The Kinetic Energy of Large-Scale Atmospheric Motion in Wavenumber-Frequency Space: II. Mid-Troposphere of the Southern Hemisphere

S.-K. KAO[1] AND R. L. JENNE

National Center for Atmospheric Research,[2] Boulder, Colo.

AND J. F. SAGENDORF

Dept. of Meteorology, University of Utah, Salt Lake City

(Manuscript received 27 March 1970)

ABSTRACT

The wavenumber-frequency spectra of the kinetic energy of the zonal and meridional components of the motion in the mid-troposphere of the Southern Hemisphere show a definite spectral domain of wave activities. This spectral domain is generally oriented from a region of low wavenumbers and low frequencies to a region of high wavenumbers and negative frequencies designated for waves moving from west to east. The wavenumber-frequency spectra of the large-scale motion indicate that wave activities in the summer have the same intensity as in the winter in the Southern Hemisphere, whereas in the Northern Hemisphere the wave intensity in summer is about 50% of that in winter.

The frequency spectra of the kinetic energy of the zonal and meridional components of the motion show similar distributions at all latitudes and seasons for the respective components of the motion. In the high-frequency range, the frequency spectra of both the zonal and meridional motion are approximately proportional to the -1 power of the frequency.

The wavenumber spectra of the kinetic energy of the zonal and meridional motion also show a similar distribution at all latitudes and seasons for the respective components of the motion. In the high-wavenumber range, the spectra of both the zonal and meridional components of the motion are approximately proportional to the -3 power of the wavenumber, which is characteristic of the wavenumber spectrum for the two-dimensional flow of an incompressible viscous fluid. The fact that the wavenumber and frequency spectra are proportional to different powers of wavenumber and frequency indicates that Taylor's transformation does not apply to the large-scale motion in the atmosphere.

The mean kinetic energy of the zonal motion in the mid-troposphere of the Southern Hemisphere shows a maximum near 40S in winter and 50S in summer, with 75% of the kinetic energy of the zonal motion being associated with the stationary mean zonal motion and 25% with the zonal component of the moving waves.

The mean kinetic energy of the meridional component of the motion shows a maximum at 50S for both the summer and winter seasons. Practically all the kinetic energy of the meridional motion is associated with the moving waves.

1. Introduction

In Kao and Wendell (1970), referred to as I, the wavenumber-frequency spectra of the kinetic energy of large-scale atmospheric motion at the 100-, 200- and 500-mb levels in the Northern Hemisphere were analyzed. It was found that: 1) a definite spectral domain of wave activities exists in the atmosphere; 2) in the high-frequency range, the spectra of both the zonal and meridional components of the motion are approximately proportional to the -1 power of the frequency; 3) in the high-wavenumber range, the spectra are approximately proportional to the -3 power of the wavenumber; 4) the kinetic energy associated with wave activities is about 30% of the total kinetic energy; 5) the kinetic energy associated with the moving waves is of the same order of magnitude as that with stationary waves; 6) the wave activity in winter is generally greater than that in summer; 7) the resultant of the nonlinear interactions due to the longitudinal convergence of flux of kinetic energy always supplies kinetic energy to the eastward moving waves, but abstracts kinetic energy from the westward moving waves; 8) the nonlinear interactions due to the latitudinal convergence of the flux of kinetic energy are generally smaller than and of the opposite sign to those due to the longitudinal convergence of the flux of kinetic energy; and 9) the ageostrophic mechanism always supplies kinetic energy to the westward moving waves but abstracts kinetic energy from the eastward moving waves.

In view of the difference in the land-sea distribution and the intensity of the general circulation in the Northern and Southern Hemispheres, the question arises as to what are the differences and similarities in

---

[1] Permanent affiliation: The University of Utah, Salt Lake City.

[2] The National Center for Atmospheric Research is sponsored by the National Science Foundation.

the spectral characteristics in the two hemispheres. The purpose of this paper is to investigate these characteristics.

The horizontal velocities used in this analysis are derived from the 1957–58 International Geophysical Year (IGY) isobaric height analyses at 500 mb. The basic grid point data were on a diamond latitude-longitude grid with points at each 5° of latitude and 10° of longitude read to the nearest 10 *m* of geopotential height. The height data were first interpolated to complete the grid for each 5° of latitude-longitude. Geostrophic winds were then calculated from height differences taken over 5° of latitude or longitude, centered about the desired location of the wind. In these calculations, "Spring" is the period September, October, November 1957, "Summer" is December 1957, January, February 1958, "Autumn" is March, April, May 1958, and "Winter" is June July, August 1958.

2. Wavenumber-frequency spectra of the zonal and meridional components of velocity

In order to investigate the effect of waves of various wavelengths and frequencies on the distribution of the

