NOTES AND CORRESPONDENCE

On Cumulus Entrainment and One-Dimensional Models

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1. Introduction

In a recent article, Warner (1970b) is severely critical of one-dimensional cumulus models on the following grounds:

1) They assume entrainment from the sides only, while, in fact, cumulus entrainment takes place into cloud tops.
2) They are based on an inverse-radius entrainment law which observational evidence contradicts.
3) They cannot predict jointly cloud tops and liquid water contents.
4) They must be adjusted to fit each class of cloud environment by an after-the-fact adjustment of disposable constants.

For these asserted reasons, Warner states that this class of cumulus model is physically unsound and of only very limited usefulness. In the case of the EMB[1] series of one-dimensional models (Simpson and Wiggert, 1969, 1971), it will be shown here that the above four points are either invalid or inapplicable. Warner's negative arguments are based in part upon a misinterpretation of these models, in part on problems inherent in present-day aircraft observations, and in very large part upon his neglect of precipitation particles, which virtually always form in tropical maritime cumuli of 2–3 km vertical thickness or more.

2. The EMB model series

The EMB models are not steady-state models postulating entrainment from the sides only but treat an actively rising cloud tower in a quasi-Lagrangian framework; the tower is envisaged as entraining roughly comparable amounts of environment air from the sides and top. Without reliance on similarity arguments, the equation for W, the rate of rise of the tower, has been shown to be (cf. Levine, 1959, 1965; Simpson and Wiggert, 1969)

$$\frac{dW}{dt}=W\frac{dW}{dz}=\frac{gB}{1+\gamma}-\frac{1}{M}\frac{dM}{dz}W^2-K_dW^2, \qquad (1)$$

where gB is the buoyancy per unit mass, γ the apparent mass coefficient due to acceleration of the surrounding fluid, M the mass of the vertically circulating tower, and K_d a constant of proportionality between aerodynamic drag and W^2. The vertical integration is to be performed in a coordinate system fixed to the tower center. With our models, the plots of W and other cloud parameters as functions of altitude z do not represent in-cloud profiles but rather the mean properties of the active tower as it rises through that level. This important point was overlooked by Warner. Therefore, the model he tested (which he said was ours) actually resembled ours only in choice of constant radius, which, in fact, is not a necessary requirement in the EMB series.

The entrainment relationship is, however, crucial in any cumulus model. The major impact of entrainment upon cumulus dynamics lies in the dilution of buoyancy by the incorporation of drier outside air. To specify the entrainment rate, we have postulated an inverse dependence on tower radius, namely

$$\frac{1}{M}\frac{dM}{dz}=\frac{2\alpha}{R}, \qquad (2)$$

where α is a constant of proportionality. Again without requiring similarity, this relationship can be either empirical or based on the dimensional argument that entrainment should be proportional to the ratio of surface area to volume. The present evidence for and against this relation will be evaluated at the end of this

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[1] Experimental Meteorology Branch, NOAA.

note. Thus, for the dynamic aspect of the modelling we have to specify γ, α and K_d in Eq. (1). In our work these have been all taken in advance from laboratory experiments (e.g., Turner, 1962) to be 0.5, 0.1 and zero, respectively, as explained in detail by Simpson and Wiggert (1969); there has been no adjustment to force predicted cloud tops or other parameters to agree after the fact with observations.

Another vitally important negative contribution to cumulus buoyancy is the weight of the liquid water carried; this is the main mechanism of interaction between the physics and dynamics of the cloud. As little as 1 gm m^{-3} can subtract more than the equivalent of 0.5C from the buoyancy, which in small cumuli is often as large as the buoyancy itself. In the early EMB models (1965 series; Simpson *et al.*, 1965, 1967) the treatment of precipitation fallout was so crude that after the fact "calibration" of the dynamics was necessary. Beginning with the 1968 series (Simpson and Wiggert, 1969, 1971) the treatment of the growth and fallout of precipitation in liquid clouds has been sufficiently improved to obviate this. The autoconversion equation of Berry (1968) is used. The collection equation follows Kessler (1965) as does the Marshall-Palmer spectrum, the terminal velocity law for precipitation, and the equation for radar reflectivity. The expression for the fractional fallout in each height interval follows logically from the postulation of the vertically circulating rounded plume cap or thermal. In liquid clouds, all cloud physics constants have been taken directly from the work of Berry or Kessler and have also been tested with measurements by us and others (Simpson and Wiggert, 1971). Only in seeding subroutines and freezing clouds, where the needed parameters are not known from either theory or observation, does any after-the-fact calibration remain. Freezing clouds are outside the area of disagreement between our work and Warner's.

