Heat Transfer by Symmetrical Rotating Annulus Convection

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ABSTRACT

Results of heat transfer measurements in a differentially heated annulus of fluid for both the non-rotating and rotating cases are given. (In the latter case the flow is in the upper symmetric regime.) In all cases the upper surface of the fluid is free. The non-rotating heat transfer is essentially the same as that of vertical slot convection, whereas rotation modifies the heat transfer; the resulting main effects appear to be exerted through a decrease in the Ekman layer thickness.

1. Introduction

The total heat transfer rate of a fluid, commonly expressed as a Nusselt number Nu (the ratio of the total heat transfer rate of the system to the equivalent conductive transfer rate), is one of the most important properties of free convection flows in a rotating annulus. In particular, the variation of Nu with rotation can give important information regarding physical processes in the fluid; it is also one of the easier quantities to determine experimentally. At the same time there appears to be some disagreement between different theoretical and experimental investigators on this variation in the region of upper symmetrical flows. The data to be given here were carefully obtained for one annulus geometry and two fluids, and appear to be internally consistent enough so that definite conclusions can be drawn concerning the behavior of Nu with the rotation rate Ω.

2. Apparatus and procedure

The apparatus used for the measurements is nearly the same as that used for the measurements in Kaiser (1969). The only modifications were a change in the heat flux measuring thermocouples to eliminate several errors which occurred in the earlier data. The annulus is formed by two concentric cylinders, the outer having a radius of 4.898 cm and the inner 2.458 cm. The outer cylinder is always warmer than the inner. The temperature of each cylinder is maintained by well-stirred thermostated water baths. Each cylinder is within 5% of the total temperature difference of being isothermal and each is steady to within less than 0.01C. The fluid depth is 13.00 cm and the upper surface is free.

The heat flux is measured from the temperature rise ΔT_{cs} of the water circulated through the inner bath by two copper-constantan thermocouple junctions, one referenced against the other. The heat flux measuring system was calibrated against an electrical heater inserted in the inner water bath. The amount of power dissipated by the heater was read on laboratory grade watt meters, while the temperature difference of the thermocouples was read off a strip-chart recorder driven by a Keithley model 148 nanovoltmeter. The flow rate of water being supplied to the inner bath was obtained from a flowmeter which was calibrated to within 1% for several different water temperatures. The heat flux calibration was made at several different power levels, flow rates, and mean fluid temperatures; in all cases the heat dissipation measured by the thermocouples was within 5% of the wattmeter reading. For the specific flow rate used for the experimental determinations, the calculated heat fluxes were within 2% of the wattmeter reading. The temperature difference generated across the cylinders by the water baths was measured at three heights by thermocouples imbedded in the cylinders. The mean of these three, ΔT, was used for the parameters.

The measurements were carried out by holding the temperature difference across the cylinders fixed and changing the rotation rate approximately every 20 min. When the rotation rate was changed, the heat flux reached its new steady value within 10 min in all cases, so the data represent a true steady-state condition. For each value of the temperature difference used, the heat transfer rate was measured with air only in the annular gap. This heat transfer rate was under 10% of the corresponding value with water in the gap, except at temperature differences <1C. At 0.1C the "air value" was 40% of the "water value." The air transfer rate, in addition to including the actual transfer by the air in the gap, presumably is the sum of the transfer rates through the base and lid of the apparatus and the heat gain or loss of the fluid in the cold inner bath to

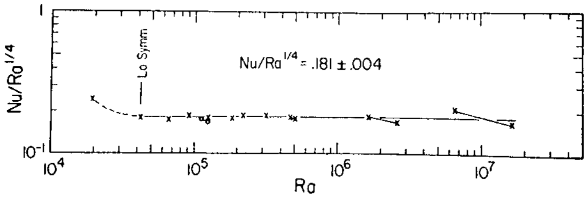

FIG. 1. The heat transfer rate in a non-rotating annulus with a free upper surface.

the environment. The actual heat transfer through the air itself was always less than 2% of the water transfer (Jakob, 1949, pp. 534–539). The actual heat transfer rate is then the difference between the water and air rates for the same temperature difference. Radiative transfer rates are also very small compared to the water transfer rates. If the cylinders have an emissivity of 1, the radiative component would be $\leqslant 5\%$; however, the cylinders, being lightly oxidized brass, have an emissivity[1] of $\lesssim 0.1$. In any case, since the water is extremely opaque to radiation in these infrared regions, all radiative transfers can be neglected in the data.

