Collision Efficiencies in Washout[1]

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(Manuscript received 5 May 1970)

ABSTRACT

Collision efficiencies E have been determined from particle trajectories for the case of a 1-mm glass sphere and 6–20 μ spherical glass particles falling in still air. An empirical formula for the dependence of E upon scavenger size, scavenger velocity, and particle terminal velocity has been derived.

1. Introduction

A preceding publication (Berg *et al.*, 1970), referred to henceforth as Part I, described an experimental technique for the determination of the trajectory of a small particle in the vicinity of a large drop when the particle and the drop were falling freely in still air. The following parameters were measured and controlled:

Drop: diameter D, fall velocity v_D, charge Q

Particle: diameter d, terminal velocity v_{p_0}, charge q.

Examples of trajectories were given for $D=2.5$ mm and $d=20$–$30\ \mu$.

The same technique has been employed further, and this paper reports determinations of the collision efficiency E from particle trajectories. Most of the experiments were conducted with a glass sphere instead of a drop. Its diameter was ~ 1 mm, its fall velocity ~ 155 cm sec^{-1}, and its charge zero. The particles were glass spheres in the range 6–20 μ. They were charged to $\sim 10^{-5}$ esu. It was ascertained in the previous work that the particle charge had no effect, unless the drop, too, was charged.

2. Theory

In view of the large number of variables it is necessary to derive some simple function of them, in terms of which E can be expressed. The equations of motion (Part I) have a form such that the solutions of the homogeneous differential equations contain an exponential factor $gz/(v_D v_{p_0})$, when g denotes the acceleration of gravity and z is the vertical distance between particle and drop center. This suggests that

$$A = \frac{gD}{v_D v_{p_0}} \tag{1}$$

is a pertinent variable in terms of which E may be expressed. This quantity, or rather its inverted value was introduced by Langmuir (1948) in his calculations of E, and on similar grounds. It has since been used by several investigators, e.g., Barth (1959), Walton and Woolcock (1960) and Engelmann (1965) for the presentation of experimental data. Those presentations show that E is a monotonic function of A. A few interesting conclusions follow from this result.

Fig. 1 shows the ratio D/v_D, calculated from the data of Gunn and Kinzer (1949), and plotted as a function of D. The minimum at $D=0.5$ mm gives a corresponding maximum value for E in agreement with observations

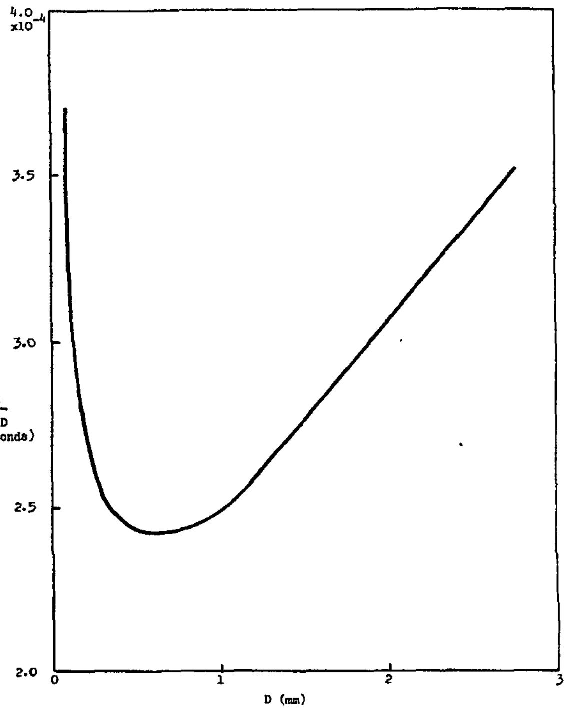

FIG. 1. The ratio D/v_D as a function of D.

