Wavenumber-Frequency Spectra of Temperature in the Free Atmosphere

S.-K. KAO[1]

National Center for Atmospheric Research,[2] Boulder, Colo.

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ABSTRACT

An analysis of the wavenumber-frequency spectra of temperature in the free atmosphere is made. It is found that a striking similarity exists between the spectrum of temperature and that of the large-scale wind velocity in the free atmosphere. The wavenumber-frequency spectrum of temperature shows a preferred spectral domain of wave activities, oriented primarily from a region of low wavenumbers and low frequencies to a region of high wavenumbers and negative frequencies assigned to waves moving from west to east. In the high-wavenumber range, the wavenumber spectrum of temperature is approximately proportional to the -3 power of the wavenumber. In the high-frequency range, the frequency spectrum of temperature is approximately proportional to the -1 power of the frequency. These indicate that the structure of the temperature field in the free atmosphere is essentially affected by the large-scale two-dimensional turbulent motion. It is also found that most of the sensible heat is associated with the stationary zonal mean motion, and that there is more sensible heat associated with nonstationary waves than with stationary waves in the atmosphere.

1. Introduction

Studies of the spectral characteristics of turbulent-scalar quantities have mostly been confined to the field of isotropic, homogeneous turbulence (Obukhov, 1949; Yaglom, 1949; Corrsin, 1951). It is found in such a field of turbulence that similarity between the spectrum of temperature and that of velocity exists.

The large-scale motion in the atmosphere is generally anisotropic. Similarity between the spectrum of potential temperature and that of velocity cannot generally be expected. However, if a certain degree of similarity between the two can be established, predictability of the spectra of scalar quantities, such as radioactive debris, in the free atmosphere would be greatly increased.

In a recent paper (Kao and Wendell, 1970), the wavenumber-frequency spectra of the zonal and meridional components of the velocity in the troposphere and stratosphere were computed and investigated. It was found that there exists a preferred wavenumber-frequency domain of wave activities in the atmosphere, and that in the high-wavenumber range the velocity spectrum is approximately proportional to the -3 power of the wavenumber, and in the high-frequency range to the -1 power of the frequency. The question is to what extent the velocity field would affect the structure of the field of temperature in the atmosphere. The purpose of this paper is to make such an investigation.

[1] Permanent affiliation: University of Utah, Salt Lake City.
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2. Analysis and data source

In order to make a spectral analysis of temperature θ in the wavenumber-frequency domain, we make use of the Fourier transform

$$T(k,\pm n)=\frac{1}{4\pi^2}\int_0^{2\pi}\int_0^{2\pi}\theta(\lambda,t)e^{-i(k\lambda\pm nt)}d\lambda dt, \quad (1)$$

where λ, t, k and n are, respectively, the longitude, time, wavenumber and frequency; $\theta(\lambda,t)$ is a real, single-valued function, which is piecewise differentiable in a normalized domain, $0\leq\lambda,\ t\leq 2\pi$; and T is the complex coefficient of the Fourier transform. The positive and negative signs are assigned to waves moving toward the west and east, respectively.

For convenience of computation in this study, we express

$$T(k,\pm n)=T_r(k,\pm n)+iT_i(k,\pm n), \quad (2)$$

where T_r and T_i are, respectively, the real and imaginary part of the complex coefficient.

It has been shown (Kao, 1968) that

$$\frac{1}{4\pi^2}\int_0^{2\pi}\int_0^{2\pi}\theta^2(\lambda,t)d\lambda dt =\sum_{k=0}^{\infty}\int_0^{\infty}[E_{TT}(k,+n)+E_{TT}(k,-n)]dn, \quad (3)$$

and that

$$\left.\begin{aligned}E_{TT}(0,\pm n)&=T_r^2(0,\pm n)+T_i^2(0,\pm n)\\ E_{TT}(k,\pm n)&=2[T_r^2(k,\pm n)+T_i^2(k,\pm n)],\quad k=0\end{aligned}\right\}, \quad (4)$$

where $E_{TT}(k,+n)$ and $E_{TT}(k,-n)$ are the wavenumber-frequency spectra of θ due to eddies of wavenumber k and frequency n, moving toward the west and east, respectively. Eq. (3) indicates that the total contribution of θ^2 is equal to the power spectrum of θ integrated over all frequencies and over all wavenumbers. In (3) the summation of $E_{TT}(k,\pm n)$ with respect to the integer wavenumbers is the consequence of the cyclic distribution of $\theta(\lambda,t)$ along latitude circles.

