Observations of Diffusional Ice Crystal Growth in Clouds

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ABSTRACT

The observed size of ice crystals obtained by sampling in Formvar is compared to the theoretical size an ice crystal should attain when growing by diffusion. The variables which are not fixed by environmental conditions are adjusted for the best fit. For reasonable values of these parameters the agreement between observation and calculation is good, except in the case where phloroglucinol was the nucleating agent.

The sizes of the ice crystals sampled after the same apparent time and under the same growth conditions varied over a range of 4 to 1; this range is discussed.

The observations, all in natural cloud conditions, are not in complete accord with diffusional growth. The differences are discussed with respect to weather modification.

1. Introduction

There have been many investigations of the growth of ice crystals by diffusional process (Houghton, 1949). The growth characteristics have been calculated from theoretical considerations and demonstrated in laboratory studies. Ice crystals collected from natural clouds often exhibit single-crystal characteristics, but at other times show evidence of more complex growth forms where some other types of collection processes in addition to diffusion have occurred. Nakaya and Terrada (1935) pioneered in observing diffusional growth for several crystalline shapes. Detailed studies of the effects of ice crystal shapes on their growth were performed by McDonald (1963) using an electrostatic analogy. Hallett (1965) has disucssed more aspects of the growth and presented some observations.

The diffusional growth of an ice crystal can be readily computed, provided such factors as turbulence, ventilation and variations of shape are neglected. These factors are not well documented or quantitatively known. As the ice crystal grows larger, some of these factors become increasingly important, as does growth by accretion.

The intent of this paper is to report three series of observations of ice crystal growth. The crystals which were collected appear to have resulted from diffusional growth alone. The crystals in each case were regular, either hexagonal plates or prisms.

The observations, all in natural cloud conditions, serve to demonstrate that the usual diffusional growth equations do not adequately explain all the observations. Some suggestions are advanced which may, after more detailed studies, lead to a better understanding of the growth of ice crystals in supercooled clouds.

2. Observational data

The observations of ice crystal growth were made in a turbulent roll cloud, supercooled fogs, and wave clouds. All the data were analyzed using the same techniques.

In Case I (9 March 1966), a roll cloud in the vicinity of Reno, Nev., was seeded with CO_2 pellets from a Beechcraft airplane at a rate of 5 kg per kilometer of flight. The temperature at cloud base was -8 ± 0.5C at an altitude of 3660 m. The cloud particles were replicated in Formvar on an airborne sampler similar to that described by McCready and Todd (1964).

Fig. 1 shows cloud droplets prior to intercepting the seeded plume, and the ice crystals that were collected, the ice crystals coexisting with water droplets. After passing through the seeded plume, droplets with similar sizes and concentration as Fig. 1a were again observed. The growth time was computed from the wind velocity obtained by chaff tracked with radar. This was done throughout the seeding and sampling periods. Using these data, and neglecting small-scale turbulence, the most probable ice crystal growth time was computed to be close to 180 sec. The aircraft tracks and wind velocity are shown in Fig. 2.

Two further sets of natural cloud observations (Cases II and III) were made while one of the authors (J. A. W.) was participating in the seventh Yellowstone Field Research Expedition (Schaefer, 1967). Opportunities existed for seeding slowly drifting supercooled fogs in the Old Faithful basin, at an altitude of 2195 m.

The air within the basin was saturated by the hot springs and geysers, and during the early morning hours when the air was stable, supercooled drainage fogs existed. The measurements of crystal growth were made on several occasions at different locations within the