[1] Research sponsored by the Fallout Studies Branch, U. S. Atomic Energy Commission, under Contract AT(04-3)-673.

by Kinzer and Cobb (1956), Walton and Woolcock, and Engelmann. The mininum is fairly flat, and $D/v_D = 2.8\times10^{-4}\pm10\%$ in the range $D=0.2-2.0$ mm, i.e., over the entire range of raindrop sizes. When drops fall, or are carried by a downdraft in a cloud, they grow by collision with cloud droplets and shrink by evaporation. Growth predominates only for the drops that have close to maximum collision efficiency, whereas drops outside this range evaporate. The collision efficiency is not a function of the Reynolds number, which is proportional to the product Dv_D, but is a function of the ratio D/v_D.

Fig. 2 presents the data of Barth, and Walton and Woolcock, and also a few of Langmuir's calculated values. The data show that

$$E=E_0e^{-kA}, \tag{2}$$

where E_0 and k are constants. Fig. 3 is an enlargement of the region where $A<1$. Both sets of data give the same value of k, i.e., 0.0784, in the range $A>2$, but they differ in the range $A<1$, where Barth's data give a value $k_B=0.410$, while Walton and Woolcock's data give $k_{WW}=0.514$. Barth states in his paper that his data represent $E^{\frac{1}{2}}$ and not E. We think that this statement is erroneous because E would otherwise be absurdly small; in addition, the plots in Figs. 2 and 3 are linear in both cases, the values of k differing by a factor of 2.

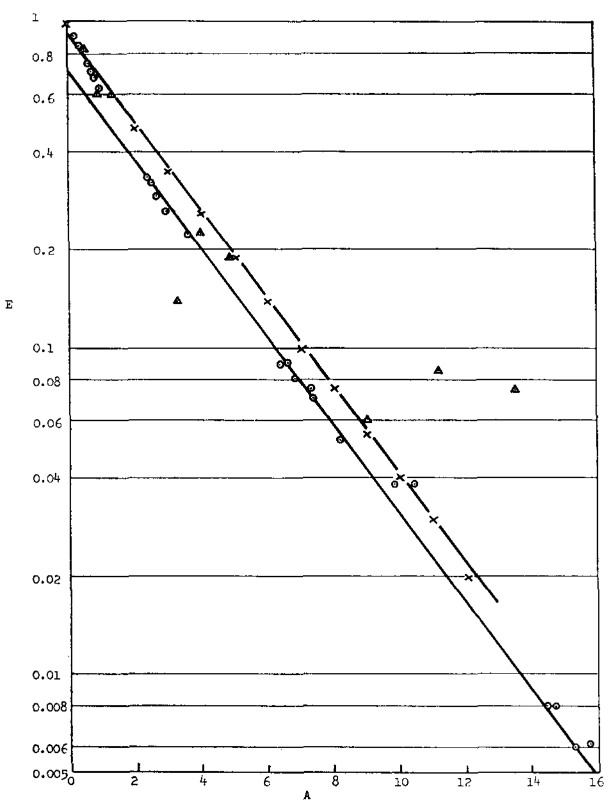

FIG. 2. Log$E$ as a function of A for the data of Barth (crosses) and Walton and Woolcock (circles). The triangles represent values calculated from Langmuir's theory.

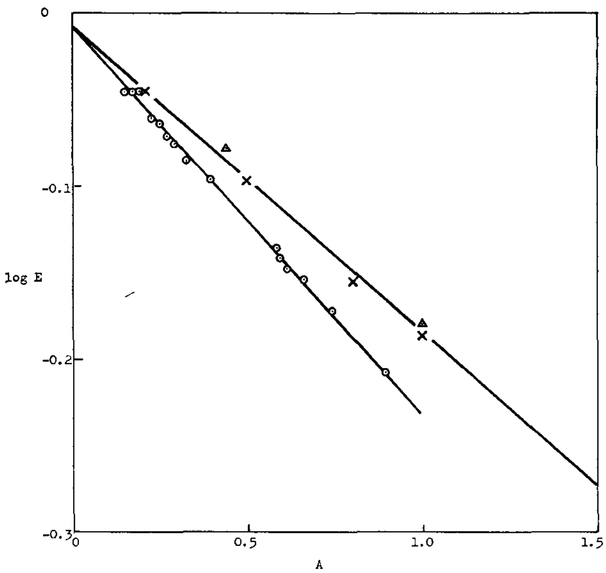

FIG. 3. Enlargement of the region $A<1$ in Fig. 2.