The wavenumber spectrum of θ due to eddies of wavenumber k and all frequencies, moving, respectively, toward the west and east, is therefore

$$E_{TT}(k,\pm)=\int_0^\infty E_{TT}(k,\pm n)dn, \tag{5}$$

and the frequency spectrum due to eddies of frequency n and all wavenumbers, moving, respectively, in the direction of decreasing and increasing longitude, is

$$E_{TT}(\pm n)=\sum_{k=0}^{\infty} E_{TT}(k,\pm n). \tag{6}$$

The temperature data used in this analysis were obtained from the 1964 National Meteorological Center (NMC) temperature analyses at the 100-, 200-, and 500-mb surfaces. The temperature distributions are available at 12-hr intervals except at the 100-mb surface for the summer where a 24-hr interval is used. The temperature data are interpolated from the grid devised by the Joint Numerical Weather Prediction unit

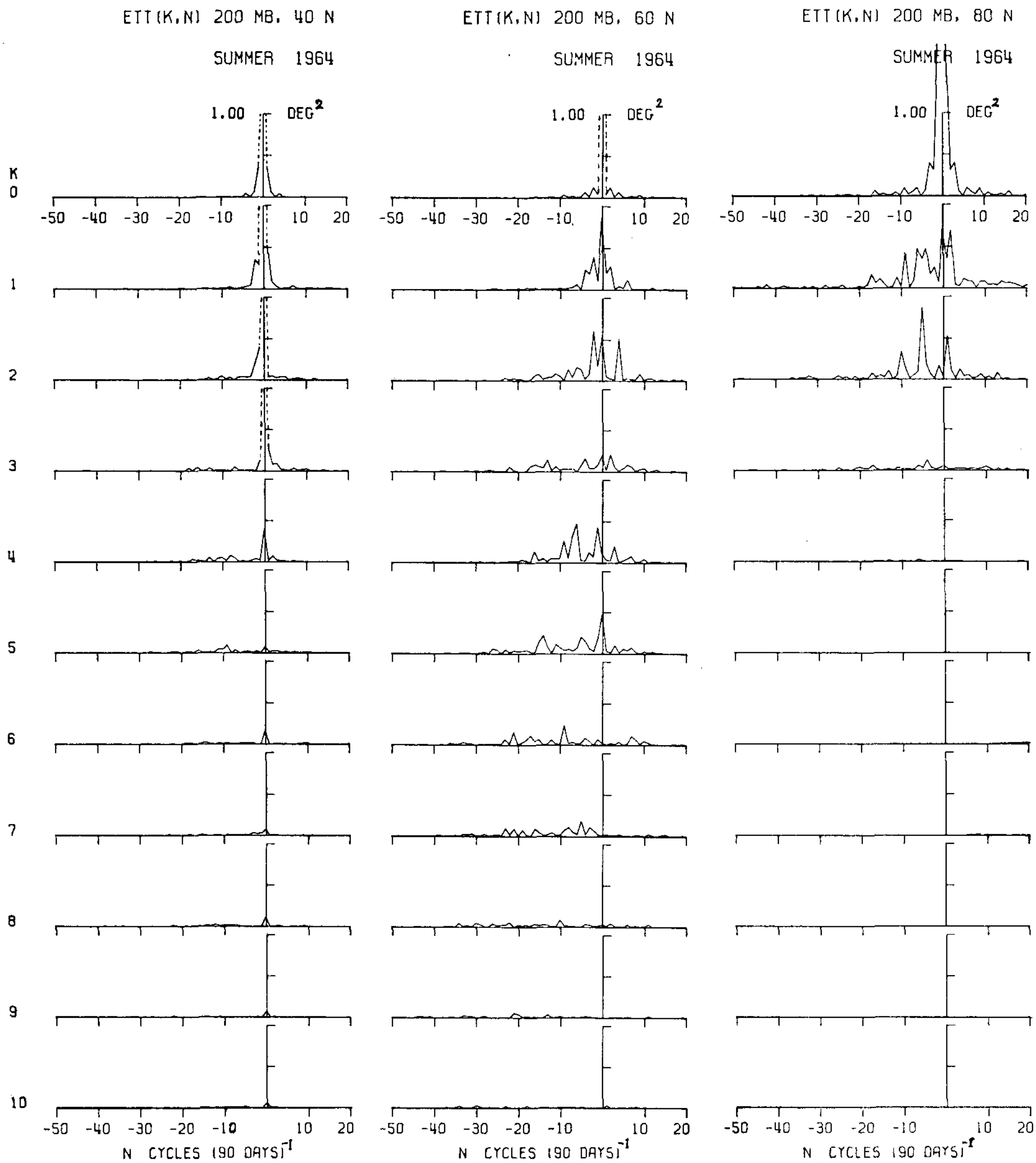

Fig. 1. Wavenumber-frequency spectra of temperature at 200 mb, summer, 1964 [units: (°K)²].