3. Precipitating towers and the disagreement with Warner's work

Warner (1970b) asserts that no existing one-dimensional cumulus model makes a joint correct prediction of top heights and liquid water contents. He then constructs a set of one-dimensional steady-state models which either predict correct tops and about 60% too high water contents or correct water contents and tops that are half or less as high as observed. The problem is that Warner excludes any fallout of precipitation-sized hydrometeors so that the total amount of liquid water condensed is carried upward with the rising air.

This difficulty is a self-compounding one. If the correct entrainment rate is used, the model cloud will be so overloaded with water that it will both fail to reach the correct height and will overpredict the liquid water content. If a small enough entrainment rate is taken to permit the overloaded model cloud to attain the observed height, the reduced dilution will compound further the overestimation of liquid water. Conversely, if the entrainment is increased drastically enough to reduce the total water condensed to agree with measured water contents, then the buoyancy is so reduced by overdilution that the cloud peters out at unrealistically low levels.

It should be noted from Warner's diagrams (Figs. 2 and 3) that the entrainment rate necessary for the "correct top" prediction is a factor of 6–10 smaller than the entrainment rate necessary for the "correct water content" prediction. Clearly, then, Warner could have used his in-cloud temperature measurements to show that the "correct top" entrainment rate gives temperatures exceeding the observational uncertainty ($\sim$0.5C), while the "correct water content" entrainment rate gives temperatures less than the uncertainty by an even greater amount. This point is independent of any model and is based merely on trial entrainment calculations on typical tropical and subtropical soundings using the range of entrainment rates cited. The result would appear to add to the inconsistencies arising in his model application. However, it will now be shown that these problems are simultaneously resolved by allowing a realistic amount of hydrometeors to fall out from the actively rising tower. This does *not* imply that there is any rain reaching the ground or even appearing below cloud base, as will be amplified later. It is, however, well known (Byers and Hall, 1955) that tropical oceanic cumuli reach 100% probability of a precipitation radar echo when their tops exceed 11,500 ft (3.5 km); furthermore, Levine (1965) found that towers of warm trade cumuli commonly had about half their liquid water content in precipitation-sized drops.

In our model, a fairly realistic treatment of precipitation formation and fallout from the tower avoids the problem encountered by Warner and permits accurate joint prediction of top heights and water contents. Its height predictions have been checked with both warm and supercooled clouds, the latter in randomized seeding experiments in 1965 in the Caribbean and in Florida in 1968. Only in a minority of cases was vertical cloud growth halted by a stable layer. The constant radius assumption proved adequate even when towers combined or widened at high levels because dilution by entrainment[2] becomes rather small above 400 mb.

Of more importance, however, is the fact that it was possible to make some satisfactory water comparisons with the 1965 data, and several with those from 1968 (Simpson and Wiggert, 1971). In the latter cases the model's partitioning of water in broad size categories could frequently be tested, at least roughly,

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[2] The low temperatures in the upper troposphere reduce the difference between saturation and actual mixing ratios, thus reducing the effectiveness of entrainment in drying out the cloud.

since the penetration aircraft had a total water content meter (Levine, 1965), a Johnson-Williams hot wire, a foil hydrometeor sampler, and a continuous Formvar replicator. In addition, individual components of the model were tested by special measurement programs. For example, the Marshall-Palmer spectrum and parameters were verified in active towers (Mee and Takeuchi, 1968) and rise rates were checked in numerous cases by photogrammetry (Simpson and Woodley, 1969; Woodley and Powell, 1970). Predicted vs observed radar reflectivities compared favorably on the few occasions when the aircraft penetrated an active tower which was simultaneously being observed by the University of Miami's calibrated 10-cm ground radar.