The collision efficiency is defined in terms of a critical distance r_0 from the vertical line through the center of the drop, within which the particle must be in order to collide with the drop. Thus,

$$E=\left(\frac{r_0}{D/2}\right)^2. \tag{3}$$

We may calculate r_0 from (2) in the form

$$r_0=\frac{D}{2}E_0^{\frac{1}{2}}e^{-\frac{1}{2}kA}=\frac{E_0^{\frac{1}{2}}v_Dv_{p0}}{2g}Ae^{-\frac{1}{2}kA}. \tag{4}$$

It is apparent that r_0 has a maximum when $dr_0/dA=0$, i.e., when

$$A=\frac{2}{k}. \tag{5}$$

The corresponding value of r_0 is

$$r_{0\max}=\frac{E_0^{\frac{1}{2}}v_Dv_{p0}}{kge}. \tag{6}$$

The function $r_0=f(D)$ is presented in Fig. 4 for the case $E_0=1$, $k=0.514$, and $A=1$ for $D=0.1$ cm. The difference between this function and the linear function $r_0=D/2$ is the deflection Δr_0. Approximately,

$$r_0+\Delta r_0=D/2. \tag{7}$$

Thus,

$$\Delta r_0=\frac{D}{2}(1-E_0^{\frac{1}{2}}e^{-\frac{1}{2}kA}). \tag{8}$$

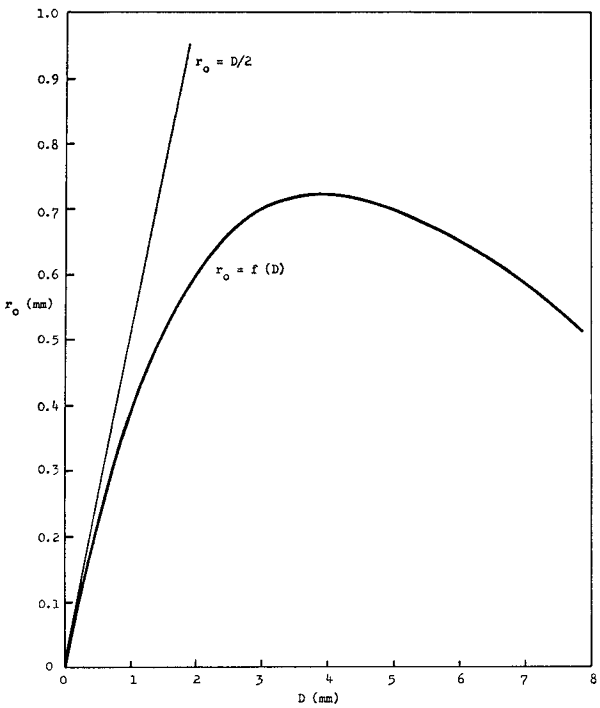

FIG. 4. Critical collision distance r_0 as a function of D for constant values of v_D and v_{p0} from Eq. (4) for the case $E_0=1$, $k=0.514$, and $A=1$ for $D=0.1$; and the linear function $r_0=D/2$.

Taking $E_0=1$ and expanding, we obtain

$$\Delta r_0=\frac{D}{2}\frac{1}{2}kA\left(1-\tfrac{1}{2}kA+\cdots\right)\approx\frac{gk}{4v_{p0}}\frac{D^2}{v_D}. \qquad (9)$$

Thus, to this approximation, Δr_0 is proportional to D^2 for constant values of v_D and v_{p_0}. But for a falling raindrop D/v_D is almost constant, and Δr_0 is then proportional to D.