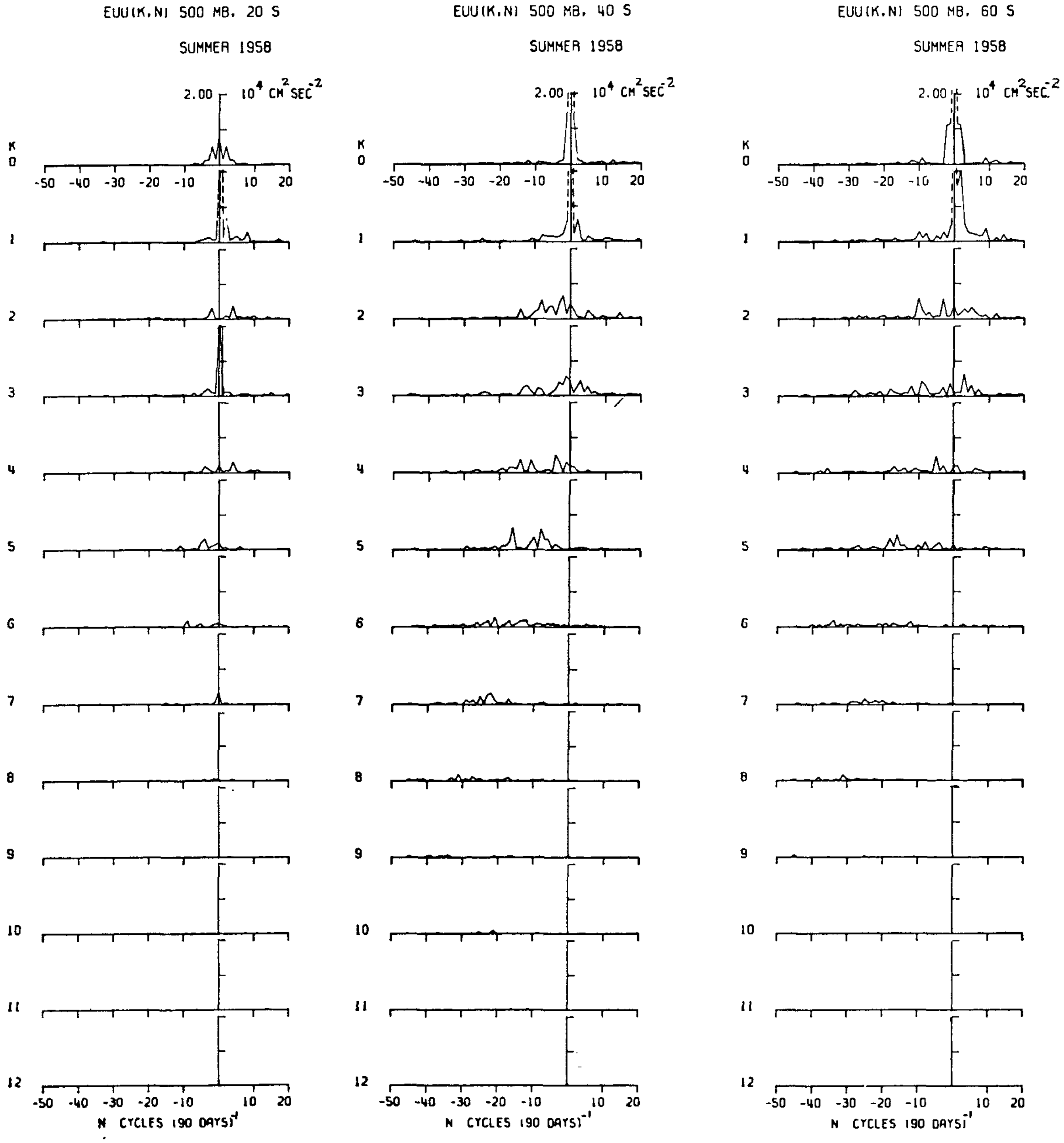


FIG. 1. Wavenumber-frequency spectra of the zonal velocity at 500 mb, Southern Hemisphere, summer 1958.

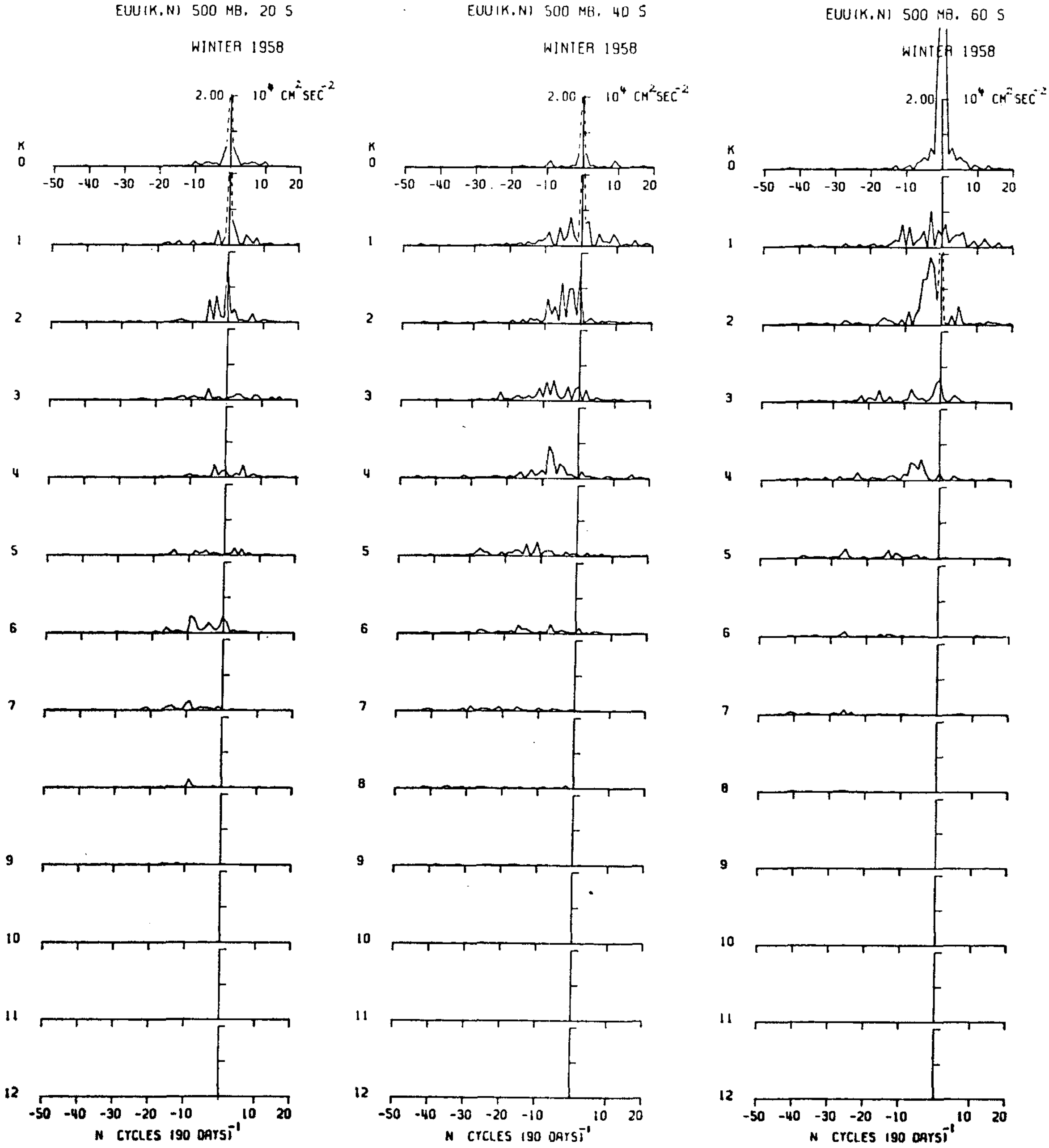


FIG. 2. Same as Fig. 1 except for winter.

kinetic energy of large-scale motion in the atmosphere, it is necessary to analyze the longitude-time spectra of the zonal and meridional components of the motion. To do so the wavenumber-frequency spectra of the zonal and meridional components of the velocities at 20, 30, 40, 50, 60 and 70S at the 500-mb level for the four seasons of 1958 have been computed. Because of the limited space only the power spectra of the velocities at 20, 40 and 60S for the summer and winter seasons are presented here in Figs. 1–4. In these figures, the vertical axis represents the spectral eddy kinetic energy, and the horizontal axis the frequency in units of cycles (90 days)$^{-1}$, the positive and negative frequencies being designated to waves moving from east to west and from west to east, respectively.

It is seen from Figs. 1–4 that there exists a preferred spectral band in the power spectra of the zonal and meridional components of the velocities at various latitudes, which indicates the wavenumber-frequency domain of wave activities in the Southern Hemisphere. In middle and high latitudes, the spectral band is oriented in a domain extending from a region of low wavenumbers and frequencies to a region of high wavenumbers and negative frequencies. It may be noted that in the Southern Hemisphere the intensity of the wave activities in the summer is about same as in the winter. This is not so in the Northern Hemisphere in which the intensity of the wave activity in summer is about half of that in winter as shown in I.

Comparing the wavenumber-frequency spectra of the zonal and meridional components of the velocity at 500 mb in the Southern Hemisphere with those in the Northern Hemisphere, it is found that there is more kinetic energy associated with the fast eastward moving waves in the Southern Hemisphere than in the Northern Hemisphere. It is also found that the kinetic energy associated with the long stationary waves ($k=1$, 2) in the winter of the Southern Hemisphere is less than in the Northern Hemisphere. It may be noted that most of the spectral energy of the zonal component of the motion appears at low wavenumbers, whereas that of the meridional component occurs at medium wavenumbers. This indicates that long waves associate more with the zonal component of the motion, and the medium size waves with the meridional motion.