Here we will confine our attention to old and new tests of the model on small, warm, oceanic tropical cumuli since Warner's observations and model tests were run on such clouds. An excellent set of suitable measurements were provided by Saunders (1965) who kindly loaned us all his original data. He studied about 20 small cumuli over the ocean near Barbados, West Indies, using a calibrated 3-cm radar, visual observations and photography. He measured the radii of the active towers as well as their top heights and rise rates. He also documented the behavior with time of the radar echoes. Cloud tops were always between 3–4 km above sea level, similar to those studied by Warner; tower radii ranged between 350–650 m.

The first important point is that all clouds displayed a strong 3-cm radar echo (peaking between 34–46 db) although less than one-third had a visible shower from the base. The model was tried on all the completely documented cases, using the Barbados sounding (usually within 2–3 hr of the cloud observations) and Saunders' measured radii and cloud base levels as input. The average absolute error between predicted and observed tops was 600 m, with the predicted model results running higher in two-thirds of the cases. For the radar echoes, one-half were predicted within ±2 db and all the remainder but one within ±5 db, with no systematic over or underprediction. The first error margin corresponds to ±26% in precipitation water content while the latter corresponds to ±67%. Saunders' estimate of a predominant precipitation drop diameter of nearly 0.5 mm is surprisingly consistent with the model-predicted volume median diameter of ~1 mm, which according to the Marshall-Palmer spectrum should be roughly $2\frac{1}{2}$ times the predominant size. Tower rise rates nearly always agreed within 2 m sec^{-1}, or 30–50%, with the model predictions consistently higher than the measurements.

All these departures are within the accuracy of the observations. Saunders' rough triangulations of heights are not better than 20% nor are rise rates better than ~50%; he estimates the absolute accuracy of his radar reflectivities as ±5 db. Furthermore, experimentation with the model calculations shows that errors of 100 m in cloud base frequently lead to discrepancies this large, while errors of 100 m in radius virtually always do. The crucial result is that all these small clouds had precipitation-sized particles, first found within a diameter of tower top and then observed to descend, and that the model predicted their amount and size distribution fairly well.

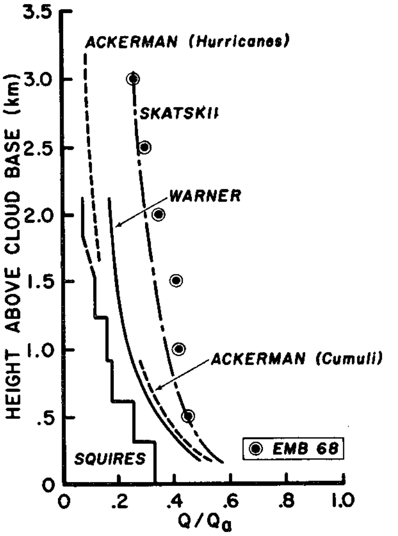

Fig. 1. The ratio of the mean liquid water content to the adiabatic value, Q/Q_a, at a given height above cloud base. The observational curves are after Warner (1970b). The circles are values computed with model EMB 68 for the tropical cumuli studied by Saunders (1965).

Next, the ratio of model-predicted total water content to adiabatic values was computed as a function of height above cloud base for the same clouds. The results of averaging at each level are shown as the circles in Fig. 1 in comparison to the observations of Warner and others. The points fall very close to the curve of Skatskii, thus disproving Warner's allegation that one-dimensional models do not account quantitatively for the upward decrease of the ratio Q/Q_a while at the same time yielding correct height predictions. It should be emphasized that the water contents predicted by the model are for active towers as they rise. It is surprising and interesting that these are so little larger than values measured during cloud profiles, even though it is unlikely that vertically coherent buoyant updrafts are common in such small clouds. This result would seem to suggest the usefulness of a steady-state concept, at least for some purposes.