3. Results

The parameters which are useful here are the Nusselt number Nu, a Rayleigh number Ra, and an Ekman layer thickness δ_E. These are defined as

$$\mathrm{Nu} \equiv \frac{\rho c_p Q(\Delta T_{cs})}{2\pi K\,(\Delta T)/\ln(r_0/r_i)},$$

$$\mathrm{Ra} \equiv gE_r(r_0-r_i)^3/(\nu\kappa),$$

$$\delta_E \equiv (\nu/\Omega)^{\frac{1}{2}},$$

where ρ, c_p and Q are the density, specific heat, and flow rate of the inner bath water, respectively; K, ν and κ the thermal conductivity, kinematic viscosity, and thermometric conductivity, respectively, of the experimental fluid evaluated at the cold bath temperature; r_0 and r_i the respective outer and inner radii of the annulus; g the local gravitational acceleration; and E_r the fractional density difference from the inner to the outer walls of the annulus. The inner source temperature is used to evaluate the experimental fluid parameters because, as will be discussed, the lower Ekman layer in the fluid is thought to exert a major control on the heat transfer, and this layer has a temperature very close to that of the cold bath.

a. No rotation

In addition to the measurements at different rotation rates, a few values of the heat transfer rates were measured for a non-rotating annulus. This case is very similar to the standard vertical slot convection case (see, e.g., Jakob, 1946; Eckert and Carlson, 1961), except here the slot is curved upon itself (which in effect eliminates one end correction). Also, in most of the vertical slot work, the hot wall is maintained by an electrical heater which produces a constant heat flux per unit area, or a constant normal gradient of temperature as the boundary condition. In our case, the hot wall is isothermal. These differing boundary conditions produce boundary layers which have some slight dynamic differences. As for the vertical slot case, we find that $\mathrm{Nu} \propto \mathrm{Ra}^{\frac{1}{4}}$ over most of the range of Ra; this is shown in Fig. 1. Here Ra is the product of the standard Grashof and Prandtl numbers used for heat transfer work. The three pairs of points on the right-hand end of the line were run at large enough temperature differences so that the viscosity of the water was significantly different on the inner and outer cylinders. The lower-right point of each pair represents values of the parameters using values of ν and κ computed for the hot cylinder temperature (the other point is for the cold cylinder temperature). Ignoring the two largest and smallest Ra values, the mean value of $\mathrm{Nu}/\mathrm{Ra}^{\frac{1}{4}}$ is 0.181 ± 0.004. This is the same form of relation determined for the vertical slot from theory and experiment where most of the measurements have been done for air. In Kaiser (1969) it was stated that $\mathrm{Nu}/\mathrm{Ra}^{\frac{1}{4}} = 0.194$ for the non-rotating case. The relation given here is more accurate since the 1969 value did not take account of the corrections used here.

The one point at $\mathrm{Ra} = 2 \times 10^4$ is for a small temperature difference (0.08C) and is in a range where the flow is in a transition zone between conductive and convective regimes (Eckert and Carlson, 1961). The vertical line marked "Lo Symm" denotes the approximate value of Ra for which the baroclinic wave regime in rotating experiments has its onset from the low temperature side. The two circles represent one data point obtained for a 60% by volume of glycerol in water solution with a 2C temperature difference.

b. With rotation

The data for all of the measurements in the upper symmetric regime are given in Fig. 2. Water having temperature differences from 0.6 up to 40C and a 60% by volume of glycerol in water solution having a 2C difference were used. The set of points for $\Delta T = 40$C has a correction applied to it. For that data $\mathrm{Nu}/\mathrm{Ra}^{\frac{1}{4}} = 0.209$ using the cold bath temperature (5C) to evaluate the parameters; thus, $\mathrm{Nu}/\mathrm{Ra}^{\frac{1}{4}}$ was multiplied by 0.181/0.209 to normalize it to the rest of the data. For very small values of Ω the Nu ratio is the same as in the non-rotating case. As Ω is increased sufficiently, the Nu ratio starts decreasing and eventually becomes proportional to δ_E and hence to $\Omega^{-\frac{1}{2}}$ (the slanted lines of slope +1 through the sets of data points represent this proportionality). The two sets of data points for a free surface and rigid lid marked "B & E" are recalculated from

[1] Polished, slightly tarnished brass has an emissivity of 0.045 (Jakob, 1949, p. 126.)

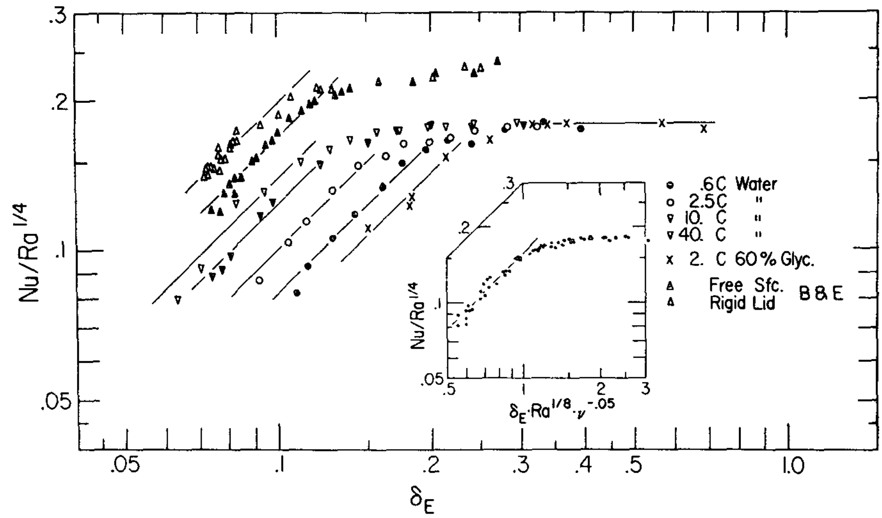

FIG. 2. The variation of heat transfer rate with rotation in an annulus. The inset shows the data scaled with respect to a modified Ekman thickness. The data points marked "B & E" are from Bowden and Eden (1965).