The question may arise as to whether the shorter

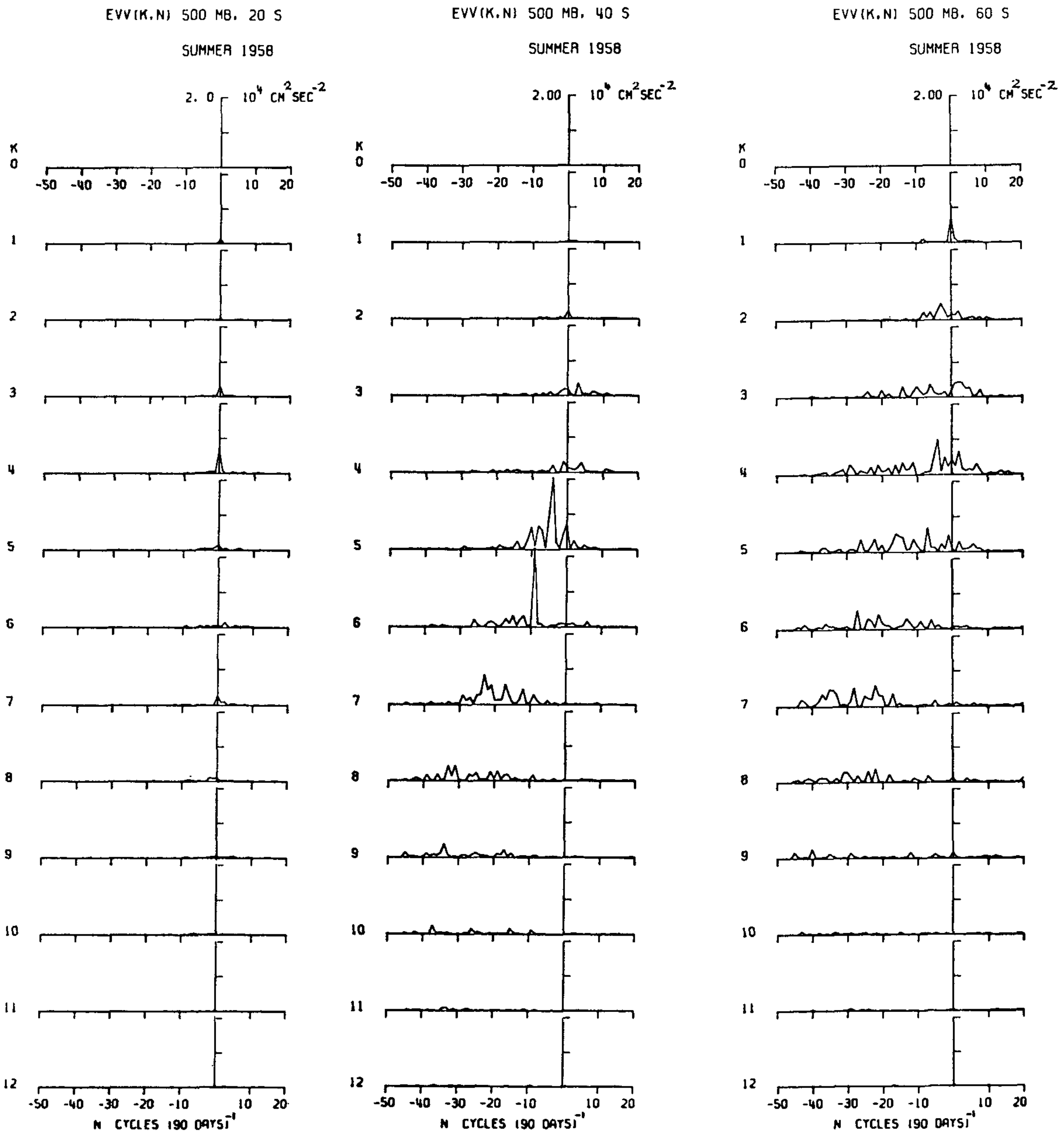


FIG. 3. Wavenumber-frequency spectra of the meridional velocity at 500 mb, Southern Hemisphere, summer 1958.

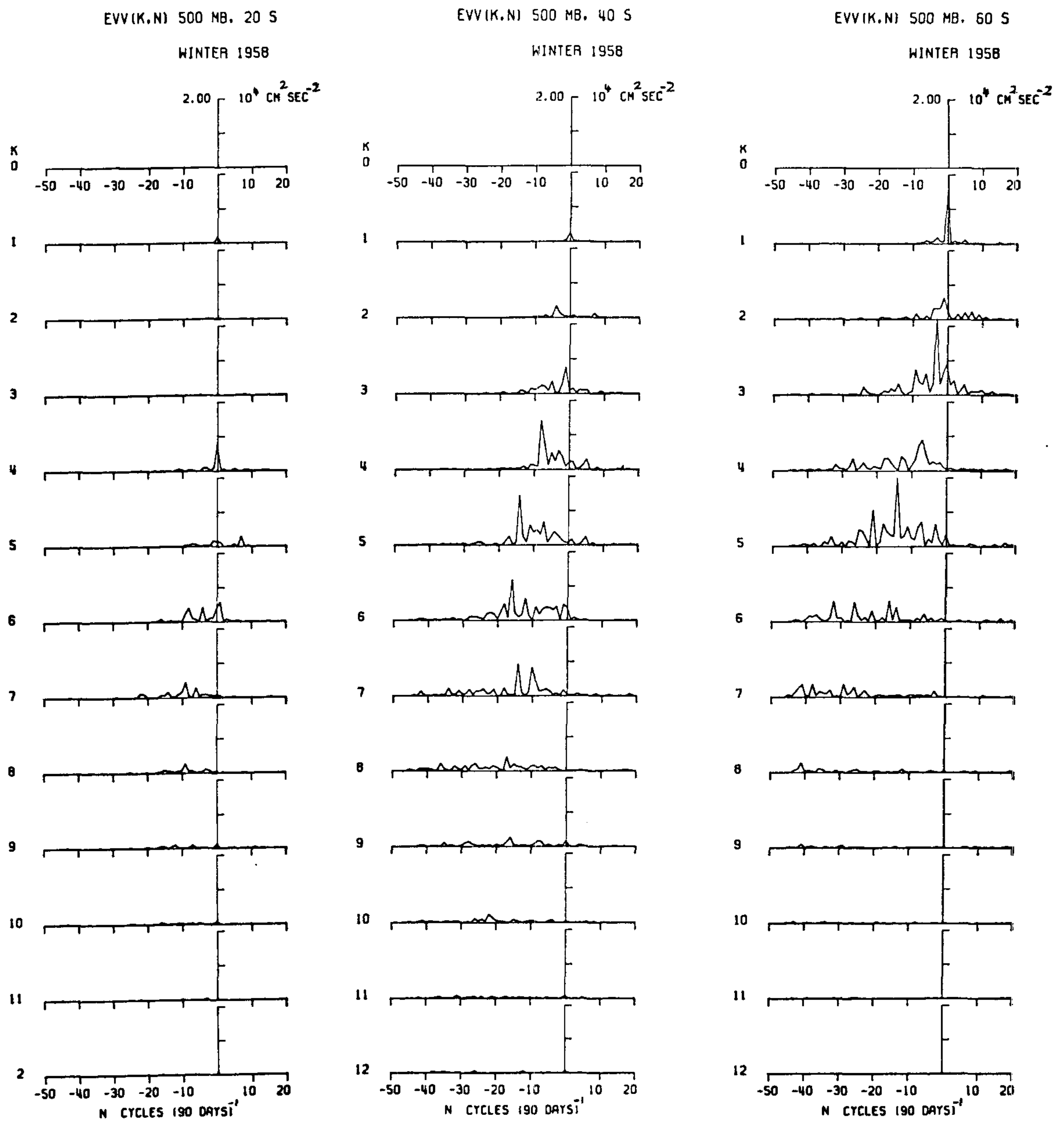


FIG. 4. Same as Fig. 3 except for winter.

waves are strongly smoothed and the longer waves enhanced in the limited data areas of the Southern Hemisphere. The 500-mb analyses were built up by thickness addition from sea level pressure where the data were more plentiful. In addition, the analyses were done late enough that continuity could be applied both backward and forward in time, and the characteristic size of the pressure systems was preserved between points having observations. Furthermore, calculations of 24-hr height changes for every grid point indicate that only a small amount of the amplitude of the systems has been lost in the regions between observing stations. For these reasons it seems likely that waves as short as wavenumber 12 may, on the average, be smoothed somewhat, but that waves any longer are little affected in the analyses.

