) \sinh \mu_{n1} \cosh \mu_1, \\ G_{12} &= \sinh \mu_{n1} \sinh \mu_1 + (2n+1) \cosh \mu_{n1} \cosh \mu_1, \\ H_1 &= \sinh \mu_1 - (2n+1) \cosh \mu_1. \end{aligned}$$

Equating coefficients of  $P_n$ 's leads to a set of finite difference equations,

$$\left. \begin{aligned} \alpha_0 &= \alpha'_0 \\ \alpha_n &= n\alpha'_{n-1} + (n+1)\alpha'_{n+1} \end{aligned} \right\}, \quad (11)$$

with a similar set for droplet 2. These sets may be solved stepwise exactly up to as many terms as desired. The difference equation requires knowledge of  $F_0^{(1)}$  for its solution. This is derived from the limiting condition mentioned earlier,

$$\lim_{\mu_1 \rightarrow \infty} T_i^{(1)} = F_0^{(1)} / \sqrt{2} = \frac{\rho_{\infty} - c + (K_e/LD) T_{\infty}}{b + (K_e/LD)}.$$

It should be mentioned that the values of  $F_0^{(1)}$  and  $F_0^{(2)}$  are critical to the convergence of the series, and must be calculated accurately. We found that computer solutions gave reasonable results only if double precision was used.

The above solutions permit an immediate calculation of the instantaneous mass growth rate from Fick's law, i.e.,

$$\frac{dm}{dt} = -D \int_{\eta=0}^{\pi} \nabla \rho|_{\text{surface}} \cdot dS. \quad (12)$$

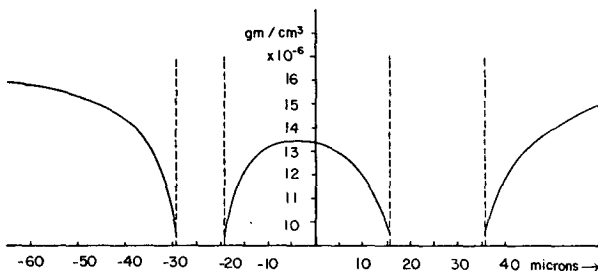


FIG. 1. Vapor density profiles of a two-droplet system:  $a_1 = 5 \mu$ ,  $a_2 = 10 \mu$ ,  $d_0 = 50 \mu$ ,  $\rho_{\infty} = 17.56 \times 10^{-6} \text{ gm cm}^{-3}$ .

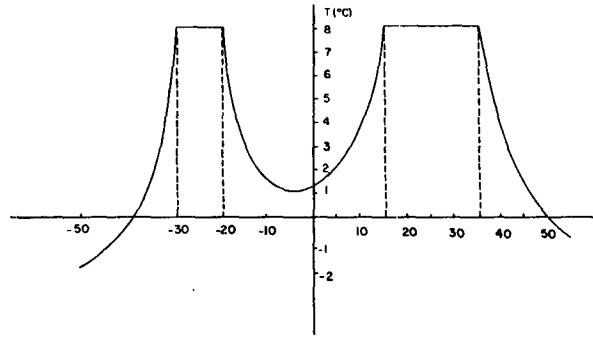


FIG. 2. Temperature profiles around two droplets of 5 and 10  $\mu$  in radius and 50  $\mu$  apart ( $T_{\infty} = -6.2^{\circ}\text{C}$ ).

The integrand can be expressed as products of Legendre polynomials and integrated term by term using the orthonormality of these functions. The result for mass growth is

$$\begin{aligned} \pm \frac{4\pi a D}{\sqrt{2}} \sum_{n=0}^{\infty} \exp[-(n+\frac{1}{2})|\mu|] \\ \times [(D_n \cosh \mu_n \pm C_n \sinh |\mu_n|) \\ \pm (C_n \cosh \mu_n + D_n \sinh |\mu_n|)], \quad (13) \end{aligned}$$

where the formula has been arranged so that the minus sign must be used for the lower droplet and the plus sign for the upper. It is evident that the same rate will be obtained if  $|\mu_1| = |\mu_2|$  (for this case,  $D_n = 0$ ).