Fig. 3 of Bowden and Eden (1965). Both sets show quite definitely that $\mathrm{Nu} \propto \Omega^{-\frac{1}{2}}$ for a fixed value of Ra. This variation of Nu with Ω is quite similar to that given in Kaiser (1969); there $\mathrm{Nu}/\mathrm{Ra}^{\frac{1}{4}} \propto \Omega^{-0.42}$.

4. Conclusions

The Ω dependence of the Nu ratio suggests that the Ekman layer thickness δ_E might control the heat transfer for sufficiently large values of Ω. The parameter which seems to scale the data reasonably well, however, is not δ_E, but a more complicated number $\delta_E \mathrm{Ra}^{1/8} \nu^{-0.05}$, which is the abscissa of the inset of Fig. 2. That the heat transfer rate should be strongly controlled by δ_E arises from the following argument. The bulk of the meridional flux of fluid in the free surface case is up along the hot outer cylinder, in across the interior region of the fluid, down the cold cylinder, and out across the base in the lower Ekman layer. The flux of fluid out in the Ekman layer is proportional to $\delta_E U_E$, where U_E is some radial velocity component in the Ekman layer, and since Nu is also proportional to this outward flux, it should be strongly controlled by δ_E. Further, since U_E is proportional to V_E, the zonal velocity at the top of the Ekman layer, the fact that Nu is directly proportional to δ_E would suggest that V_E (and of course U_E) must be nearly independent of rotation in a suitable range of rotation rates.[2] When Ω is sufficiently small the fluid from the cold wall is transported outward on the base through a boundary layer which is much thinner than δ_E. In this case $\mathrm{Nu}/\mathrm{Ra}^{\frac{1}{4}}$ is independent of Ω and the Ekman layer dynamics no longer govern the lower boundary layer. Only when Ω is sufficiently large so that δ_E is comparable to the small Ω boundary layer thickness does $\mathrm{Nu}/\mathrm{Ra}^{\frac{1}{4}}$ start decreasing and eventually at a rate proportional to $\Omega^{-\frac{1}{2}}$ or δ_E.

Very little theory applicable to the flow in an annulus with a free upper surface in the symmetric regime is available. Hunter (1967) looked at the free surface case, but for Rossby numbers ($\propto \Delta T/\Omega^2$) much smaller than for the results presented here. He finds $\mathrm{Nu} \approx 1$ for small Ω. Robinson (1959) and McIntyre (1968) both considered the rigid upper surface case. Robinson's theory is valid only for small Rossby numbers as is Hunter's, but McIntyre's results are valid for a portion of the upper symmetric regime. McIntyre's theory is limited to fluid depths less than twice the radial gap. McIntyre finds no simple power law relating Nu to ΔT and Ω, but for the geometry used for the measurements, McIntyre's results can be approximated as $\mathrm{Nu} \propto \Omega^{-\frac{3}{4}}$. Williams (1967a) finds that $\mathrm{Nu} \propto \Omega^{-1}$ for a free surface; but this is from only two points in $(\Omega, \Delta T)$ space and hence is not representative of the majority of the upper symmetric regime.

Williams (1967b) also found that $\mathrm{Nu} \propto \Omega^{-\frac{1}{2}}$ for five *rigid* lid cases; however, for the rigid lid case he had $\mathrm{Nu} \propto \Delta T^{\frac{1}{3}}$ in contrast to our relationship $\mathrm{Nu} \propto \Delta T^{3/8}$ for the free surface case. Williams' result is from five points in $(\Omega, \Delta T)$ space; two points are in the upper symmetric regime, two in the lower symmetric regime, and one within the wave regime neutral curve (but still symmetric). Even so, the five points all seem to fit the same power law closely. There is some evidence in these annulus experiments that the free surface does not act like a classical no-stress boundary. Surface tension and other forces tend to inhibit the radial velocity component at the free surface (as evidenced by the fact that particles of an aluminum powder tracer on the free surface follow trajectories which are circular and do not spiral inward as they would if the free surface radial

[2] This was pointed out in a private communication to the author from Prof. Michael McIntyre.

velocity was nonzero; notice also in Fig. 2 that Bowden and Eden's Nu ratios for both the free surface and rigid lid cases are essentially the same for $\delta_E > 0.12$ cm. Thus, one might expect that the heat flux for the "free surface" experiments to be somewhat intermediate between that calculated for a no-stress upper boundary and a no-slip upper boundary.

Since the heat transfer is one of the more important and reliably measured quantities, the discrepancies between the theoretically and the experimentally determined dependency on Ra, and particularly on the rotation rate, appear quite serious and must represent some lack of correspondence between theory and experiment.

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