3. Spectra of the zonal and meridional components of velocity in the wavenumber and frequency domains

The distribution of the power spectra of the zonal and meridional components of velocity in the wavenumber domain at 500 mb in the Southern Hemisphere for the summer and winter seasons of 1958 have been computed

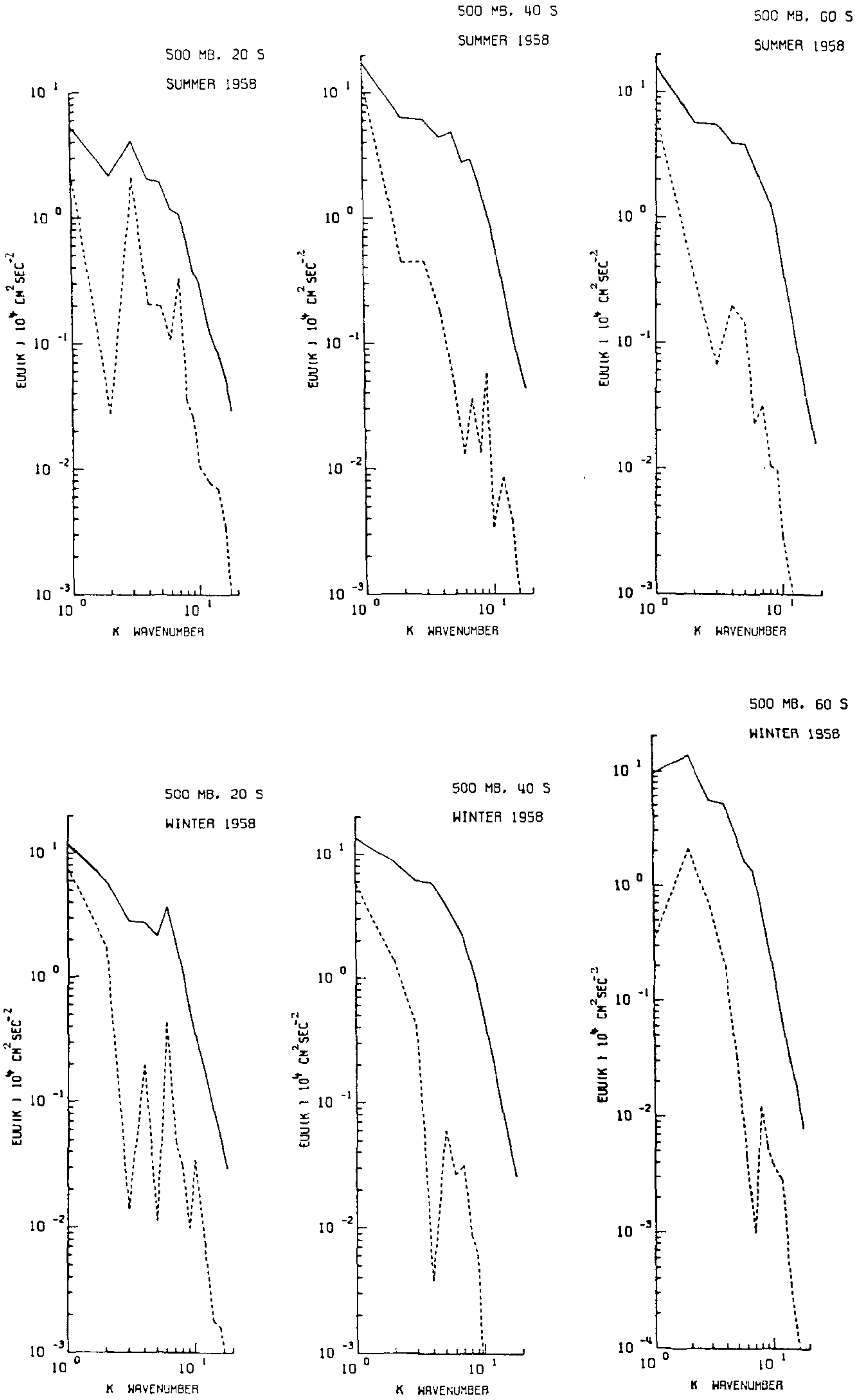


Fig. 5. Wavenumber spectra of the zonal velocity at 500 mb, Southern Hemisphere, 1958. The solid curves represent the spectra due to both the stationary and moving waves, the dashed curves those due to the stationary waves only.

with the use of Eqs. (6) and (8) of I, and are shown in Figs. 5 and 6. In these figures, as a visual aid, the points of the line spectra in the wavenumber domain are connected by solid and dashed lines. The solid lines represent the contribution made by both the stationary and nonstationary waves, whereas the dashed lines represent that made by the stationary waves. It is seen that most of the kinetic energy is associated with moving waves.

The wavenumber spectra of the zonal component of the velocity as shown in Fig. 5 generally decrease with