### 3. Numerical results and discussion

For illustrative purposes, we present in Figs. 1 and 2, respectively, a plot for the vapor density and temperature along the lines of centers of two drops under cloud chamber conditions (Allen and Kassner, 1969). In this example we have taken 5 and 10  $\mu$  drops 50  $\mu$  apart (between centers). The profiles show clearly the expected constancy of the interior temperature (this is true for drops as close as a few radii between centers), and the asymmetry, resulting from the interaction term, of the exterior solution. The extent to which the drops are able to heat up the region between them appears rather striking, although the effect is, of course, at its maximum along the line of centers.

In order to illustrate the effect that juxtaposition may have on droplet growth, we take a specific example which is appropriate to typical warm cloud conditions with  $T = 300\text{K}$  and a supersaturation of 0.5% (Fletcher, 1962). Plots of the ratio of both areal growth rates and radial growth rates as a function of distance between drop centers are shown in Fig. 3 for a drop pair of 5 and 10  $\mu$ .

Now, neglecting curvature effects, it is a known fact that, at constant supersaturation, conventional diffusion theory predicts the areal growth of an isolated

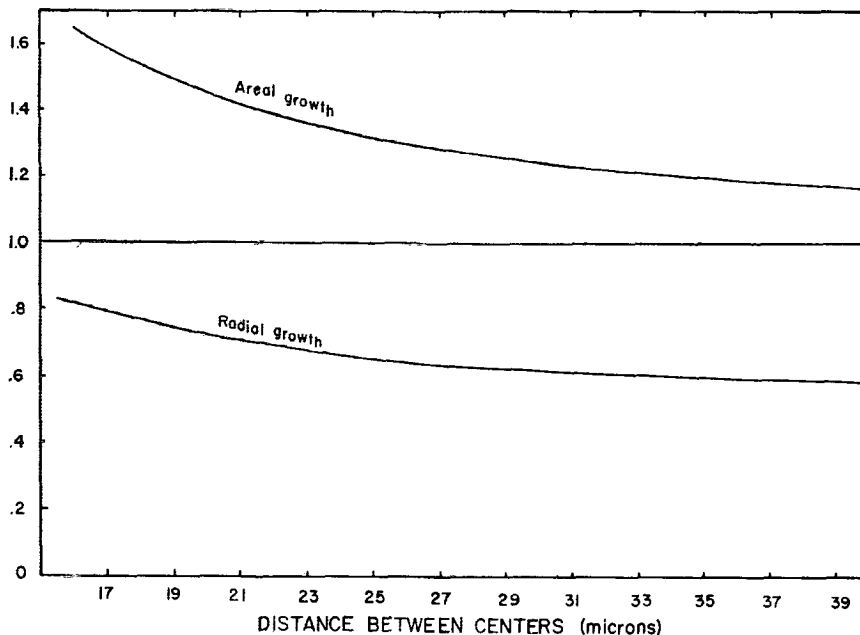


Fig. 3. The ratio of areal and radial growth rates of a 10 μ drop to those of a 5 μ drop vs the interdroplet distance  $d_0$ .

drop to be a constant. Hence, the areas of isolated drops grow at the same rate, and it is not hard to deduce that this leads to a narrowing of the radial size distribution. It is clear from Fig. 3 that the ratio of areal growth rates is less than unity, so that this narrowing in the distribution must be retarded. But it is also clear by looking at the radial rates of each drop pair that the tendency toward monodispersity, while retarded, is never reversed; that is, one always has

$$1 < \frac{da_1/dt}{da_2/dt} < \frac{a_2}{a_1}$$

Of course, the retardation is significant only when the two drops are rather close together, say less than a distance  $\ell$  (which might, for example, denote the distance at which the areal growth ratio is diminished by 10%). If we designate two drops whose centers are separated by a distance  $\leq \ell$  as a "pair," then any overall effect that direct competition might have on the diffusional growth of an aerosol will depend, at least in part, upon the number of such pairs in the system, and hence upon its number density  $n$  (i.e., the number of drops per unit volume), and the total volume occupied by it.