(JNWP) on latitudes 20, 40, 60 and 80N, at longitude intervals of 5°.

The basic computation is the numerical calculation of the real and imaginary parts of Fourier coefficients for the temperature. This involves the numerical evaluation of the integrals (1) and (2). The data intervals used in the integrations are 5° of longitude and 12 hr except at 100 mb where 24 hr is used for the summer season. The time integrations are carried out over periods of 90 days for the winter and summer seasons of 1964, which are normalized to 2π. Computations of Fourier coefficients are carried out for a wavenumber range 0–21, and for a frequency range 0–90 cycles (90 days)$^{-1}$. The power spectra of temperature are computed with the application of (4) and with the use of the computed Fourier coefficients. In these calculations, "Winter" is the period December 1963, January, February 1964, while "Summer" is June, July, August 1964.

It may be remarked that the 100-mb analyses were based on rawinsonde data as received at the National Meteorological Center without additional corrections for radiation errors. This can produce areas of the grid where the temperature is nearly 2C warmer than it should be and with 100-mb heights up to 50 m too high. Analyses at 0000 GMT were used at 100 mb. At this time the sun is over the Pacific Ocean where United

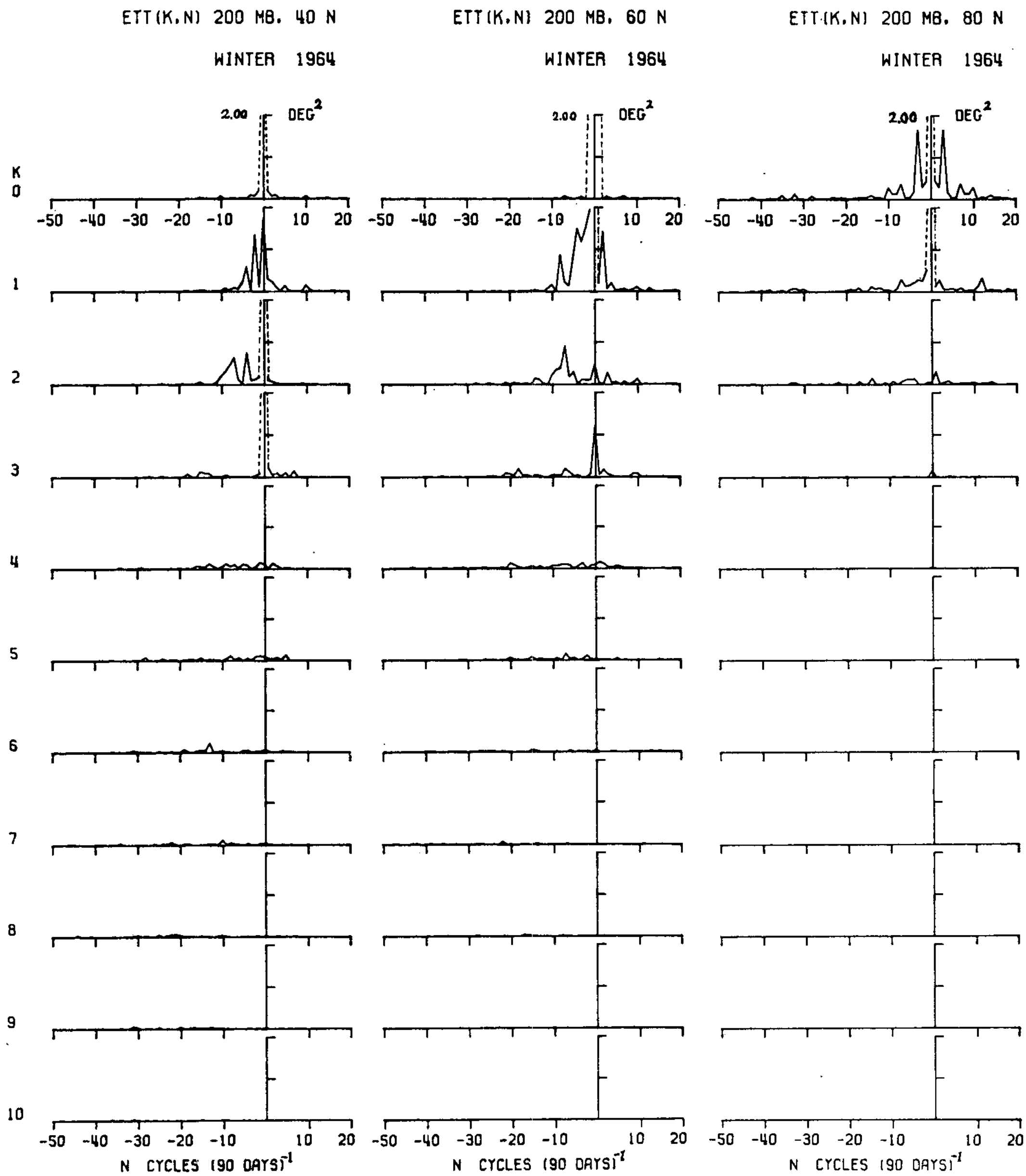

FIG. 2. Same as Fig. 1 except for winter.

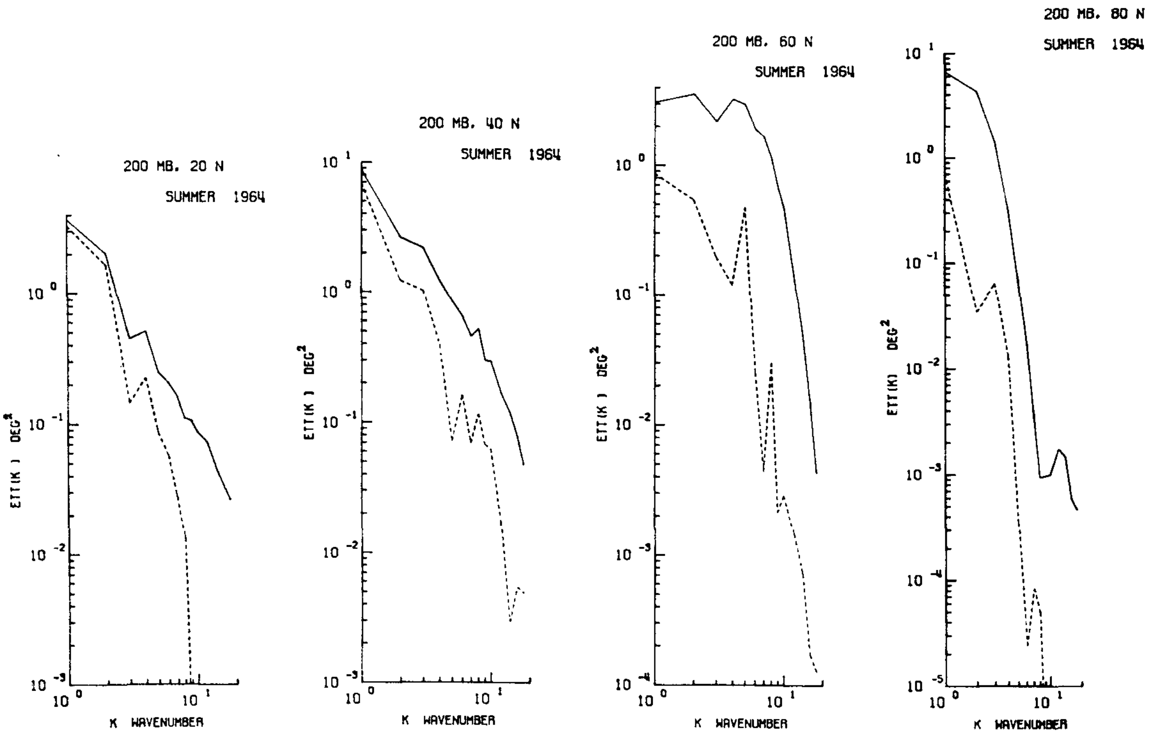

FIG. 3. Wavenumber spectra of temperature at 200 mb, summer, 1964. The solid curves connect the line spectra contributed by both the stationary and moving waves, the dashed curves those contributed by the stationary waves only.