The model Saunders clouds had a mean total water content of 1.5 gm m^{-3} as they neared their maximum vertical development (2.5–3.0 km above cloud base) of which 32% was in precipitation (diameter $\gtrsim 200\ \mu$) sizes. By the time these levels had been reached an amount comparable to ~68% of its water content had fallen out of the cloud tower. If Warner had introduced a similar precipitation growth and fallout scheme into his model, a tower (between his two extreme radii) could have achieved the observed top with the correct water content. While Coff's Harbour is farther poleward (30S) than Barbados or Puerto Rico, our measurements on summer oceanic cumuli near Florida (26–27N) and Bermuda (32N) differ in no perceptible way from those in the Caribbean. It is difficult to believe that oceanic cumuli in the Southern Hemisphere differ drastically from those of the same size, latitude and season in the Northern Hemisphere, particularly since Warner's previous papers (1969, 1970a) show updraft structures, water contents and droplet concentrations not detectably different from existing measurements in these areas. Nevertheless, it is recommended that Warner repeat his Coff's Harbour observations with a hydrometeor foil sampler, a 3-cm radar and a time-lapse camera.[3]

4. On the evidence concerning the inverse radius dependence of entrainment rate

The postulated $1/R$ dependence of entrainment was introduced by us (Levine, 1959; Malkus, 1960) in early modelling efforts and has since then formed an important cornerstone in our models and those of other workers (e.g., Weinstein, 1970). For this reason it is very important to carefully examine the range of validity, if any, of this relationship. Warner claims, based on some entrainment calculations reported by Sloss (1967) and some liquid water measurements of his own (Warner, 1955), that no such relationship exists.

It will now be shown that even if this relationship prevailed perfectly in real clouds, the errors in Sloss' measurements and assumptions, conservatively estimated, would completely mask it. Although Sloss stated that his results showed no correlation between $1/R$ and entrainment rate, McCarthy[4] later re-analyzed his data with computerized, more accurate entrainment calculations and found a $1/R$ dependence which would, however, account for only about 17% of the variance of the entrainment rate.

Sloss used data obtained by a single research aircraft on about 40 passes through the active portions of cumuli at levels above cloud base ranging between 1.6 and 3.2 km. The aircraft made temperature measurements inside the clouds and in their environment from cloud base up to penetration level. The in-cloud humidity was assumed to be saturated, while the environment humidities had to be interpolated from radiosonde measurements at about 100 and 280 mi away. Cloud base level was measured by the aircraft on its ascent; cloud base temperature was assumed equal to ambient with saturation inside the cloud.

The following errors exist and may affect the calculations: 1) departures of cloud base temperature from ambient due to terrain inhomogeneities, such as the many lakes in the Bemidji area; 2) sampling or instrumental errors in cloud temperature; 3) errors in radius determination; and 4) errors in environment humidity due to use of distant radiosondes. The effects of each of these upon calculated entrainment will be considered using 12 Florida model clouds with radii ranging from 750–1300 m (described in detail by Simpson and Wiggert, 1971). The model used is EMB 68 in which Eq. (2) is an inherent component.

Normally the model is run with cloud base temperature equal to ambient, with the cloudy air saturated. To investigate the effect on calculated entrainment, a series of cloud base temperature departures (ranging from −0.8 to +1.0C in 0.2C steps) is introduced through a sequence of random numbers, one departure to each cloud. The model is rerun with the new base temperature (and saturation) for all 12 clouds. Then a one-step entrainment calculation is made with the new results as "observations" but assuming no cloud base temperature departure from ambient, exactly as done by Sloss. The correlation between entrainment rate E and $1/R$ is reduced to 0.2, which is barely significant at the 25% level. The best fit straight line on the plot of E vs $1/R$ is constructed; use of it reduces the variance in E by just 5%. The equation of this best fit straight line is

$$E\equiv\frac{1}{M}\frac{dM}{dz}=0.076+\frac{0.078}{R}, \tag{3}$$

so that the entrainment constant α, if evaluated from this "experiment," would be less than half its "correct" value in Eq. (2)!