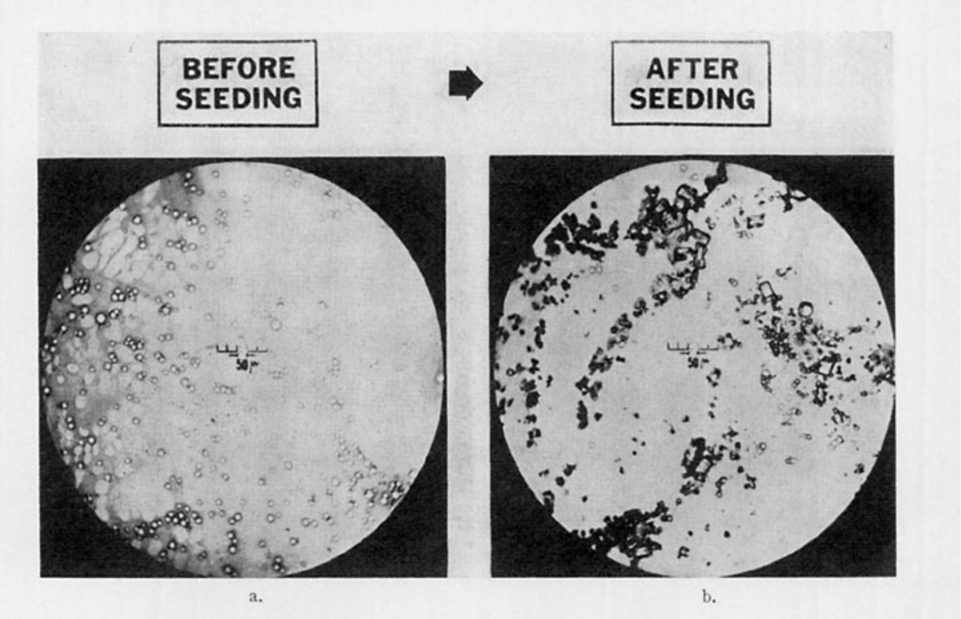

FIG. 1. Ice crystals and droplets collected by the sampler when penetrating the cloud.

basin. The temperature 20 ft above the ground, during the period of observations reported here, was −12±1C.

The growth times for the ice crystals in these cases were computed from a knowledge of the time difference between the start of seeding and the arrival at the observing point of the first ice crystals. Since virtually no natural ice crystals occur in these conditions, the effects of seeding were very easily observed. Replicas of the ice crystals were made using the technique described by Schaefer (1964). A typical replica is shown in Fig. 3.

In Case II the seeding agent was silver iodide, while in Case III, phloroglucinol nucleated the ice crystals.

Two other Cases, IV and V, consisted of observations made in wave cloud flights conducted near Reno, on 15 November 1966. Again the wind velocity was determined by radar observations of chaff, dropped before and after seeding. This velocity was used to predict the interception of the seeded parcel by the sampling aircraft.

In Case IV the transit time of the seeding material in the cloud prior to sampling is questionable.

In Case V the growth time was reasonably well known, but since few droplets were observed with the ice crystals, saturation vapor pressure with respect to water was not as valid an assumption as in the other cases. In Case IV the cloud was seeded with AgI flares at an altitude of 5180 m, where the temperature was −14±0.2C. The average liquid water content in the cloud was less than 0.2 gm m^{-3} and the turbulence was extremely light. (This is the case for 90% of all wave cloud penetrations that have been made by this laboratory over the Eastern Sierra.)

In Case V the seeding was performed with CO$_2$ pellets at 5030 m and the temperature was −13±0.2C. On this penetration the liquid water content was very low (<0.1 gm m^{-3}), and again the turbulence was very light.

3. Growth calculations

The size of an ice crystal growing by diffusion can be affected by many factors. Among these are: 1) variations in the density of the crystal due to gaps, 2) variations in the ratio of the length or side of the ice crystal to its thickness, 3) the ventilation of the ice crystal, and 4) the "capacitance" of the ice crystal when its shape during growth is not hexagonal. This has been experimentally determined by McDonald (1963).

Those factors whose influences are well known can be used directly in the diffusive equation for ice crystal growth as given by Byers (1965), i.e.,

$$\frac{dm}{dt}=\frac{4\pi C[S_e-1]}{[L_s{}^2M_w/(KRT^2)]+[RT/(DM_wp_eT)]}, \qquad (1)$$

where C is the shape factor, S_e the ratio of the vapor pressure to that of the ice, L_s the latent heat of sublimation, K the thermal conductivity, D the diffusivity, p_e the saturation vapor pressure with respect to water at temperature T, and the other symbols have their standard meanings.