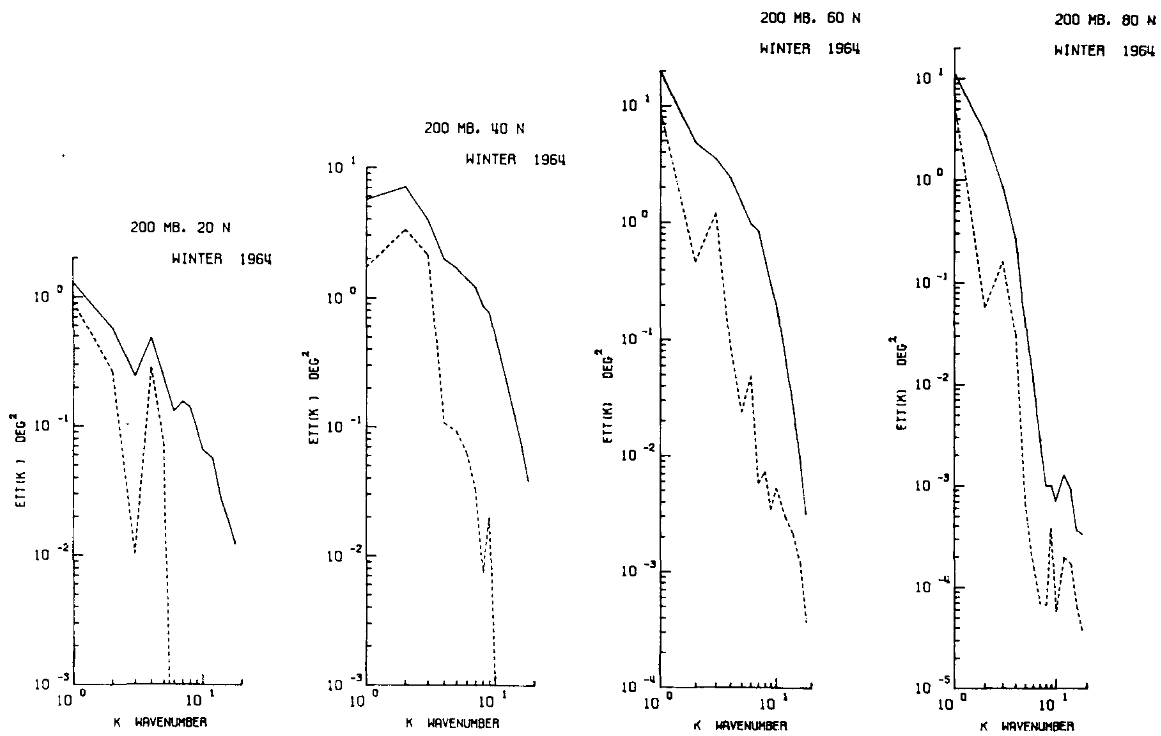

FIG. 4. Same as Fig. 3 except for winter.

States rawinsondes, which need a correction of ~0.7C, are generally used. Over data-poor areas the forecast (which is used as the first guess analysis) strongly influences the analysis. The shorter waves tend to be smoothed out in both the forecast models and in the analysis smoothing. Wavenumber 12 represents a wavelength of 30° of longitude which at 40° latitude is about 7 grid distances. Waves of this wavelength and longer should be little affected by the analysis procedures.

3. The wavenumber-frequency spectra of temperature

In order to investigate the effect of waves of various sizes and periods on the distribution of temperature or sensible heat divided by ρc_p in the atmosphere, it is necessary to analyze the wavenumber-frequency spectrum of temperature in the atmosphere. To do so the wavenumber-frequency spectra of temperature at 20, 40, 60 and 80N at 100, 200 and 500 mb for the summer and winter seasons of 1964 have been computed with the use of (1), (2) and (4). Because of the limited space only the temperature spectra at 40, 60 and 80N at 200 mb are presented in Figs. 1 and 2. In these figures, the vertical axis represents the spectral density of the temperature, and the horizontal axis the frequency in units of cycles per 90 days, the positive and negative frequencies being assigned to waves moving from east to west and from west to east, respectively. It may be noted that the magnitude of the spectral density for the summer is about one-half that for the winter.

It is seen from Figs. 1 and 2 that there exists a preferred spectral domain, which indicates the domain of wave activites. In middle latitudes, the spectral band is oriented from a region of low wavenumbers and low frequencies to a region of high wavenumbers and negative frequencies. In low latitudes, however, there exist two domains; in addition to the one similar to that in middle latitudes, a second domain occurs in the high-wavenumber region confined to a narrow band centered near zero frequency.