Next, the effect of instrumental and sampling errors in the in-cloud temperatures alone are examined. Following Warner's assessment that instrumental uncertainties in temperature are about ±0.5C in the presence of water, we arbitrarily assigned (with random numbers) each cloud temperature an error of either ±0.5C. When the entrainment calculation is repeated by the method of Sloss, the correlation between E and $1/R$ is 0.38, significant at the 10% level. This time the $1/R$ dependence reduces the variance in E by 8%. A second type of temperature error was tested, i.e., with departures ranging from −0.6 to +0.6C in steps of 0.2C being assigned to each cloud by another sequence of random numbers. In this case the correlation between E and $1/R$ was reduced to 0.1, which had no significance, and the $1/R$ dependence of E reduced its variance by less than 1%.

In these entrainment calculations, the most difficult variable to determine accurately is R. Sloss sometimes measured R from the length of aircraft penetrations in active clouds and sometimes from photographs. He points out that the latter method gives dimensions ranging from two-thirds to one and one-half times the former for the same tower. Also, we have found that

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[3] The fallout of the larger particles from the rising tower is easily seen visually by this method, particularly into the clear air on the downshear side in a moderate shear field.

[4] Personal communication; to be submitted to the JOURNAL OF APPLIED METEOROLOGY.

it is very easy to interpret two or more adjacent towers as one or, if flying too close to the cloud, to identify as an independent tower on a photograph a sub-element which is one of three to five cauliflower-like protuberances from the main tower. The wide variations in Sloss' measured diameters, from 0.2–4.1 km, strongly suggest that all these problems occurred. We have measured tower diameters as great as 4 km only in an intense hurricane (Malkus, 1960) and we have never measured diameters as small as 0.2–0.3 km at these levels even in tropical air; our model predicts that these diameters would not reach 2 km elevation even in an environment much wetter than that treated by Sloss.

For a conservative range of radius errors we assigned to the 12 model clouds by random numbers 10 errors of ±200 m (in 100 m steps) and one error each of one-half and one-third (that is, the correct radius of the tower was divided by two and three, respectively). With these errors alone the E vs $1/R$ correlation is reduced to 0.3 with a significance of 25%. Combining the radius and in-cloud temperature errors, we get a zero correlation with the first kind of temperature error and a slight negative correlation with the second. Actually, one of the apparently more serious shortcomings of Sloss' work, namely, the interpolation of environment humidity from distant radiosondes, does not seem to lead to major problems if the specific humidity error is ±1 gm kg^{-1} or less; it becomes disastrous if the humidity error approaches ±2 gm kg^{-1}. An important demonstration from these tests is that useful deductions regarding the functional dependences of entrainment are difficult to draw from an aircraft program with these types of uncertainties in measurements and assumptions and with the crude graphical one-step entrainment calculation. Hence, Sloss' conclusions must be discarded as invalid. That McCarthy's careful recomputation of entrainment from Sloss' data did reveal the $1/R$ dependence is encouraging, but even so it would be risky to deduce a constant of proportionality from data of this sort.

The next question on entrainment determination concerns liquid water measurements and Warner's (1955) finding that the ratio of actual to adiabatic values did not vary with cloud width. For this evaluation we tried a series of experiments with the same 12 model Florida clouds. If the water content and tower radius measurements were perfect and Eq. (2) correct, the correlation between Q/Q_a and the radius would amount to 0.7, with a significance of 0.5%. If the smallest conceivable instrumental and sampling error in Q [maximum error ±0.4 gm m^{-3}, with a mean absolute error 0.3 gm m^{-3} (or less than 10%)] is introduced in steps of ±0.1 gm m^{-3} by a set of random numbers, the correlation is reduced to 0.4 with a significance of barely 10%. If these small errors are compounded with the same type of radius errors considered in Sloss' case, again applied by another set of random numbers, the Q/Q_a correlation with R is reduced to 0.09, which is completely insignificant. If more reasonable-sized instrumental and sampling errors in Q are applied [namely, a maximum error of ±0.6 gm m^{-3}, with a mean absolute error <0.5 gm m^{-3} ($\sim$17%)], the correlation between Q/Q_a and R is reduced to 0.03, or complete insignificance, even with perfect determinations of R. Actually, Warner (1955) made no attempt to correlate Q/Q_a with active tower radius but compared it with cloud body width which is only very roughly related to tower radius.