For a given set of environmental conditions, this can be rewritten as

$$\frac{dm}{dt}=4\pi CG(e,p,T), \qquad (2)$$

where G is a thermodynamic function representing the terms of Eq. (1) in brackets.

The ventilation coefficient which, physically, represents the transport of mass and heat away from the

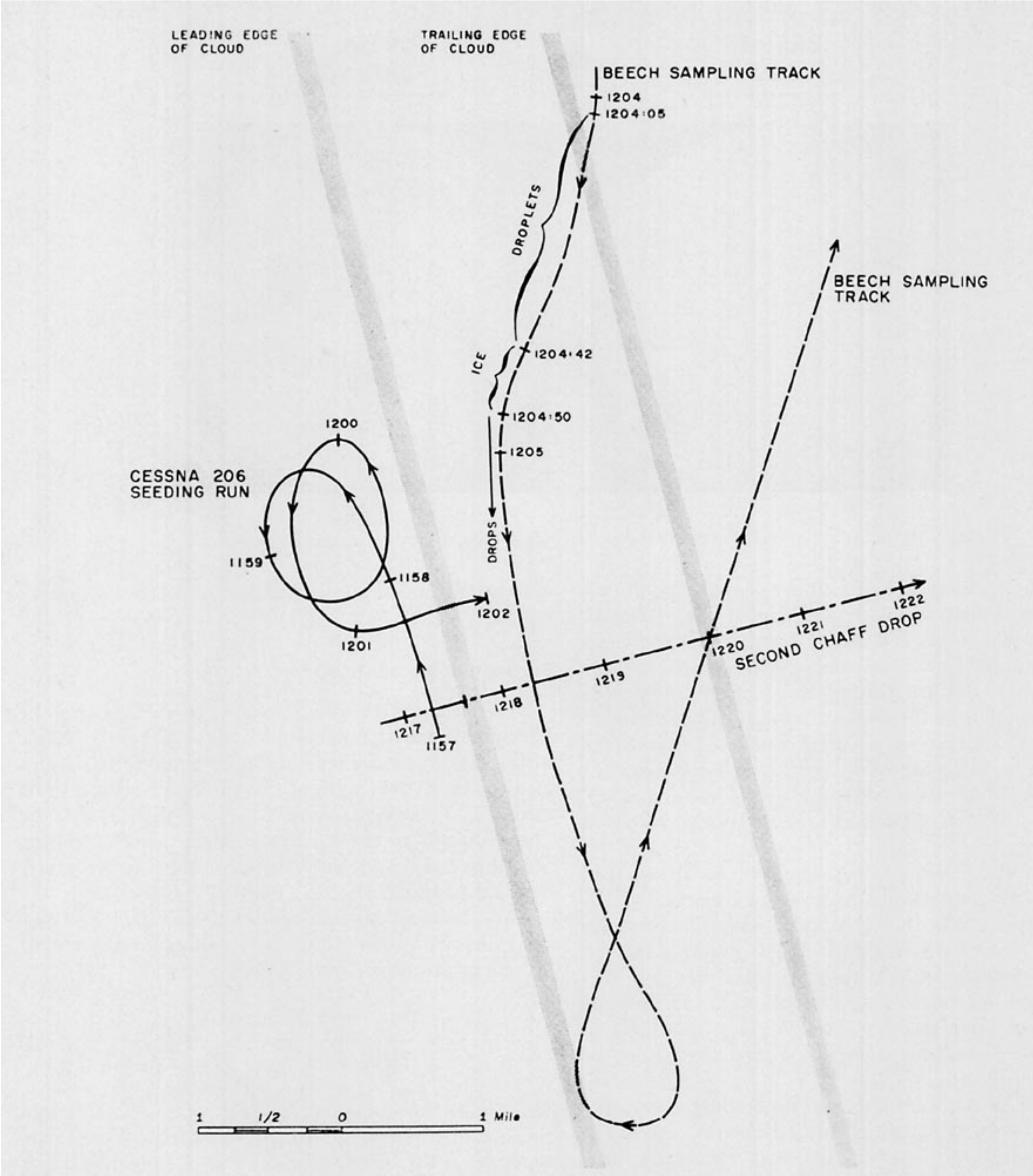

FIG. 2. The track of the seeding and sampling aircraft for Case I. This analysis technique is typical.

growing surfaces of the crystal due to its motion through the air, will typically take values from 1.0–1.3 for spherical drops in free fall.