It may be noted that the wavenumber-frequency spectra of temperature as shown in Figs. 1 and 2 are very similar to the distribution of the sum of the spectra of the zonal and meridional components of the velocities in the atmosphere (Kao and Wendell, 1970; Kao *et al.*, 1970). This indicates that the distribution of the wavenumber-frequency spectra of temperature is primarily affected by the speed rather than direction of the wind.

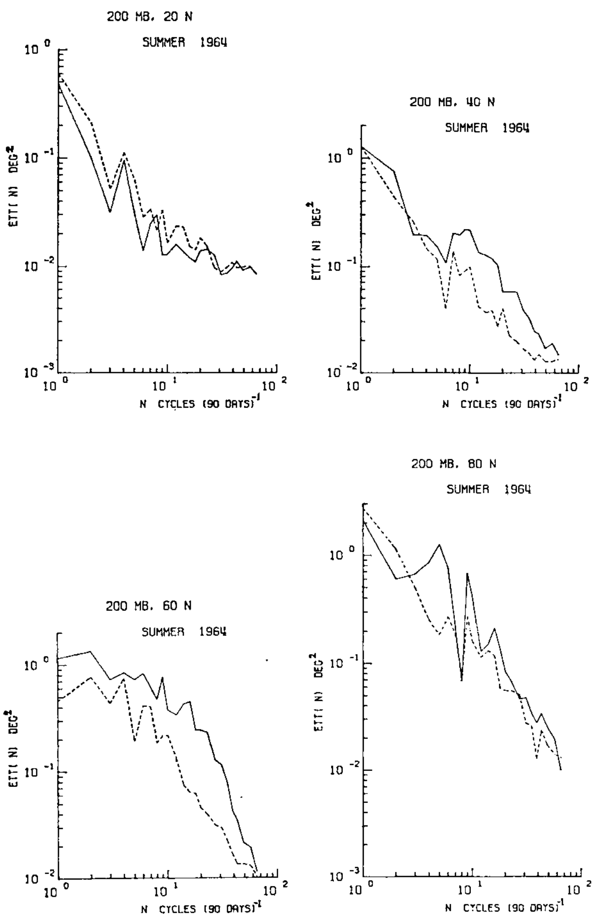

FIG. 5. Frequency spectra of temperature at 200 mb, summer, 1964. The solid and dashed curves represent the spectra contributed by the waves moving from west to east and from east to west, respectively.

4. The wavenumber and frequency spectra of temperature

The wavenumber spectra of temperature at 20, 40, 60 and 80N, at 100, 200 and 500 mb for the summer and winter seasons of 1964 have been computed with the use (4) and (5). Because of the limited space, only the spectra at 200 mb are shown in Figs. 3 and 4. In these figures, as a visual aid, the points of the line spectra in the wavenumber domain are connected by solid and dashed lines. The solid lines represent the contribution made by both the stationary and nonstationary waves, whereas the dashed lines represent that made by the stationary waves only. The distribution of the temperature spectra is very similar to that of velocity (Kao and Wendell, 1970; Kao *et al.*, 1970). In the high-wavenumber range, the spectra generally decrease with increasing wavenumber, and are approximately proportional to the −3 power of the wavenumber. It has been found that in the high-wavenumber range the spectra of the zonal and meridional components of the velocity in the atmosphere are also proportional to the −3 power of the wavenumber (Benton and Kahn, 1958; Saltzman and Fleisher, 1962; Kao and Wendell, 1970). This indicates that the structure of the large-scale atmospheric turbulent motion is essentially two dimensional (Kraichnan, 1967; Leith, 1968). Therefore, the structure of the temperature field in the atmosphere is primarily affected by two-dimensional turbulent motion.

The distribution of the frequency spectra of temperature at 200 mb, computed with the use of (4) and (6), are shown in Figs. 5 and 6. In these figures, the solid and dashed curves represent the frequency spectra of temperature contributed by waves moving from west to east and from east to west, respectively. These two figures indicate that there is generally more temperature, i.e., sensible heat, associated with waves moving from west to east than with waves moving from east to west, except that at 20N in summer the reverse is true.

Figs. 5 and 6 also show that the frequency spectra of temperature generally decrease with increasing frequency, and that in the high-frequency range the spectra are approximately proportional to the −1 power of the frequency at all latitudes and seasons. A similar result has been found by Chiu (1960) from analyzing temperature data at selected stations over North America. It has been found in the high-frequency range that the frequency spectra of the zonal and meridional components of velocity are also proportional to the −1 power of the frequency (Kao and Wendell, 1970; Kao *et al.*, 1970). This again indicates that the frequency spectra of temperature in the free atmosphere are primarily affected by the large-scale two-dimensional turbulent motion.