3. Determination of E from trajectories

To the critical distance r_0 corresponds a critical trajectory that barely misses or barely hits the drop. This trajectory is rarely encountered experimentally, and when it is, it is hard to recognize. It is better to use trajectories that miss by a small distance. Such a trajectory has the initial distance $r>r_0$ from the center line and a deflection $\Delta r<\Delta r_0$.

If E were independent of D, which it is not, we could take

$$E=\left(\frac{r}{r+\Delta r}\right)^2, \qquad (10)$$

but since Δr depends on r and D we must calculate a new value of D or new values of r and Δr. To a first approximation we may use (9) and take

$$\Delta r_0=\left(\frac{r+\Delta r}{D/2}\right)^2\Delta r. \qquad (11)$$

In conjunction with Eq. (7), we now have

$$E=\left[\frac{r}{r+\left(\frac{r+\Delta r}{D/2}\right)^2\Delta r}\right]^2, \qquad (12)$$

corresponding to a drop diameter $r+\Delta r_0$. A more accurate value of Δr_0 is obtained from (8) instead of the approximate formula (9). Using (8), we obtain

$$\Delta r_0=\Delta r+\frac{d\Delta r}{d(D/2)}\left[r_0+\Delta r_0-(r+\Delta r)\right], \qquad (13)$$

with the derivative

$$\frac{d\Delta r}{d(D/2)}=1-E_0^{\frac{1}{2}}e^{-\frac{1}{2}kA}+\tfrac{1}{2}E_0^{\frac{1}{2}}kAe^{-\frac{1}{2}kA}. \qquad (14)$$

Since the average value of the derivative should be used in (13), the mean value of A should be used in (14). Since $r_0+\Delta r_0$ is not known, we may take $r+\Delta r-D/2$ instead of $r_0+\Delta r_0-(r+\Delta r)$ in (13). The constants E_0 and k may be determined by successive approximations, as may the value of $r_0+\Delta r_0-(r+\Delta r)$. It turns out that E_0 is close to unity for $A<1$, and that k is close to $k_{WW}=0.514$. We may then take $E=1$ and $k=0.514$ and use $r+\Delta r-D/2$ in (13). For the average value of A we may take the average of the values corresponding to $D/2$ and to $r+\Delta r$ as calculated from Eq. (11). The first step

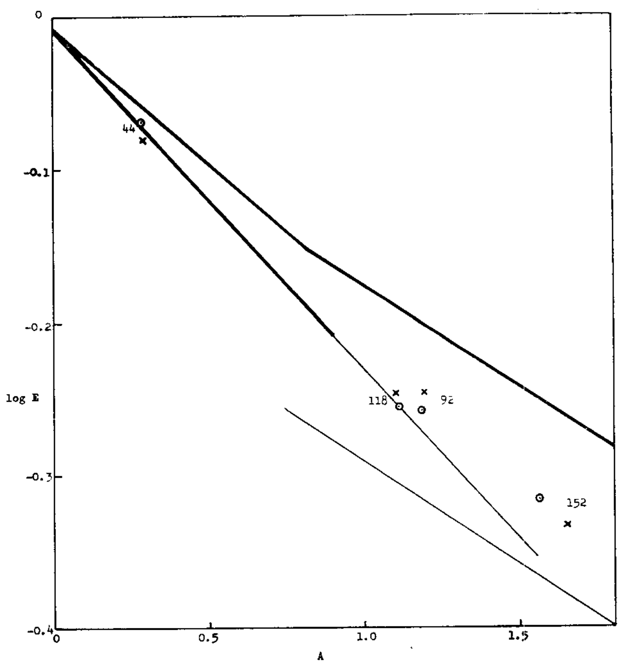

FIG. 5. Collision efficiences calculated from observed trajectories using Eqs. (12), crosses, and (13), circles. The numbers refer to the columns in Table 1. The lines are those in Figs. 2 and 3.

of the successive approximations then gives a sufficiently accurate value of E. To this value of E corresponds one of A in which $2(r+\Delta r_0)$ [Δr_0 from Eq. (13)] is taken for D.