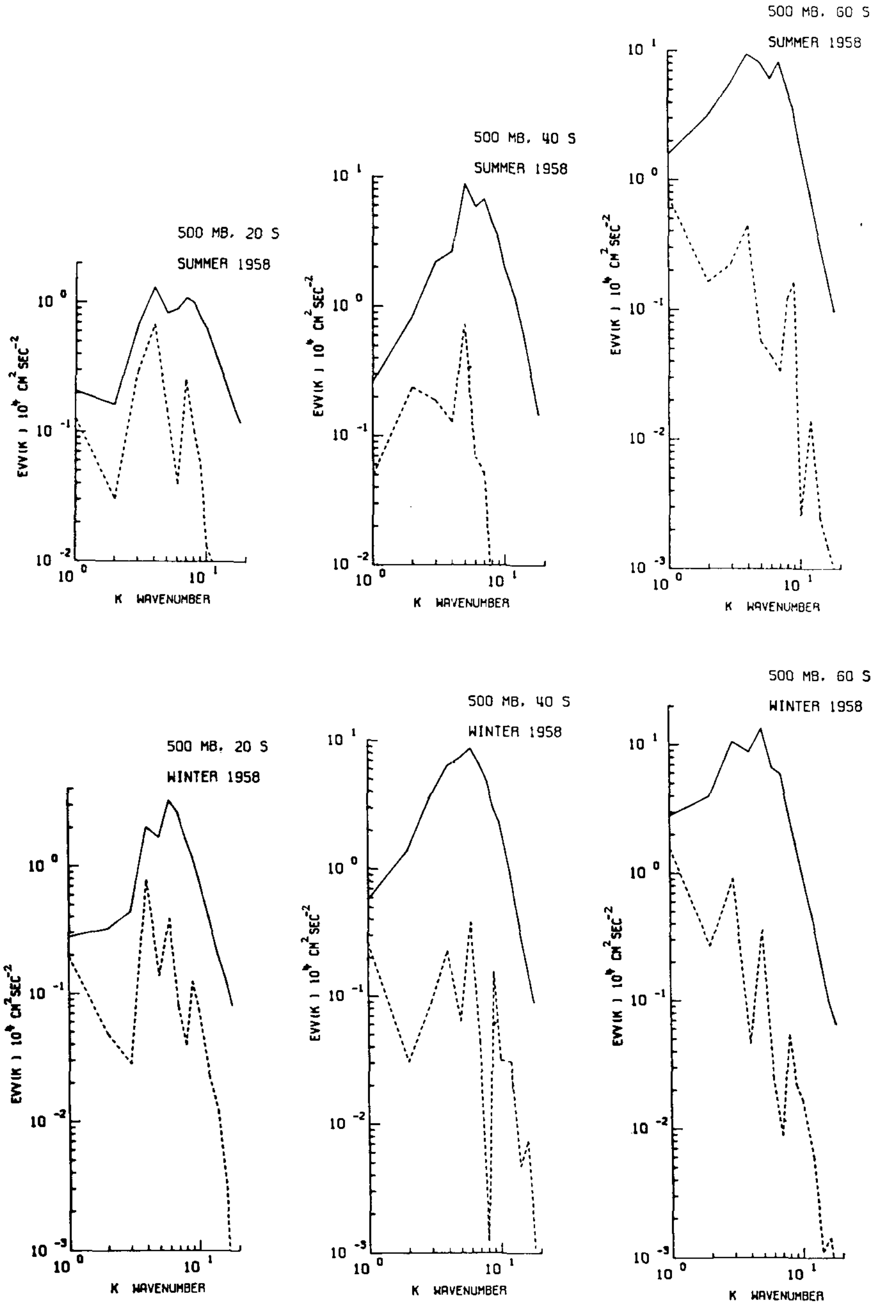


FIG. 6. Wavenumber spectra of the meridional velocity at 500 mb, 1958. The solid and dashed curves have the same representation as in Fig. 5.

increasing wavenumber, whereas those of the meridional component as shown in Fig. 6 show an energy peak near wavenumber 6. It may be noted that there is no kinetic energy involved in the power spectra of the meridional velocity at zero wavenumber, since the zonal mean of the meridional velocity computed from the isobaric height is zero.

In the high-wavenumber range, the energy spectra of both the zonal and meridional components of the velocity are approximately proportional to the -3 power of the wavenumber. In their studies of the nonrotating flow of an incompressible viscous fluid, Kraichnan (1967), Leith (1968) and Lilly (1969) found that the enstrophy of the motion is transferred to the

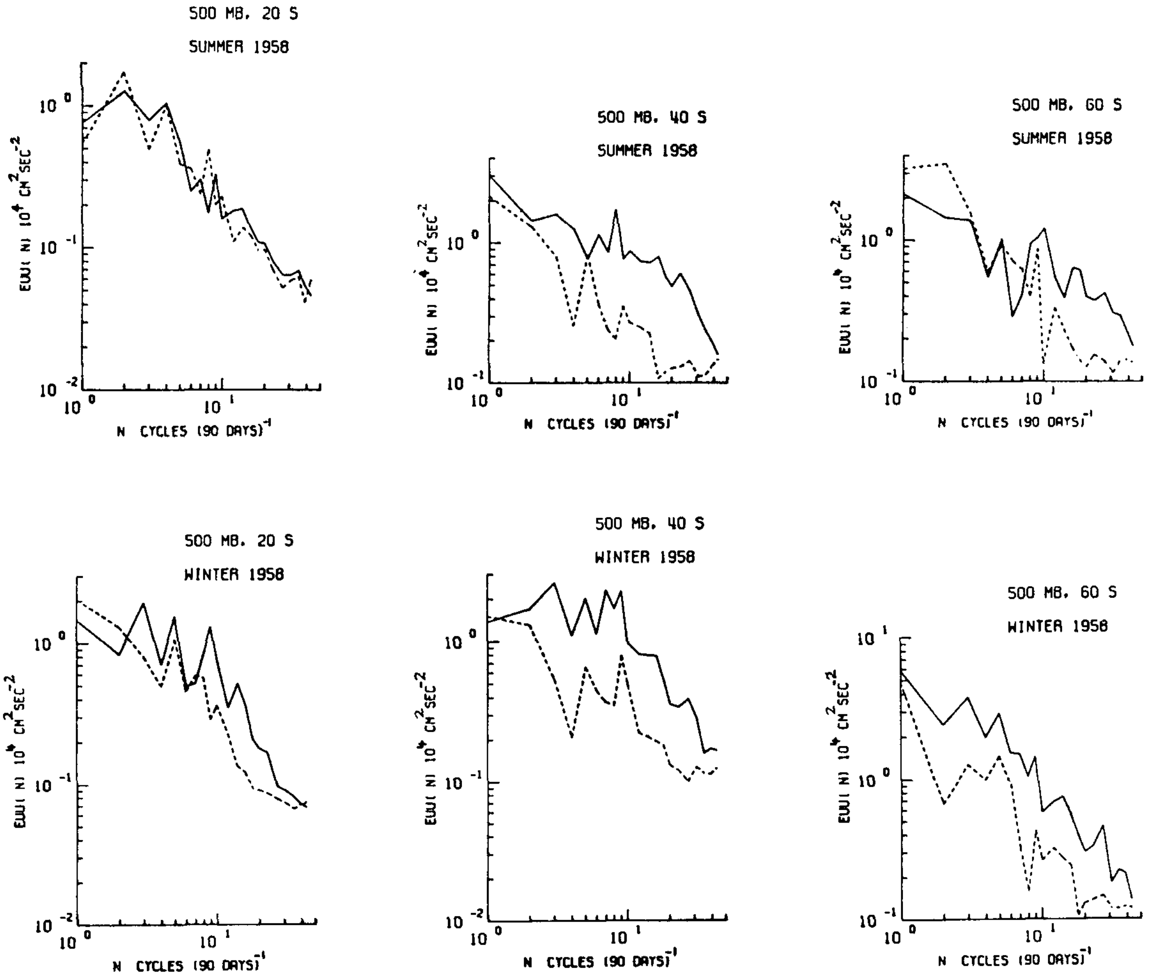


Fig. 7. Frequency spectra of the zonal velocity at 500 mb, Southern Hemisphere, 1958. The solid and dashed curves represent the spectra contributed by the waves moving from west to east and from east to west, respectively.

higher wavenumber through a k^{-3} spectrum, whereas the kinetic energy of the motion is transferred to the lower wavenumbers through a $k^{-5/3}$ spectrum. The spectral distribution in the high-wavenumber range is very similar to those shown in Figs. 5 and 6. As shown in I, the spectra of long atmospheric waves generally depend on the latitudinal variation of the Coriolis parameter. However, in the high-wavenumber range the value of the latitudinal variation of the Coriolis parameter is small, and the spectra of the motion in the rotating system thus behave similarly to those in a nonrotating system.