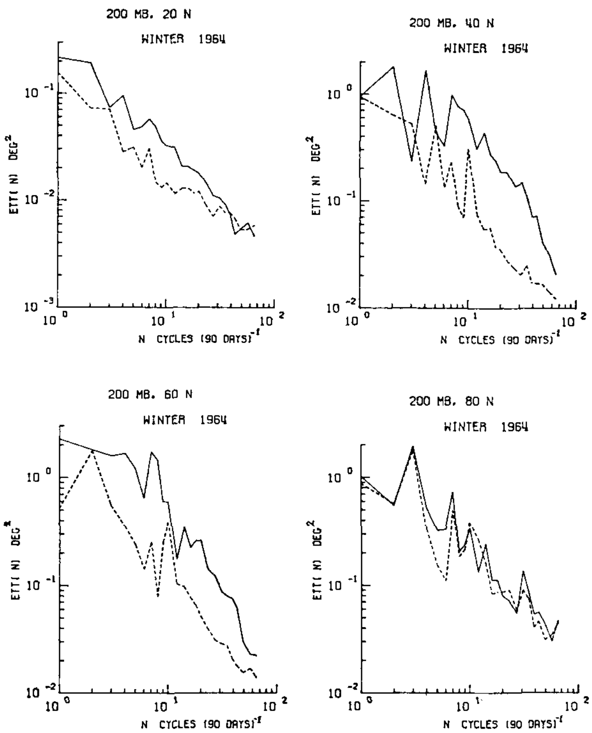

Fig. 6. Same as Fig. 5 except for winter.

5. The mean temperature associated with the motion in the atmosphere

To examine the characteristics of the distribution of the mean temperature associated with the motion in the atmosphere, it is convenient to express the square of the

Table 1. Mean square of temperature (°K) in the Northern Hemisphere.

Season	Level (mb)	Latitude (N)	$\sum_k \sum_n E_{TT}(k,n)$	$E_{TT}(0,0)$	$\sum_{(n)} E_{TT}(0,n)$ $(n\neq0)$	$\sum_{(k)} E_{TT}(k,0)$ $(k\neq0)$	$\sum_{(n)} \sum_{(k)} E_{TT}(k,n)$ $(n\neq0, k\neq0)$
Winter	100	80	22107.9	22086.7	8.5	7.5	5.2
		60	23344.6	23322.2	0.5	10.5	11.4
		40	22918.4	22911.8	1.1	1.9	3.6
		20	20457.0	20449.6	3.6	1.8	2.0
	200	80	22804.2	22791.7	4.9	2.7	4.9
		60	23351.7	23333.8	0.4	5.2	12.3
		40	23666.8	23653.0	0.7	3.7	9.4
		20	23663.6	23661.6	0.2	0.8	1.0
	500	80	26943.2	26926.2	5.2	2.0	9.8
		60	28725.5	28706.4	0.9	8.9	9.3
		40	31127.1	31112.9	0.8	4.8	8.6
		20	35005.0	35001.5	0.2	1.5	1.8
Summer	100	80	26518.5	26516.0	1.3	0.2	1.0
		60	25189.0	25187.0	0.1	0.3	1.6
		40	22271.2	22264.5	1.6	1.3	3.8
		20	19979.1	19975.4	0.2	1.8	1.7
	200	80	26348.8	26339.2	3.3	0.4	5.9
		60	24816.0	24804.8	0.4	1.2	9.6
		40	24207.6	24197.7	0.5	5.0	4.4
		20	24185.1	24180.7	0.4	2.8	1.2
	500	80	30943.7	30935.4	3.3	0.6	4.4
		60	32491.0	32483.2	1.3	1.5	5.0
		40	34673.1	34667.3	2.1	0.9	2.8
		20	35806.9	35804.7	0.1	1.3	0.8