Values of E were calculated from (12) and (13) and are plotted in Fig. 5. The lines drawn in Fig. 5 are those of Figs. 2 and 3, but the data points have been left out in order not to clutter the graph. The difference between the two approximations is approximately 4% of the calculated value. The experiment numbers are marked in Fig. 5 and refer to Table 1, in which the parameters and the observed and calculated variables are listed. The values of E are close enough to the line for the data of Walton and Woolcock to justify the use of $k=0.514$ in Eq. (14).

It was pointed out in Part I that the vertical component of the aerodynamic force exerted by the drop upon the particle reverses its sign and is directed upward in the sector $\pm 35°$ to the horizontal through the drop center. This may turn the trajectory upward as shown in an illustration in Part I, or merely turn it horizontal as in Fig. 6. This effect increases with increasing D, and with decreasing v_D and v_{p0}, i.e., with increasing A, and also with increasing r. Consequently, small particles with large values of r experience a force that is not experienced by large particles and by particles with small values of r. Particles and trajectories showing this effect are not suitable for the calculation of E, giving values which are too large. However, when $E>1$, $r>D/2$, e.g., in cases of strong electrostatic attraction, this usually anomalous effect is normal and contributes to the large value of E. For comparison, the normal type of trajectory is shown in Fig. 7. In order to show the force reversal effect, a few E values were calculated from trajectories of the type shown in Fig. 6. They are listed in Table 2.

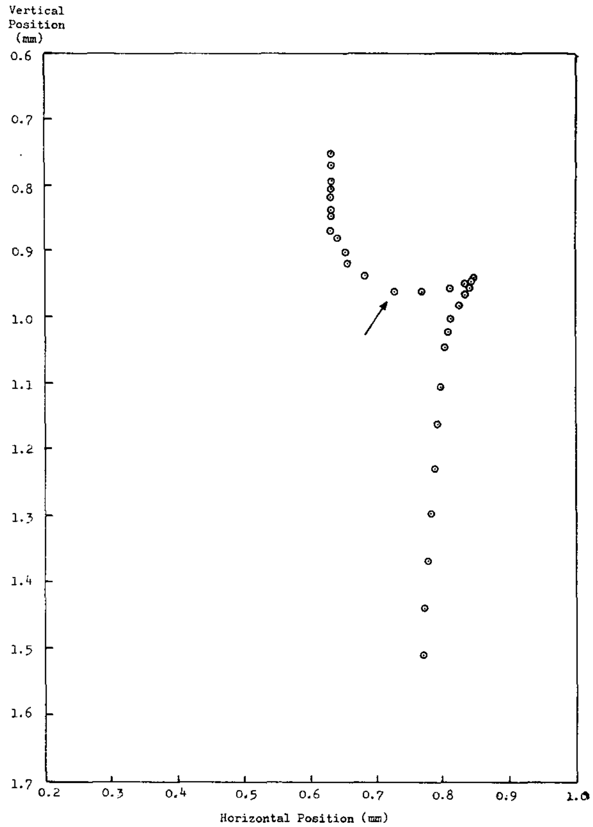

Fig. 6. The trajectory (Experiment 88) in a fixed coordinate system of a 6.75 μ particle, $v_{p0}=0.56$ cm sec^{-1}, in the vicinity of a 1-mm drop, showing the effect of the reversal of the vertical aerodynamic force. The arrow marks the instant when the drop center is level with the particle.

Table 1. Observed and calculated* parameters for particles unaffected by sign reversal of vertical force.