A shape factor multiplier can be used to correct for changes in capacitance from a standard shape. McDonald (1963) shows the variation to range from 0.6–1.0.

4. Prismatic crystal calculations

One of the cases under study involved hexagonal prism crystals. The standard shape factor for these crystals is given by $C=a[\ln(2a/b)]^{-1}$, where a is the half-length of the major axis and b the length of the hexagonal side of the prism. After putting $\tau=a/b$ and defining ρ_i as the mean ice crystal density, (2) becomes

$$\frac{1.164\ln(2\tau\rho_i)a\,da}{\tau^2}=G(e,p,T)dt. \qquad (3)$$

On integration when ρ_i and τ are held constant, this yields

$$a_2^2-a_1^2=\frac{2\tau^2G(e,p,T)(t_2-t_1)}{\rho_i\ln 2\tau} \qquad (4)$$

Eq. (4) was solved for different values of ρ_i with $\tau=1$, the observed ratio. The results of these computations are plotted in Fig. 4.

5. Hexagonal plate calculations

In the other four cases under study the ice crystals were hexagonal plates.

In order to solve Eq. (2) for this case it is necessary to make an assumption concerning the ratio of the thickness of the crystal to its side length. The simplest tractable assumption is that this ratio is a constant over short time intervals. Letting δ be this value and substituting for the other parameters, Eq. (2) becomes

$$S_2^2-S_1^2=\frac{2.1429G(e,p,T)(t_2-t_1)}{\rho_i\delta}, \qquad (5)$$

where S is the length of a side of a hexagonal crystal.

Unfortunately there are two unknowns, S_2 and δ, in this equation. It was solved by the following procedures:

1) The environmental conditions along with a value for ρ_i were put into the equation with δ having an arbitrary value of 1.

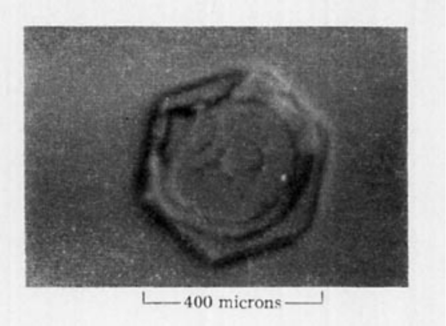

FIG. 3. Typical replica collected in Yellowstone National Park.

2) The hexagonal side length S_2 was computed.

3) The average value of S was computed over the time increment.

4) The thickness of the ice crystal was computed using the relationship, $d=0.01\ (1.82S)^{0.43}$, obtained from the data of Ono (1969) on the thickness of ice crystals as related to their diameter.

5) A value of δ was computed and used to recompute S_2. This procedure was continued until two subsequent values of S_2 agreed to within 2μ.

Time steps of 10 and 40 sec were used. There was no difference in the computed size of the length of a side of the ice crystal. The results of the computation are given in Fig. 5.