However, dismissal of evidence against a postulate does not demonstrate that the postulate is correct. What evidence exists to support the proportionality between entrainment rate and $1/R$? The earliest strong evidence came with the comparison between the entrainment results of Stommel (1947) and others of the Woods Hole group (Malkus, 1954, 1955) on small clouds and those of the Thunderstorm Project (Byers and Braham, 1949) on much larger clouds. The average entrainment rate computed by Stommel was a little above 1×10^{-5} cm^{-1} or about 100% per kilometer, for clouds about the same size as those studied by Saunders (1965), whose radii averaged between 400–500 m.

In the Thunderstorm Project, entrainment was evaluated in two ways. The first of these was a direct method involving the tracking of rawinsonde balloons. In their Florida work in 1946, they had ten rawinsonde stations within 120 mi^2. Balloon releases were made simultaneously into a thunderstorm cloud or into its nearby area. Their data showed the wind direction and velocity for each 1000-ft level, as well as the location of the balloon and the time when it reached each level. During all observations a large PPI radar was operated and the scope photographed so that the winds could be plotted relative to the cloud echo. Seventeen good cases were obtained with wind measurements surrounding the cloud. These measurements leave no doubt whatsoever that lateral entrainment into the sides of cumulus clouds occurs. The Thunderstorm Project's wind results gave a mean rate of lateral entrainment of 100% in about 6 km.

We cannot, however, assume that this was the total entrainment into these clouds, since additional entrainment into the cloud tops may have been occurring. A very revealing set of computations was presented in the Thunderstorm Project's report (Byers and Braham, 1949, p. 37) in which in-cloud temperature excesses were computed from an assumed hierarchy of entrainment rates. These diagrams show that their clouds could just barely remain buoyant up to their observed tops[5] if they were entraining at a rate of 100% in about 3 km. This suggests a roughly equal amount of entrainment from top and sides similar to laboratory plumes and thermals and to the postulates of our model.

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[5] Our observations over the years show that the level of the cloud top usually coincides within a few hundred meters with the level of zero buoyancy. Large "overshooting" is quite rare with tropical clouds.

Our tower radius measurements in Florida [tabulated in part by Simpson and Wiggert (1971)] show that towers naturally growing there to small and medium cumulonimbus stature have radii in the 1200–1500 m range, which is just three times the mean radii of the trade cumuli studied by Stommel and Saunders.

There are two other classes of evidence supporting the $1/R$ entrainment relation. First there is the strong correlation between penetration and tower width. These observations were cited here only for the hurricane case (Malkus, 1960) but were reconfirmed many times over in the extensive photogrammetric studies reported by Malkus and Riehl (1964). Second, our model, based on this postulate, makes joint predictions of top heights, cloud temperatures and water contents which are correct within current abilities to measure these parameters, under widely varying cloud size ranges.

5. Conclusions

It has been shown that the one-dimensional parameterized cumulus tower model is, as far as it goes, physically sound and gives joint prediction of cloud variables that are within the state of the measurement art, without arbitrary calibration or adjustment of "disposable" constants. The apparent negative results of Warner's study have been shown to stem mainly from his testing a model series that omits a vital ingredient (fallout of hydrometeors from the towers), dooming the tests to failure.

It has also been shown that the aircraft evidence cited by Warner to invalidate the inverse relation between entrainment rate and cumulus dimension is inadequate for a test of this relationship and that his and Sloss' negative deductions regarding this postulate are not warranted. On the contrary, several types of evidence have been cited here to indicate that the entrainment is, to first order, inversely related to tower dimension.