Parameter		Experiment No. 44	92	118	152
D(mm)		0.98	1.04	1.095	1.06
v_D(cm sec^{-1})		150	155	155	153
$d(\mu)$		22.1	7.75	8.83	8.00
v_{p0}(cm sec^{-1})		3.27	0.690	0.696	0.570
A		0.193	0.954	0.997	1.192
r(mm)		0.659	0.479	0.454	0.500
Δr(mm)		0.032	0.127	0.131	0.154
Δr_0(mm)	(12)	0.065	0.172	0.150	0.234
A	(12)	0.290	1.193	1.099	1.651
E	(12)	0.830	0.542	0.566	0.463
$\log E$	(12)	−0.081	−0.266	−0.247	−0.334
Δr_0(mm)	(13)	0.056	0.166	0.156	0.220
A	(13)	0.285	1.183	1.111	1.560
E	(13)	0.851	0.552	0.554	0.482
$\log E$	(13)	−0.070	−0.258	−0.256	−0.317
A	(2)	0.285	1.183	1.111	1.560
E	(2)	0.845	0.533	0.553	~0.450
$\log E$	(2)	−0.073	−0.273	−0.257	~−0.350

*From Eqs. (12), (13) and (2).

The point marked 152 in Fig. 5 may be too high as a result of this effect, but we think that it is probably correct, and that it is located in a transition zone between the two straight lines on the data of Walton and Woolcock.

The point just made, that the critical trajectory is difficult to recognize when encountered, is illustrated by the trajectory in Fig. 8. If this trajectory were taken as the critical trajectory, it would give $r_0=0.382$ and $E=0.550$. The values calculated from (2) are $r_0=0.411$ and $E=0.637$. Thus, not only was the particle seen to collide, but it would have collided at a distance that is 7% larger. Since r_0 is squared in E, the relative error is doubled and E comes out too low by 13%.

4. Discussion

The experimental technique used in this work permits almost perfect control of the conditions, and still air is a simple and controllable condition. The technique, in fact, was designed for this purpose, whereas accuracy

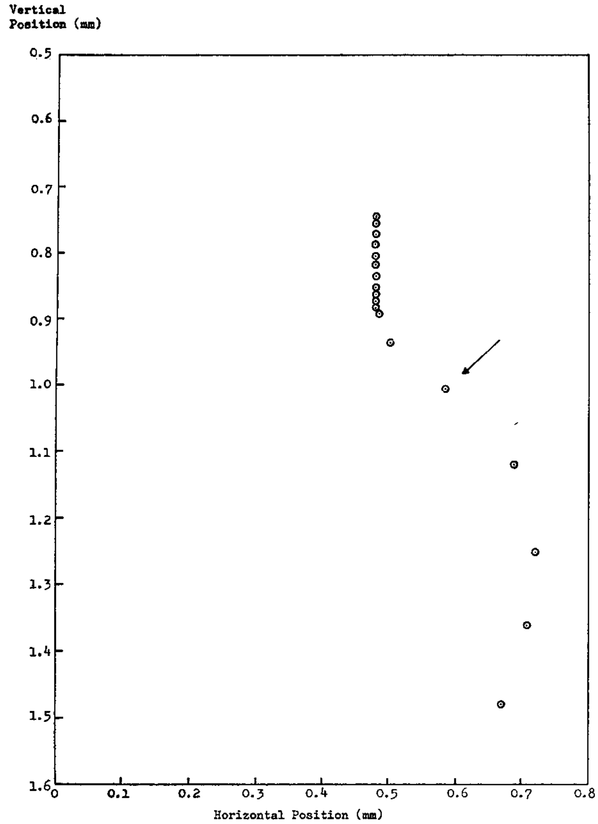

FIG. 7. Same as Fig. 6 except for Experiment 92 with a 7.75 μ particle, $v_{p0}=0.69$ cm sec^{-1}, showing little effect of the reversal of the vertical aerodynamic force.

of the determined E values was of secondary concern. Nevertheless, a comparison of the E values among themselves and with those of Walton and Woolcock shows

TABLE 2. Observed and calculated* parameters for particles affected by sign reversal of vertical force.