For drop populations  $< 2000 \text{ cm}^{-3}$ , the number of pairs  $n_2$  is given approximately by

$$n_2 = \left(\frac{n}{2}\right) \frac{4}{3} \pi (r^3 - 8a^3) n,$$

with

$$2a < r < 15a, \quad a \leq 20 \mu,$$

where  $r$  is the distance from the center of one drop to another and the factor  $\frac{1}{2}$  arises to avoid double counting. Thus for, say, 10 μ drops ( $\ell \approx 100$ ), with  $n = 500 \text{ cm}^{-3}$ , we have an average distance between drops of 780 μ and 0.5 pair  $\text{cm}^{-3}$ , while for  $n = 2000 \text{ drops cm}^{-3}$  we have an average distance between drops of 490 μ and 8 pairs  $\text{cm}^{-3}$ . Evidently the pair probabilities are such that, in volumes appropriate to clouds, direct droplet growth interaction is a likely event for reasonable drop populations even though it is an improbable event when compared with the number of drops.

The above results, however, must also be viewed in the context of drop fall. For droplets of 5 and 10 μ the relative fall rate is about  $9000 \mu \text{ sec}^{-1}$ . Hence, two droplets of such different radii could hardly be expected to remain in mutual proximity long enough for appreciable growth interaction to take place. It is then evident that, for growth interaction to persist, we must consider drops of equal, or nearly equal, size. Of course, diffusion theory already suggests such a consideration, but it should be pointed out that for two drops of, say, roughly 5 μ to interact for even 1 sec they can differ in size by only 0.037 μ. Thus, a "falling" aerosol ("falling" in the sense of a downward velocity relative to its ambient atmosphere) must be rather strongly monodisperse in order to satisfy the criteria for significant growth interaction. Departures from monodispersity then necessarily occur for a small number of (otherwise isolated) pairs of drops whose growth is retarded. The number of more or less equal size pairs depends on the history of the aerosol, but once that is known, the theory here presented affords a means of

evaluating the significance of direct "growth interaction."

Another consequence of fall must be mentioned, and that has to do with ventilation. It is well known that falling drops with radii  $< 20 \mu$  grow at nearly the same rate as their stationary counterparts (Squires, 1952). Recently, Abraham (1968) has suggested that droplets in this range (and even above) carry with them, so to speak, their stationary profiles despite the relative bulk motion of the surrounding gas. On the basis of the above argument, it is therefore reasonable to suppose that ventilation has the effect of establishing bulk conditions somewhat closer to the drop than if the drop were at rest with respect to the bulk gas. This tends to increase the growth rate of the drop. If bulk conditions are established at an average distance  $R$  out from the drop center, and if the stationary profiles are maintained inside  $R$ , then the usual "cellular" model predicts an areal growth rate of

$$(dS/dt) = (dS/dt)_{\infty} (1 + a_0/R),$$

where  $S$  is the surface area of the drop and  $(dS/dt)_{\infty}$  is the growth rate of the drop for  $R = \infty$  (Carstens and Kassner, 1968). The areal growth of a ventilated drop, on the other hand, is given by (Squires, 1952)

$$(dS/dt) = (dS/dt)_{\text{stationary}} (1 + 0.24 \text{Re}^{1/2}),$$

where  $\text{Re}$  is the Reynolds number. Equating the perturbing terms yields a value of  $R$  in terms of  $a_0$ . For  $20 \mu$  this is  $13a_0$ , while for  $10 \mu$  it is  $37a_0$ . Our results generally refer to interactions well within this distance, so that it is plausible to regard conclusions based on stationary growth to be valid, or nearly so, under gravitational fall.