These conclusions are not intended to imply that we have reached more than a preliminary goal in cumulus modelling or that the vitally important problem of entrainment is solved, or anywhere nearly so. The range of validity of the entrainment-radius relationship has not yet been specified. It may turn out to be only a crude first-order approximation, with the actual mixing depending on other variables also, for example, perhaps on the details of the turbulent structure within the cloud and in its immediate environment. Advances in the theory of turbulent fluids may be required for a major step in resolving this problem.

To better test the entrainment-dimension relationship by aircraft will be difficult; the sampling problem remains, even if instrumental accuracy is improved. The most fruitful approach would appear to be to concentrate on two, or better three, classes of widely separated cloud sizes with the same measurement systems. Tracer techniques, such as chaff and perhaps chemicals, should be employed to examine the question of where the entrainment takes place. Furthermore, one of the penetrating aircraft will have to box the clouds at relatively constant distance in order to effect accurate tower size determinations by photogrammetry (Herrera-Cantilo, 1969). One size-class of clouds would be those studied by Stommel, Saunders, and Warner, with tops about 3 km and radii of 300–600 m. The second class should be the supercooled cumulus congestus studied in the Florida seeding programs; they top at 6–8 km with radii of about 800–1200 m. The final class should be the giant tropical thundercloud or hurricane eyewall towers topping from 12 to about 15 km, with radii in the range of about 1500–2500 km. Although flying in the most active portions of these may be both difficult and risky, the buoyancies are so large that instrumental requirements are less stringent; also much can be accomplished with radar, photogrammetry, tracers and perhaps optical techniques probing the clouds from the outside.[6]

Due largely to the incompletely resolved entrainment problem, the dynamical aspect of cumulus modelling is still crude, although it has vastly advanced since the 1940's. In addition to obvious usefulness in predicting experimental results, the one-dimensional cumulus models have provided the first framework to examine quantitatively the interaction between the dynamics and microphysics of cumuli (Cotton, 1970); they may remain the best proving ground for microphysical modelling for some time. At the very best, however, the tower model deals only with the actively growing phase of a cloud element; in the tropics its rainfall can under no circumstances be equated to that reaching cloud base or the ground, although the amounts appear to be strongly correlated (Simpson and Wiggert, 1971). So far, no model exists that adequately treats a whole cloud for its whole lifetime, although some apparently promising work is underway by Orville and his collaborators (e.g., Liu and Orville, 1969).

The greatest shortcoming of the one-dimensional models, even if the $1/R$ entrainment relationship should prove completely valid, is that cloud dimension (either assumed or measured) must be used as an initial condition. These models can never, therefore, really predict cumulus structure and development ahead of time, although on many occasions they can be employed usefully to show either that all reasonable tower radii will grow to great heights or that no reasonable radius will get above a specified low level (Woodley *et al.*, 1970). No one yet knows how to predict what horizontal cumulus dimension nature will provide at a given place and time. Although cumulus size appears to be at least roughly proportional to the horizontal convergence in the synoptic regime, what really deter-

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[6] An observational program encompassing many of these objectives is planned for the summer of 1971 by our colleagues, Roscoe Braham and John McCarthy, of the University of Chicago.

mines the scale of convection remains one of the critical unsolved problems in meteorology.

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REFERENCES

Berry, E. X, 1968: Modification of the warm rain process. *Proc. First Natl. Conf. on Weather Modification*, Albany, N. Y., Amer. Meteor. Soc., 81–85.

Byers, H. R., and R. R. Braham, 1949: *The Thunderstorm.* U. S. Dept. of Commerce, Govt. Printing Office, Washington, D. C., 287 pp.

——, and R. K. Hall, 1955: A census of cumulus-cloud height versus precipitation in the vicinity of Puerto Rico during the winter and spring of 1953–1954. *J. Meteor.*, **12**, 176–178.

Cotton, W. R., 1970: A numerical simulation of precipitation development in supercooled cumuli. Ph.D. dissertation, Dept. of Meteorology, Pennsylvania State University.

Herrera-Cantilo, L. M., 1969: Aerial cloud photogrammetry based on Doppler navigation. Final Rept., Contract 22-2-69 (N), Institute of Marine and Atmos. Sci., University of Miami, 65 pp.