Parameter		Experiment No. 88	119	163	184
D(mm)		1.00	1.04	1.07	1.04
v_D(cm sec^{-1})		155	143	157	155
$d(\mu)$		6.75	7.91	8.40	12.1
v_{p_0}(cm sec^{-1})		0.56	0.76	0.76	1.53
A		1.130	0.877	0.880	0.430
r(mm)		0.633	0.628	0.700	0.680
Δr(mm)		0.103	0.096	0.080	0.021
Δr_0(mm)	(12)	0.223	0.185	0.170	0.038
A	(12)	1.935	1.370	1.282	0.579
E	(12)	0.548	0.597	0.648	0.897
$\log E$	(12)	−0.261	−0.224	−0.188	−0.047
Δr_0(mm)	(13)	0.243	0.200	0.190	0.064
A	(13)	1.980	1.397	1.332	0.615
E	(13)	0.522	0.576	0.748	0.836
$\log E$	(13)	−0.282	−0.240	−0.126	−0.078
A	(2)	1.980	1.397	1.332	0.615
E	(2)	0.375	0.479	0.494	0.713
$\log E$	(2)	−0.426	−0.320	−0.306	−0.147

*From Eqs. (12), (13) and (2).

that the accuracy is remarkably good for experiments of this kind. The scatter of our data may be compared with that of the data obtained by Walton and Woolcock. The points marked in Figs. 2 and 3 were taken from a fitted curve drawn through the measured values of E plotted vs A^{-1}.

The experimental conditions are more accurately controlled in these experiments than in a wind tunnel, where the flow is not perfectly laminar, and where the flow may be disturbed by the support. They are much more accurately controlled than with a cloud or a stream of particles and with a spray of drops because the properties of every particle and every drop are accurately measured in our experiments. This control of the experimental conditions has been gained at a price: the experiment is slow and many experiments are required to obtain one useful trajectory. This is illustrated by the scatter diagram in Fig. 9 showing the relative positions of particle and drop center. Only the points on the vertical line have trajectories parallel to the film plane, and among these only the trajectories within a narrow region around the circle representing the drop give sufficiently accurate E values.

This emphasis upon control of the experimental conditions derives from some known and unknown pheno-

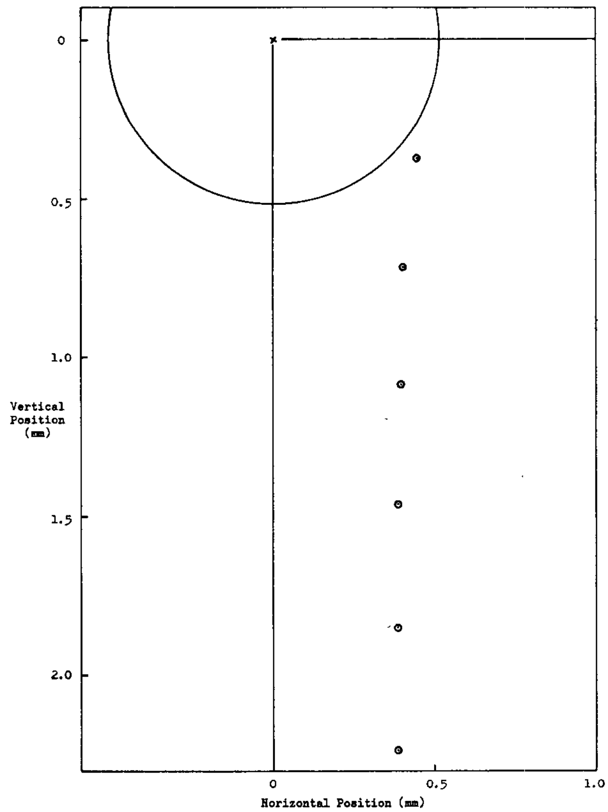

FIG. 8. The trajectory relative to the 1-mm drop of a 8.16 μ particle, $v_{p0}=0.755$ cm sec^{-1}. It is apparently close to the critical trajectory but r is actually 7% smaller than r_0.