The effect this growth interaction has on a typical droplet population of some given average size (that is, for example, a population that has grown in a convective parcel of air to a maximum of  $20 \mu$  by diffusion alone) is to displace a very small percentage of this population toward radii smaller than that of the remaining drops. Thus, a slightly less narrow distribution is predicted than that obtained by conventional diffu-

sion theory as applied to single drops. Hence, when the pairwise interactions are taken into account, a polydisperse cloud will tend to become monodisperse (by the growth process) at a slower rate than predicted by the cellular model (without drop interactions). In Fig. 4 we exhibit, for typical cloud conditions (0.5% supersaturation), a plot of the ratios of radial growth of 5, 10, 15 and  $20 \mu$  drops, at various distances between centers, to that of isolated drops of the same size. Along with these distances we have given the corresponding number of pairs for a population of 500 drops  $\text{cm}^{-3}$  of radius  $10 \mu$  ( $\ell = 100 \mu$ ). The number of pairs is clearly much less than the total number, and this inequality holds for all typical drop populations. In fact, for the total number of isolated drops to comprise, say, only half the population—constituting a significant effect on the size distribution—would require a total drop population of  $2 \times 10^5 \text{ cm}^{-3}$ . It may be concluded that this particular consequence of juxtaposition is of minor importance under typical atmospheric conditions, though it may be of importance for high populations under controlled conditions.

Of greater potential importance to the atmospheric scientist is the possibility of an attractive diffusio-phoretic force arising between two drops. Deryagin and Dukhin (1957) treated this problem by assuming the droplet to move at the Stephan flow velocity. Their calculations, however, were based on approximate diffusion fields calculated by assuming one droplet to be in a linear temperature and density field due to the other; thus, the calculation breaks down in the important case of small separation distance. Their conclusion that the (Stephan flow) interaction vanishes under quasi-steady-state conditions is not borne out by our similar calculations for a growing pair of droplets.

The Stephan flow velocity is given approximately by

$$v_s \approx \frac{M_g D \nabla \rho}{M_v \rho_0}, \quad \rho \ll \rho_0,$$

where  $M_g$  is the molecular weight of the non-condensable gas,  $M_v$  that of the vapor,  $\rho_0$  the density of the non-condensable gas, and  $\rho$  the vapor density (Fuchs, 1964). When this quantity is averaged around one of a pair of growing drops a net velocity is obtained toward the other drop, and if the drop moves at (or near) this velocity it would appear to be atmospherically significant. For example, for two  $10 \mu$  drops  $100 \mu$  apart (between centers),  $|v_s| \approx 2 \mu \text{ sec}^{-1}$ , while for a separation of  $40 \mu$ ,  $|v_s| \approx 11 \mu \text{ sec}^{-1}$ . Assuming such velocities, it is apparent that drop pairs of this size persisting for the order of a minute could collide, and perhaps coalesce. The somewhat larger drop, formed as a result of this coalescence, could now possibly scavenge other drops by falling if the diffusion forces of attraction were sufficient to overcome the usual hydrodynamic forces that tend to prevent collision below about  $20 \mu$

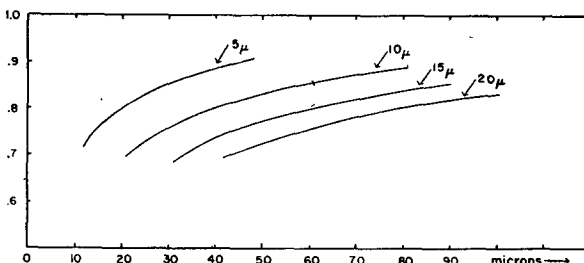


FIG. 4. The ratio of radial growth for pairs of 5, 10, 15 and  $20 \mu$  drops to those of isolated drops of the same size vs the interdroplet distance  $d_0$ . The number of pairs for  $10 \mu$  drops at various interdroplet distances are as follows: 0.03 pair at  $40 \mu$ ; 0.06 at  $50 \mu$ ; 0.11 at  $60 \mu$ ; 0.18 at  $70 \mu$ ; 0.26 at  $80 \mu$ ; 0.38 at  $90 \mu$ .