Kessler, E., III, 1965: Microphysical parameters in relation to tropical cloud and precipitation distributions and their modification. *Geofis. Intern.*, **5**, No. 3, 79–87.

Levine, J., 1959: Spherical vortex theory of bubble-like motion in cumulus clouds. *J. Meteor.*, **16**, 653–662.

——, 1965: The dynamics of cumulus convection in the trades: A combined observational and theoretical study. Ref. No. 65–43, Woods Hole Oceanographic Inst., 129 pp.

Liu, J. Y., and H. D. Orville, 1969: Numerical modelling of precipitation and cloud shadow effects on mountain-induced cumuli. *J. Atmos. Sci.*, **26**, 1283–1298.

Malkus, J. S., 1954: Some results of a trade cumulus cloud investigation. *J. Meteor.*, **11**, 220–237.

——, 1955: On the formation and structure of downdrafts in cumulus clouds. *J. Meteor.*, **12**, 350–354.

——, 1960: Recent developments in studies of penetrative convection and an application to hurricane cumulonimbus towers. *Cumulus Dynamics*, New York, Pergamon Press, 65–84.

——, and H. Riehl, 1964: *Cloud Structure and Distributions over the Tropical Pacific Ocean.* Berkeley and Los Angeles, University of California Press, 229 pp.

Mee, T. R., and D. M. Takeuchi, 1968: Natural glaciation and particle size distribution in marine tropical cumuli. Final Rept., Contract E22-30-68(N), Meteorology Research, Inc., Altadena, Calif., 71 pp.

Saunders, P. M., 1965: Some characteristics of tropical marine showers. *J. Atmos. Sci.*, **22**, 167–175.

Simpson, J., G. W. Brier and R. H. Simpson, 1967: Stormfury cumulus seeding experiment 1965: Statistical analysis and main results. *J. Atmos. Sci.*, **24**, 508–521.

—— R. H. Simpson, D. A. Andrews and M. A. Eaton, 1965: Experimental cumulus dynamics. *Rev. Geophys.*, **3**, 387–431.

——, and V. Wiggert, 1969: Models of precipitating cumulus towers. *Mon. Wea. Rev.*, **97**, 471–489.

——, and ——, 1971: 1968 Florida cumulus seeding experiment: Numerical models results. *Mon. Wea. Rev.*, **99**, 87–118.

——, and W. L. Woodley, 1969: Intensive study of three seeded clouds on May 16, 1968. Tech. Memo. ERLTM–APCL 8, ESSA, Boulder, Colo., 41 pp.

Sloss, P. W., 1967: An empirical examination of cumulus entrainment. *J. Appl. Meteor.*, **6**, 878–881.

Stommel, H., 1947: Entrainment of air into a cumulus cloud. *J. Meteor.*, **4**, 91–94.

Turner, J. S., 1962: The starting plume in neutral surroundings. *J. Fluid Mech.*, **13**, 356–368.

Warner, J., 1955: The water content in cumuliform cloud. *Tellus*, **7**, 449–457.

——, 1969: The microstructure of cumulus cloud. Part I. General features of the droplet spectrum. *J. Atmos. Sci.*, **26**, 1049–1059.

——, 1970a: The microstructure of cumulus cloud. Part III. The nature of the updraft. *J. Atmos. Sci.*, **27**, 682–688.

——, 1970b: On steady-state one-dimensional models of cumulus convection. *J. Atmos. Sci.*, **27**, 1035–1040.

Weinstein, A. I., 1970: A numerical model of cumulus dynamics and microphysics. *J. Atmos. Sci.*, **27**, 246–255.

Woodley, W. L., and R. N. Powell, 1970: Documentation and implications of the behavior of seeded cloud 17, May 30, 1968. Tech. Memo. ERLTM–AOML 6, ESSA, Miami, Fla., 29 pp.

——, J. Simpson and A. H. Miller, S. MacKay, R. Williamson and G. Cotton, 1970: Some results of single cloud pyrotechnic seeding in Florida, 1970. Tech. Memo. ERLTM–AOML 10, NOAA, Miami, Fla., 87 pp.