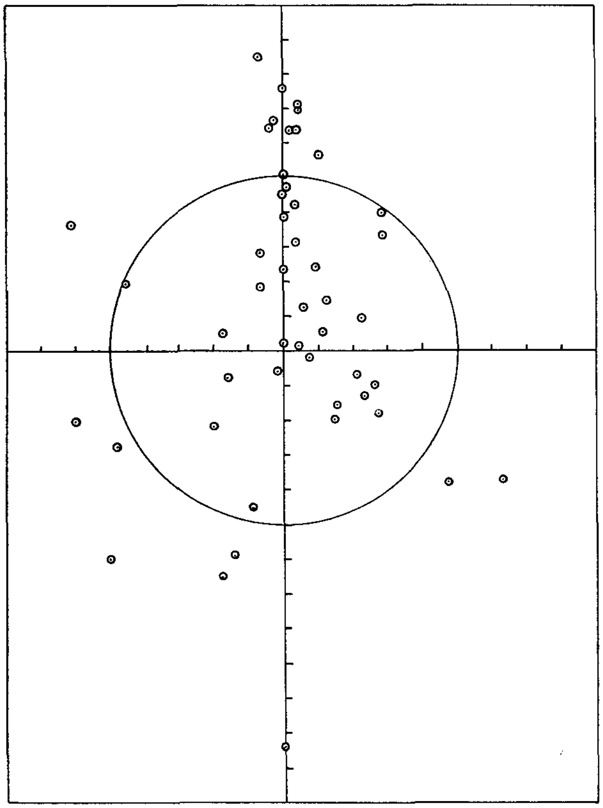

FIG. 9. Relative positions of particle and drop center in a horizontal plane in several experiments. Each scale division is 0.1 mm.

mena that affect the value of E by orders of magnitude. The effect of electrostatic charge was discussed in Part I. Several investigators have reported collection on the back side of the drop in greater amounts than the collection on the front side, e.g., Engelmann (1965) and Asset and Hutchins (1967). In the experiments of Asset and Hutchins, conducted with a glass rod and polystyrene particles in a wind tunnel, there was 131 times as much collected on the back side as on the front side at a particle size of 5 μ, a rod diameter of 2.1 cm, and a relative velocity of 8 m sec^{-1}. This ratio increased with decreasing particle size. Although several explanations for this back side collection have been offered in the literature, e.g., electrostatic attraction and wake turbulence, none have been confirmed by direct observation. Clearly, if uncontrolled electrostatic charges and turbulence may affect E by orders of magnitude, accuracy of measurement is meaningless unless one has adequate control of the experimental conditions.

5. Conclusion

From the empirical fact that E is an unambiguous and monotonic function of A, regardless of other properties of this function, it follows that E has a maximum when D/v_D has a minimum. For a falling raindrop this occurs at $D=0.5$ mm. The maximum of E is fairly flat in the region of raindrop size, $D=0.2$–2.0 mm, and raindrops in this range have a collision efficiency close to maximum. The empirical formula (2) represents the data accurately enough for practical purposes in the entire range of A and E in which it has been tested.

The validity of the relation between E and A is restricted to free fall in still air in the absence of electrostatic charges. As E decreases, especially with decreasing particle size, charge effects become increasingly important and eventually decisive. The effect of turbulence remains unknown. Collection on the back side cannot be explained on the basis of available information. Quantitative information on these two effects is lacking, but it is known that they may affect the value of E by orders of magnitude.

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