The power spectra of the zonal and meridional components of the velocity in the frequency domain are shown in Figs. 7 and 8. In these figures, the solid curves represent the spectra of the velocity associated with the waves moving from west to east, and the dashed curves those moving from east to west. It is seen that the frequency spectra of the zonal component of the velocity generally decrease with increasing frequency, and that there is more kinetic energy associated with the waves moving from west to east than with waves moving from east to west, except in the low-frequency range of the spectra.

The frequency spectra of the meridional component of the velocity (Fig. 8) show energy peaks in the frequency range 3–20 cycles (90 days)$^{-1}$, which reflects the effect of the cyclone waves. Again, there is generally more kinetic energy associated with waves moving from west to east than with waves moving from east to west.

In the high-frequency range of the spectra, the frequency spectra of both the zonal and meridional components of the velocity are approximately proportional to the -1 power of the frequency at 20 and 40S. At 60S in summer, however, the slope of the spectra is not

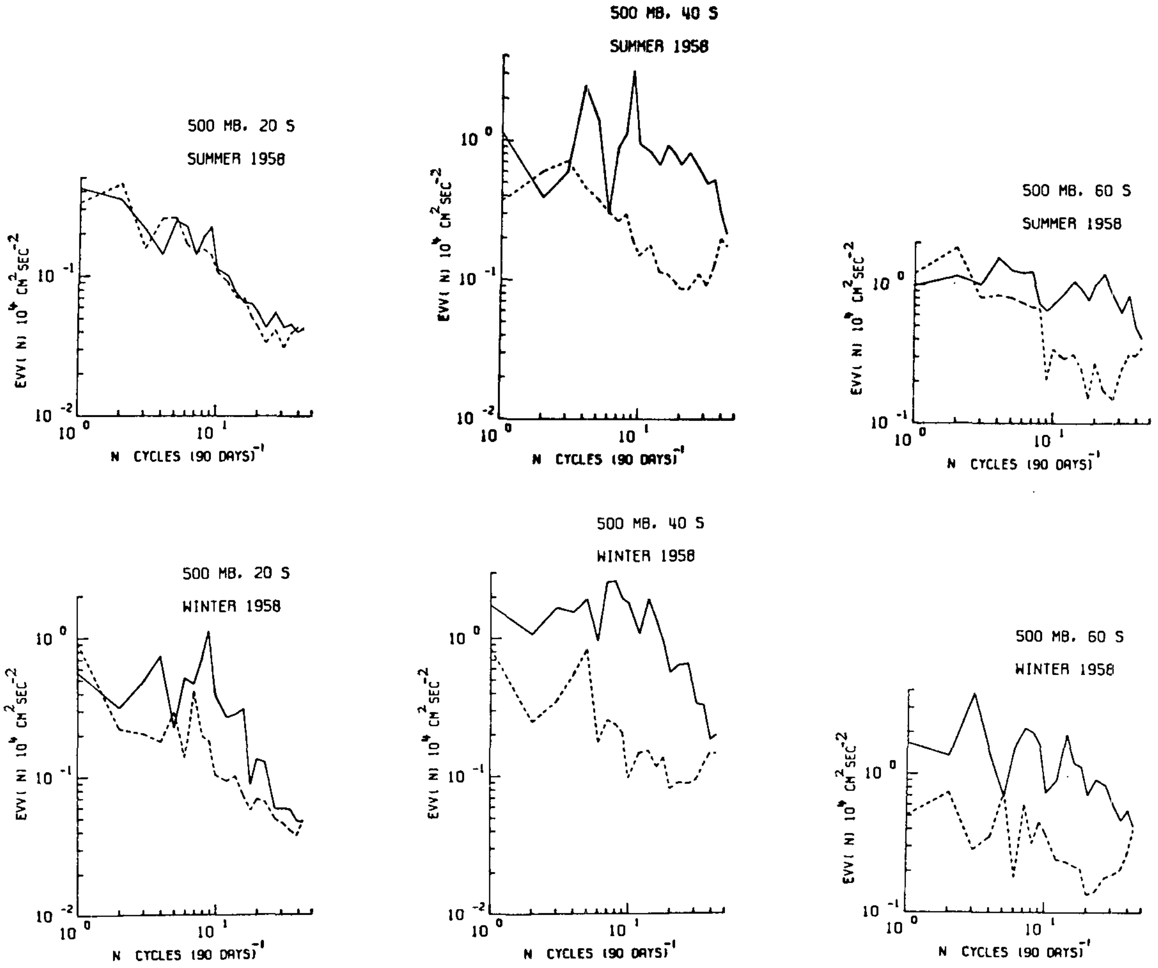


FIG. 8. Frequency spectra of the meridional velocity at 500 mb, 1958. The solid and dashed curves have the same representation as in Fig. 7.

as steep as at lower latitudes, whereas in the Northern Hemisphere the slope at higher latitudes is steeper than at lower latitudes.

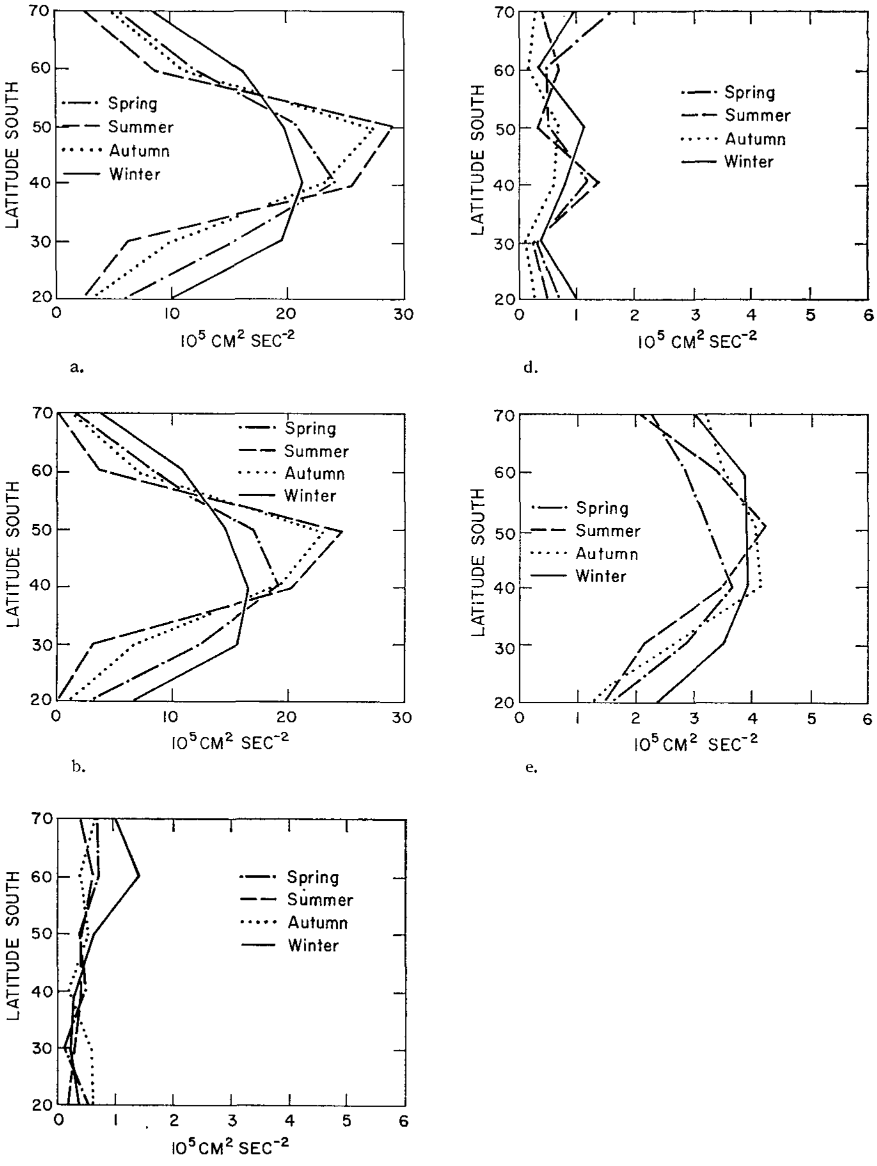
4. The mean kinetic energy of the zonal and meridional components of the velocity