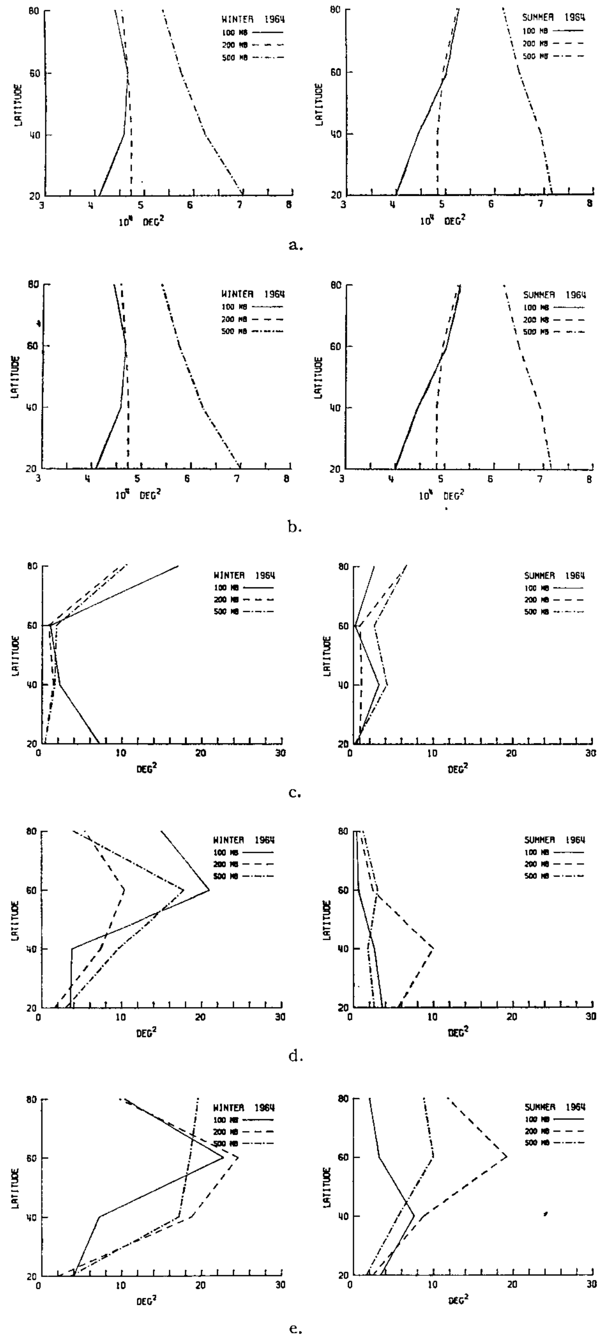

FIG. 7. Square of temperature associated with the total motion (stationary and nonstationary), a., the stationary zonal mean motion, b., the nonstationary zonal mean motion, c., the stationary waves, d., and the nonstationary waves, e.

temperature or sensible heat divided by ρc_p as

$$\sum_k \sum_n E_{TT}(k,n) = E_{TT}(0,0) + \sum_{n\neq 0} E_{TT}(0,n) + \sum_{k\neq 0} E_{TT}(k,0) + \sum_{k\neq 0} \sum_{n\neq 0} E_{TT}(k,n).$$

The term on the left-hand side represents the square of the temperature associated with the total (stationary and nonstationary) motion in the atmosphere. On the right-hand side, the first term represents the square of the temperature associated with the stationary zonal mean motion, the second term the nonstationary zonal mean motion, the third term the stationary wave motion, and the last the nonstationary wave motion. The values of these terms at 100, 200 and 500 mb are computed as shown in Table **1** and are plotted in Figs. 7a–e. It is seen from Fig. 7a that at all levels the square of the temperature associated with the total motion in summer is greater than in winter, except that at 20N, 100 mb the temperature in winter is slightly higher than that in summer. It may be noted that in summer the horizontal gradient of temperature is directed toward the north pole at 500 mb, but toward the equator at 100 and 200 mb. In winter, the gradient of the temperature is directed toward the north pole at 200 and 500 mb and north of 60N at 100 mb, but toward the equator south 60N at 100 mb.

Figs. 7b and 7c show, respectively, the distribution of the square of the temperature associated with the stationary and nonstationary zonal mean motion for the summer and winter of 1964. It may be noted that practically all the sensible heat is associated with the stationary zonal mean motion in the atmosphere.

Figs. 7d and 7e show the distribution of the square of the temperature associated with the stationary and nonstationary waves. They indicate that there is more sensible heat associated with the nonstationary than with the stationary waves. The maximum of temperature associated with the stationary waves occurs near 60N in winter, but shifts to low latitudes in summer. The maximum of the temperature associated with the nonstationary waves occurs near 60N at all levels and seasons, except that at 100 mb in summer it occurs near 40N.

It may be noted that Fig. 7b looks almost identical to Fig. 7a. This is due to the fact that the square of temperature associated with the stationary zonal mean motion represents 99% of that with the total motion.

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