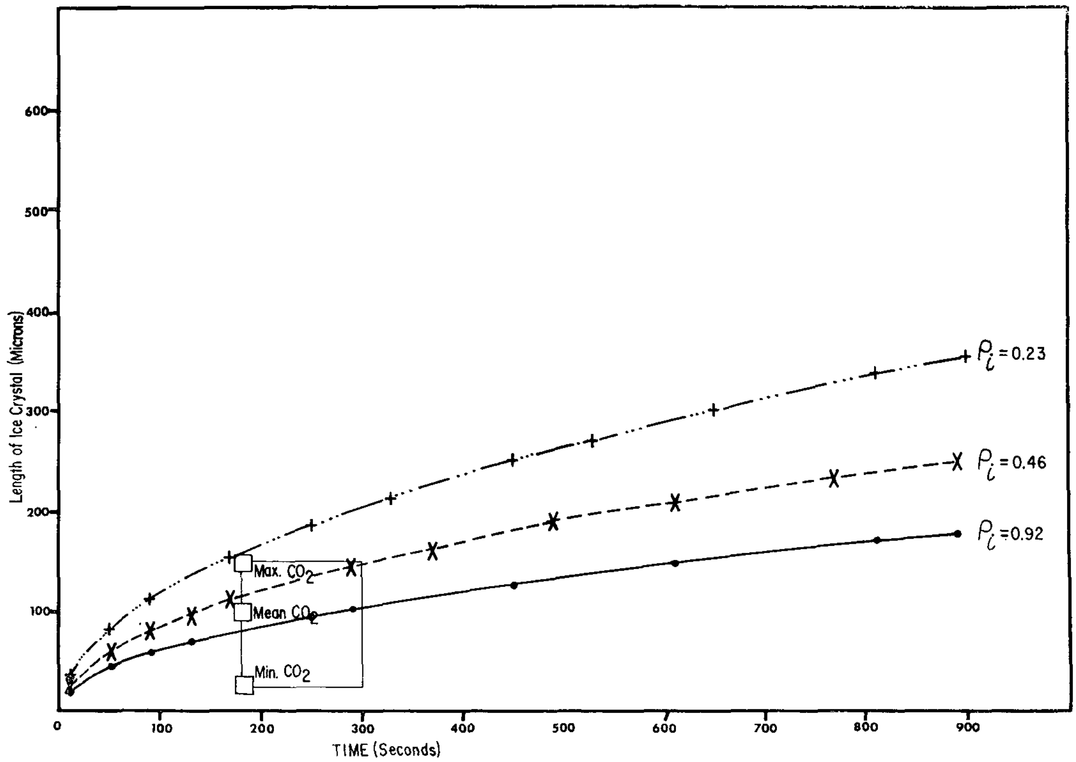

FIG. 4. Growth of a prismatic ice crystal with various densities ρ_i.