To examine the characteristics of the distribution of the mean kinetic energy in the atmosphere, it is convenient to express the mean kinetic energy of the ith component of the motion (see I) as

$$\sum_k \sum_n E_{ii}(k,n) = E_{ii}(0,0) + \sum_{\substack{n \\ n\neq 0}} E_{ii}(0,n) + \sum_{\substack{k \\ k\neq 0}} E_{ii}(k,0) + \sum_{\substack{k \\ k\neq 0}} \sum_{\substack{n \\ n\neq 0}} E_{ii}(k,n), \tag{15}$$

where $i=u, v$. The first term on the right-hand side of the equation is the kinetic energy of the ith component of the stationary zonal mean motion, the second term that of the nonstationary zonal mean motion, the third term that of the stationary wave motion, and the last that of the nonstationary wave motion. The values of these terms for the zonal and meridional components of the motion are plotted in Figs. 9 and 10. It may be noted, for the meridional component of the motion, that the first and second terms on the right-hand side of the above equation are zero, since the zonal mean of the meridional velocity computed from the geostrophic wind equation is zero. The values of the various components of (15) for the Southern and Northern Hemispheres are given in Tables 1–4.

It is seen from Fig. 9a that the mean kinetic energy of the zonal velocity in winter is slightly greater than that in summer. However, the maximum kinetic energy for the winter jet is less than that for the summer jet at this



a.

d.

b.

e.

c.

Fig. 9. Mean kinetic energy of the zonal motion, a., of the zonal component of the stationary zonal mean motion, b., of the zonal component of the nonstationary zonal mean motion, c., of the zonal component of the stationary waves, d., and of the zonal component of the moving waves, e.

level. We should note, however, that while the mid-latitude jet is weakening somewhat from summer to winter, the subtropical jet near 28S (centered near 200 mb) and the stratospheric jet near 55S are increasing markedly in strength (van Loon *et al.*, 1970). In the Northern Hemisphere, much less energy is associated with the summer jet.

The distribution of the kinetic energy of the zonal component of the stationary zonal mean motion shown in Fig. 9b is very similar to that of the total motion (Fig. 9a) for the respective season, and represents ~75% of the kinetic energy of the zonal component of the total motion.

The kinetic energy of the zonal component of the nonstationary zonal mean motion (Fig. 9c) is small as compared with that of the total motion (Fig. 9a) and of the moving waves (Fig. 9e).

The kinetic energy of the zonal component of the stationary waves as shown in Fig. 9d is also small as compared with that of the total motion and of the mov-

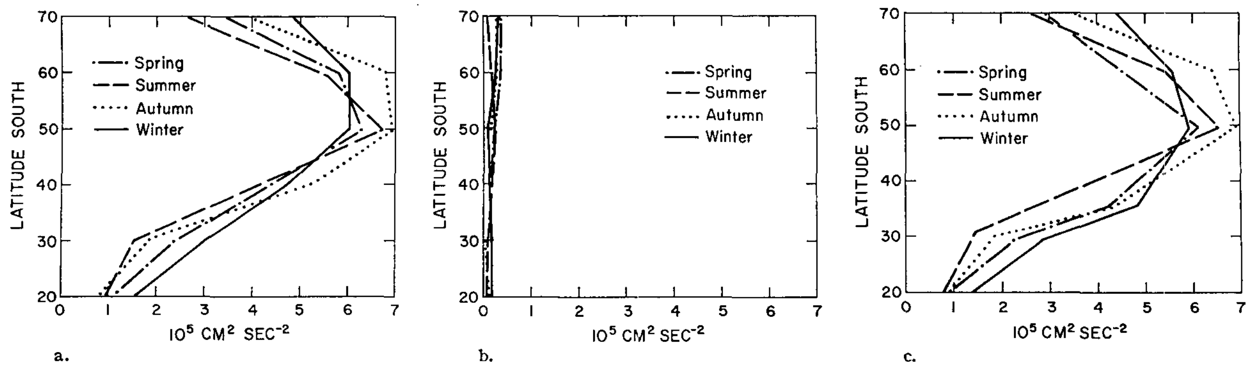


FIG. 10. Mean kinetic energy of the meridional component of the motion, a., of the meridional component of the stationary waves b., and of the meridional component of the moving waves, c.

ing waves. This is not so in the Northern Hemisphere (Kao and Wendell, 1970).

The kinetic energy of the zonal component of the moving waves (Fig. 9e) shows a more or less even distribution in the mid-troposphere with a slight maximum near 50S for summer and 40S for winter. The kinetic energy associated with the moving waves amounts to ~25% of the kinetic energy of the total zonal motion.

The kinetic energy of the meridional component of the total motion (Fig. 10a) shows similar distributions for the summer and winter seasons of 1958, with a maximum occurring near 50S. There is little kinetic energy associated with the meridional component of the stationary waves (Fig. 10b). In contrast to the Southern Hemisphere, there is a sizable amount of kinetic energy associated with the meridional component of the stationary waves in the Northern Hemisphere as shown

TABLE 1. Kinetic energy of the zonal component of the motion at 500 mb in the Southern Hemisphere: units, 10^4 cm² sec⁻².

Season	Latitude S	$\sum_k \sum_n E_{uu}(k,n)$	$E_{uu}(0,0)$	$\sum_{n\neq 0} E_{uu}(0,n)$	$\sum_{k\neq 0} E_{uu}(k,0)$	$\sum_{k\neq 0} \sum_{n\neq 0} E_{uu}(k,n)$
Spring	20	60.57	31.32	4.92	7.92	16.41
	30	157.05	123.97	1.43	3.13	28.52
	40	245.80	191.86	3.81	13.43	36.70
	50	212.10	169.19	4.44	5.52	32.95
	60	122.23	82.61	6.49	4.38	28.75
	70	61.36	15.82	6.32	16.74	22.48
Summer	20	23.11	0.81	2.48	5.31	14.51
	30	61.51	34.65	2.79	2.25	21.82
	40	261.18	206.89	4.35	14.10	35.84
	50	294.11	245.43	4.26	2.65	41.77
	60	91.09	42.60	6.38	7.47	34.64
	70	31.13	1.88	4.05	4.32	20.88
Autumn	20	34.46	12.52	5.78	2.88	13.28
	30	101.50	67.56	5.49	0.97	27.48
	40	241.71	190.51	2.13	6.74	42.33
	50	284.35	230.56	5.09	6.15	42.55
	60	114.18	73.33	4.01	1.75	35.09
	70	54.95	12.99	6.47	3.44	32.05
Winter	20	100.91	63.77	3.75	10.02	23.37
	30	194.76	154.61	1.93	2.78	35.44
	40	214.87	165.65	2.61	7.62	38.99
	50	201.54	145.51	6.14	11.48	38.40
	60	165.20	110.11	14.02	3.35	37.72
	70	89.11	40.07	10.68	8.98	29.38

in I. In view of the fact that little kinetic energy is associated with the meridional component of stationary waves in the Southern Hemisphere (Fig. 10b), practically all the kinetic energy of the meridional component of the motion is associated with moving waves as shown in Fig. 10c. It may be noted that the maximum kinetic energy of the meridional component of the moving waves occurs near the jet stream, indicating that moving waves are most active near the jet core.