(Fletcher, 1962). In brief, the possibility exists that diffusion forces may, in isolated instances, bring about a coalescence of pairs of drops falling at the same rate, and that, furthermore, they could enhance collision efficiencies of drops colliding by virtue of their different fall rates.

It must be pointed out that the above effect is merely suggestive. There is no clear cut physical reason why a drop should move at this Stephan flow velocity. While there appears to be a diffusio-phoretic force tending to draw the drops together, there is also a thermo-phoretic force acting in the opposite direction. A more complete analysis of the diffusio- and thermo-phoretic force fields for a pair of drops, using the diffusion fields presented here, is now underway in our laboratory.

#### 4. Conclusion

In conclusion, from our mathematical analysis and numerical results, we have found that in a system of two droplets:

- 1) The surface of the larger droplet increases faster than that of the smaller droplet.
- 2) However, the radius of the smaller droplet grows faster than that of the larger one, but not as fast as their isolated counterparts.
- 3) Hence, a polydisperse droplet system will tend to become monodisperse, but the approach to monodispersity is slower than predicted by the conventional theory.

*Acknowledgments.* The authors wish to thank Dr. James L. Kassner, Jr., for his many helpful suggestions

and for the interest he has shown in this work. We are also indebted to Dr. N. H. Fletcher for his constructive criticisms and helpful discussions.

#### REFERENCES

- Abraham, F., 1968: A physical interpretation of the structure of the ventilation coefficients for freely falling water drops. *J. Atmos. Sci.*, **25**, 76-81.
- Allen, L. B., and J. L. Kassner, Jr., 1969: The nucleation of water vapor in the absence of particulate matter and ions. *J. Colloid Interface Sci.*, **30**, 81-93.
- Byers, H. R., 1965: *Elements of Cloud Physics*. The University of Chicago Press, 114 pp.
- Carstens, J. C., and J. L. Kassner, Jr., 1968: Some aspects of droplet growth theory applicable to nuclei measurements. *J. Rech. Atmos.*, **3**, 33-40.
- Deryagin, B. V., and S. S. Dukhin, 1957: Theory of the interaction of evaporating or growing drops at great distances. *Dokl. Akad. Nauk S.S.S.R.*, **112**, 407-410 [*Soviet Physics Doklady*, **2**, 41-44 (English Transl.)].
- Fletcher, N. H., 1962: *The Physics of Rainclouds*. Cambridge University Press, Chap. 6.
- Fuchs, N. A., 1964: *The Mechanics of Aerosols*. New York, MacMillan, pp. 67 and 288-302.
- Happel, J., and H. Brenner, 1965: *Low Reynolds Number Hydrodynamics*. Englewood Cliffs, N. J., Prentice-Hall, p. 516.
- Morse, P. M., and H. Feshbach, 1953: *Methods of Theoretical Physics*. New York, McGraw-Hill, p. 1298.
- Panofsky, W. K. H., and M. Phillips, 1955: *Classical Electricity and Magnetism*. Reading, Mass., Addison-Wesley, p. 27.
- Reiss, H., and V. K. LaMer, 1950: Diffusional boundary value problems involving moving boundaries, connected with the growth of colloidal particles. *J. Chem. Phys.*, **18**, 1-12.
- Squires, P., 1952: Growth of cloud drops by condensation. *Australian J. Sci. Res.*, **A5**, 59-86.
- Zung, J. T., 1967a: Evaporation rate and lifetimes of clouds and sprays in air—The cellular model. *J. Chem. Phys.*, **46**, 2064-2070.
- , 1967b: Evaporation rates and lifetimes of clouds and sprays in air. II. The continuum model. *J. Chem. Phys.*, **47**, 3578-3581.