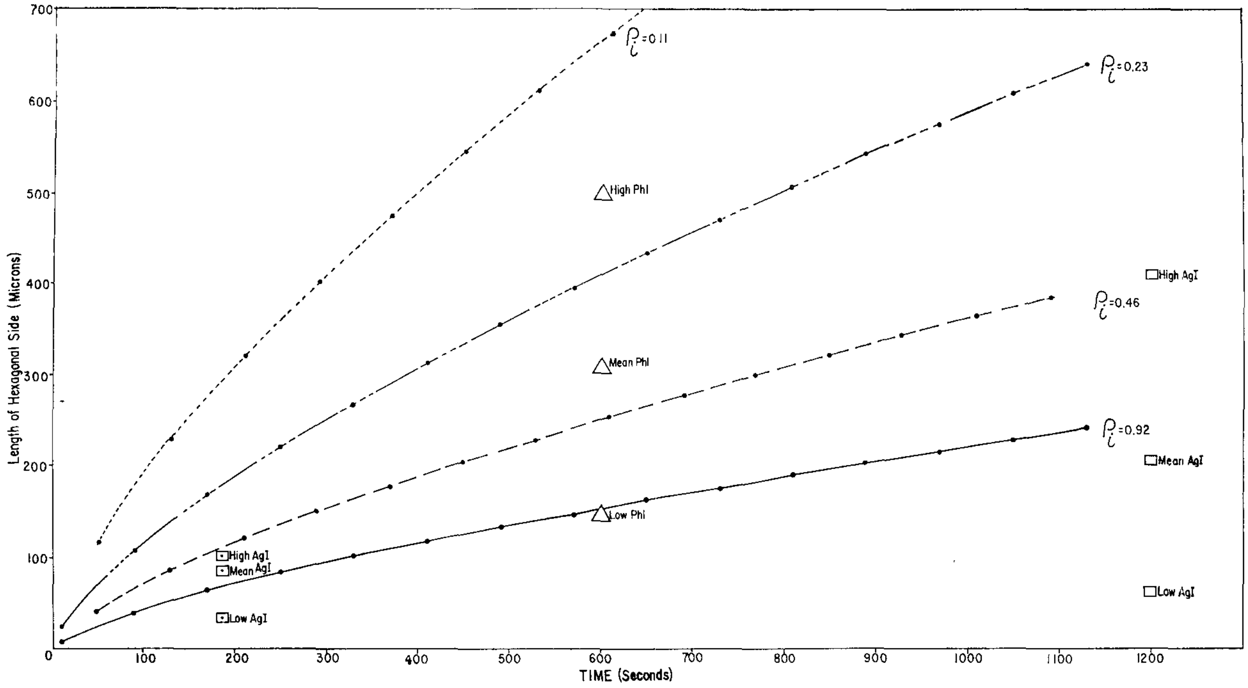

FIG. 5. Growth of a hexagonal plate ice crystal with various densities ρ_i. The thickness was calculated from Ono (1969).

The observed ice crystals grew for the indicated times. These are plotted on Figs. 4 and 5 as well as the computed growth curves which utilize ρ_i, δ and τ as parameters. In two instances involving the roll cloud, a time bracket for the observed growth is given. This bracket represents the outside limits of the uncertainty in the growth time due to turbulence or other factors within the cloud. The most probable growth is indicated by the dashed line.

6. Discussion

Figs. 4 and 5 illustrate the diffusional growth under the observed ambient conditions.

From the graphs, it is apparent that there is a large variation of ice crystal size within a sample. There also may be a difference in growth habit between ice crystals growing on different types of nuclei.

Schaefer and Cheng (1968) have studied the form of ice crystals and point out that it is possible to identify the nucleating agent from a replica of the ice crystals. Their observations coupled with these observations on ice crystal size indicate that the nucleating agents might dictate the crystalline growth habit. Thus, the phloroglucinol nucleated ice crystals would be much thinner than those nucleated by silver iodide or carbon dioxide.

The ventilation of the ice crystal, if calculable, would possibly provide for a greater growth rate. Using the Frössling (1938) ventilation equation and the fall velocity computed using Langleben's (1954) equation, the maximum effect would be 20%, insufficient to account for the observed differences. This assumes no ventilation of the other crystals which is obviously erroneous.

Another factor could be variations in the shape. The values reported by McDonald (1963) do not vary appreciably unless the crystals become radically different. These did not.

The variation of size within a given sample is not easily explained. Possibilities exist in competition, secondary ice production, and time lag in nucleating. Whatever the cause, this effect is of importance.

7. Relevance to weather modification

Variations in crystal size may not be of vital importance to large-scale cloud seeding programs where the target is thousands of square miles, since variations in the growth time would, in general, be short compared to the travel time of the ice crystal within the target area.

In the case of targeting on smaller areas, such as confined watersheds, the variations in growth could result in pronounced effects over the target area.

This can be especially important in those cases where mountains may be involved as part of the target. Variations in growth may cause considerable spreading and the experiment should be designed around less than average growth to obtain the maximum effect. Such factors should also be given consideration in the evaluation of the experimental results.

In addition, the growth rate may be highly dependent upon the number of active ice nuclei released per unit time by the generator. Of the three substances used, the phloroglucinol generator has by far the lowest output by two orders of magnitude, yet the crystals grew the most rapidly. This could be due to a greater moisture supply in the form of droplets close to the generator, or possibly to crystallographic factors.

8. Conclusion

Data have been presented concerning the growth rates of ice crystals. Computations show that the growth can be accounted for by diffusion, provided certain assumptions are made. In one case where only a few ice nuclei were released ($\sim 10^7$ sec^{-1}), at least the areal growth of the crystals was more rapid. Under such conditions, these crystals would sweep out a larger area in falling and riming than more compact crystals with the same mass.

More detailed studies of the growth in supercooled clouds should lead to seeding criteria which will optimize the factors of terrain, moisture content and cloud thickness, with the seeding agent and its dispersal rate.

At the present time, it is not possible to account for the variations in size of the ice crystals that are observed within a given set of observational data.

More information is needed on the rate of "secondary" ice production and on the ratio of thickness to area of a hexagonal plate.

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