From his study of the maintenance of the kinetic energy of the mean zonal flow in the Southern Hemisphere, Obasi (1965) found that transient eddies are the major energy sources of the zonal flow. That most of the kinetic energy is associated with the moving waves rather than with the stationary waves, as found in this analysis, indirectly supports Obasi's finding.

Acknowledgments. This research has partly been supported by the Division of Biology and Medicine of the U. S. Atomic Energy Commission, under Contract AT(11-1)-1585. The spectral computation was made with the use of UNIVAC 1108 at the Computer Center of the University of Utah.

TABLE 2. Kinetic energy of the meridional component of the motion at 500 mb in the Southern Hemisphere: units, 10^4 cm^2 sec^{-2}.

Season	Latitude S	$\sum_k \sum_n E_{vv}(k,n)$	$\sum_{k\neq 0} E_{vv}(k,0)$	$\sum_{k\neq 0} \sum_{n\neq 0} E_{vv}(k,n)$
Spring	20	11.09	1.38	9.71
	30	23.52	1.45	22.07
	40	44.10	1.29	42.81
	50	53.80	2.32	51.48
	60	48.90	3.79	45.11
	70	35.33	4.30	31.03
Summer	20	9.70	1.79	7.91
	30	15.34	1.52	13.82
	40	42.37	1.53	40.84
	50	68.94	2.61	66.33
	60	56.73	2.03	54.70
	70	27.44	0.68	26.76
Autumn	20	8.42	1.22	7.20
	30	17.92	1.16	16.76
	40	45.11	0.98	44.13
	50	71.48	1.91	69.52
	60	68.83	3.40	65.43
	70	39.51	3.25	36.26
Winter	20	16.14	1.99	14.15
	30	30.34	1.92	28.42
	40	48.60	1.40	47.20
	50	60.46	1.36	59.10
	60	59.81	3.31	56.50
	70	48.80	3.50	45.30

TABLE 3. Kinetic energy of the zonal component of the motion in the Northern Hemisphere: units, 10^4 cm^2 sec^{-2}.

Season	Level (mb)	Latitude N	$\sum_k \sum_n E_{uu}(k,n)$	$E_{uu}(0,0)$	$\sum_{n,\, n\neq 0} E_{uu}(0,n)$	$\sum_{k,\, k\neq 0} E_{uu}(k,0)$	$\sum_{k\neq 0} \sum_{n\neq 0} E_{uu}(k,n)$
Winter	100	80	101.9	38.7	20.2	12.3	30.7
		60	192.5	146.1	7.5	3.0	35.9
		40	430.7	330.2	4.4	45.9	50.2
		20	234.2	142.8	6.7	27.3	57.4
	200	80	108.5	30.8	26.5	14.9	36.2
		60	150.8	69.4	8.8	8.4	64.2
		40	655.3	464.5	8.2	80.4	102.2
		20	249.3	187.5	4.5	19.8	41.5
	500	80	66.0	9.2	21.4	7.0	28.4
		60	95.2	23.9	7.5	8.8	55.0
		40	268.3	170.6	4.8	37.6	55.3
		20	53.9	25.8	1.8	8.9	17.4
Summer	100	80	12.4	2.0	3.4	1.6	5.4
		60	21.3	5.0	3.2	1.9	11.2
		40	82.8	41.6	2.9	10.9	27.4
		20	157.3	52.5	4.8	59.4	40.6
	200	80	41.8	9.4	10.8	2.0	19.6
		60	98.0	28.9	7.8	9.5	51.8
		40	335.2	210.3	9.0	25.8	90.1
		20	70.7	0.2	6.0	29.7	34.8
	500	80	43.9	8.3	14.2	1.2	20.2
		60	58.3	10.4	4.4	8.3	35.2
		40	93.7	58.9	2.6	5.4	26.8
		20	19.6	3.6	1.0	2.5	12.5

TABLE 4. Kinetic energy of the meridional component of the motion in the Northern Hemisphere: units, 10^4 cm^2 sec^{-2}.

Season	Level (mb)	Latitude N	$\sum_k \sum_n E_{vv}(k,n)$	$E_{vv}(0,0)$	$\sum_{n \neq 0} E_{vv}(0,n)$	$\sum_{k \neq 0} E_{vv}(k,0)$	$\sum_{k \neq 0} \sum_{n \neq 0} E_{vv}(k,n)$
Winter	100	80	74.5	0.0	0.0	17.1	57.4
		60	74.2	0.0	0.0	29.1	45.1
		40	55.3	0.0	0.0	8.7	46.6
		20	60.4	0.0	0.0	17.7	42.7
	200	80	84.7	0.0	0.0	16.5	68.2
		60	103.4	0.0	0.0	35.0	68.4
		40	104.8	0.0	0.0	13.4	91.4
		20	37.8	0.0	0.0	10.1	27.7
	500	80	57.2	0.0	0.0	9.3	47.9
		60	77.1	0.0	0.0	22.7	54.4
		40	68.0	0.0	0.0	9.4	58.6
		20	15.5	0.0	0.0	2.7	12.8
Summer	100	80	12.6	0.0	0.0	3.0	9.6
		60	15.1	0.0	0.0	2.5	12.6
		40	26.0	0.0	0.0	4.9	21.1
		20	41.2	0.0	0.0	10.7	30.5
	200	80	36.8	0.0	0.0	4.3	32.5
		60	58.0	0.0	0.0	6.2	51.8
		40	92.9	0.0	0.0	9.7	83.2
		20	24.3	0.0	0.0	3.9	20.4
	500	80	35.2	0.0	0.0	1.0	34.2
		60	37.1	0.0	0.0	3.1	34.0
		40	26.9	0.0	0.0	3.5	23.4
		20	9.7	0.0	0.0	1.9	7.8

REFERENCES

Kao, S.-K., 1968: Governing equations and spectra for atmospheric motion and transports in frequency-wavenumber space. *J. Atmos. Sci.*, **25**, 32–38.

——, and L. L. Wendell, 1970: The kinetic energy of the large-scale atmospheric motion in wavenumber-frequency space: I. Northern Hemisphere. *J. Atmos. Sci.*, **27**, 359–375.

Kraichnan, R. H., 1967: Inertial range in two-dimensional turbulence. *Phys. Fluids*, **10**, 1417–1423.

Leith, C. E., 1968: Diffusion approximation for two-dimensional turbulence. *Phys. Fluids*, **11**, 671–673.

Lilly, D. K., 1969: Numerical simulation of two-dimensional turbulence. *Phys. Fluids*, **12**, 240–249.

Obasi, G. O. P., 1965: On the maintenance of the kinetic energy of zonal flow in the southern hemisphere. *Tellus*, **17**, 95–105.

van Loon, H., J. J. Taljaard, R. L. Jenne and H. L. Crutcher, 1970: Climate of the upper air, Part 1, Southern Hemisphere, Vol. II, Zonal and meridional wind components. NAVAIR 50-1C-56, Washington, D. C.