On the Effect of Fluctuations in Vertical Acceleration on the Coalescence Growth of Drizzle Drops

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The general expression for the terminal velocity for a spherical water drop of radius y is

$$U=(2g\rho y^2/9\eta)(24/\mathrm{Re}C_D), \quad (1)$$

where η is the viscosity of air, g the acceleration of gravity, ρ the density of the water drop, Re the Reynolds number, and C_D the drag coefficient. For droplets with radius $<45\ \mu$, $(24/\mathrm{Re}C_D)\approx 1$ and Stoke's law provides a fairly good approximation for the terminal velocity under a variety of environmental conditions as verified by experimental results of Beard and Pruppacher (1969). For droplets with radius $>45\ \mu$ up to a radius of $1000\ \mu$, the inverse of that factor can be expressed as a linear empirical function of the radius (Sulakvelidze *et al.*, 1967), i.e.,

$$\mathrm{Re}C_D/24=1.52\times 10^2(\rho_p/\rho_0)y, \quad (2)$$

where ρ_0 and ρ_p are the air densities, respectively, at sea level and at the cloud level with pressure p. Consider the case of a drizzle-size drop, say, with initial radius $200\ \mu$ falling through a cloud represented by an empirical Khrgian and Mazin spectrum

$$n(y)=1.45\times 10^{-6}(L/y_{\mathrm{av}}^6)y^2\exp(-3y/y_{\mathrm{av}}),$$

with liquid water content L of 1 gm m^{-3}, an average radius y_{av} of 7.5 μ, and a droplet size range from 3 to 29 μ. The coalescence growth rate of such a drizzle-size drop can be described by

$$dy_l/dt=(\pi/3)\int_0^{y_l} y_s^3 n(y_s) Y_c^2 (U_{yl}-U_{ys})dy_s, \qquad (3)$$

where y_l and y_s are the drizzle-drop and cloud droplet radii, respectively, Y_c the linear collision efficiency as defined by Shafri and Neiburger (1963), and $(U_{y_l}-U_{y_s})$ the terminal velocity difference between the drizzle drop and smaller cloud droplets. Expressing the Stokes velocity as $U_{y_s}=1.28\times 10^3 g y_s^2$, and substituting (2) into (1), the drizzle terminal velocity becomes a linear function of y_l and $U_{yl}=9.45 g y_l$, if we choose a cloud level at 900 mb with virtual temperature 0C. By introducing these expressions for the terminal velocities, (3) can be written as

$$dy_l/dt=g(\pi/3)\int_0^{y_l} n(y_s)(Ay_l-By_s^2)Y_c^2 dy_s, \qquad (4)$$

where $A=9.45$ sec cm^{-1} and $B=1.28\times 10^3$ sec cm^{-2}.

The coalescence growth history of a typical drizzle drop obtained from integrating (4) numerically as shown in Table 1 for an initial radius of 200 μ. This result can be fitted well by an exponential function, with the form of (4) suggesting the structure

$$y=y_0 \exp(gbt), \qquad (5)$$

where y and y_0 are the drizzle drop radius at time t and initial time, respectively (we are herewith omitting the subscript l), and b is a parameter determined by the least-square criterion in fitting the growth curve with (5). In general, the parameter b depends on the small droplet spectrum and for the present case $b=1.9964\times 10^{-6}$.

Spatial variation of the velocity field inside a cloud is an observed fact. For instance, updrafts and downdrafts are known to be both present in the mature stage of a thunderstorm. Vertical air velocity records from aircraft traverses of developing cumuli (Warner, 1970) also show marked small-scale fluctuations. In such a cloud environment the individual drops are likely to be subjected to the influence of fluctuating vertical accelerations.

A reasonable assumption is that the fluctuating vertical acceleration experienced by the drizzle drop has a normal distribution with mean zero and standard deviation σ. It will superpose on the acceleration of gravity with the resultant acceleration X being a random variable with the probability density function:

$$f_X(x)=[\sigma(2\pi)^{\frac{1}{2}}]^{-1}\exp[-(x-g)^2/(2\sigma^2)]. \qquad (6)$$

The large drop radius now becomes a function of X, i.e.,

$$Y=y_0\exp(btx)=h(x). \qquad (7)$$

Since $y=h(x)$ is differentiable for every x with $h'(x)>0$ for all x, and since X is a continuous random variable, then $Y=h(X)$ is a continuous random variable with probability density function given by

$$\left.\begin{aligned} f_Y(y)&=f_X[h^{-1}(y)]\left|\frac{d}{dy}h^{-1}(y)\right| \\ &=[bty\sigma(2\pi)^{\frac{1}{2}}]^{-1} \\ &\quad\times\exp\{-[\log y-bt(C+g)]^2/(2b^2t^2\sigma^2\}, \quad y\geqslant y_0 \\ &=0, \qquad \text{otherwise} \end{aligned}\right\} \qquad (8)$$

where $h^{-1}(y)=\log(y/bt)-C$ and $C\equiv(y_0/bt)$.

The expected value of the collector drop radius can be found using the important fact that the expectation of a function of a random variable is equal to the expectation of the function with respect to the random variable; therefore,

$$\begin{aligned} E(Y)&=\int_{-\infty}^{\infty} y_0\exp(btx)[\sigma(2\pi)^{\frac{1}{2}}]^{-1} \\ &\qquad\times\exp[-(x-g)^2/(2\sigma^2)]dx \\ &=[y_0\exp(gbt)]\exp[\tfrac{1}{2}(\sigma bt)^2]. \end{aligned} \qquad (9)$$

It is seen that the expected radius is that radius the large drop would reach with no fluctuations in acceleration times an amplification factor, $\exp[\frac{1}{2}(\sigma bt)^2]$. For the case of a large drop with the initial size 200 μ falling through the aforementioned cloud, if the standard deviation of the fluctuating field is equal to $g/10$, which perhaps represents the upper limit that is realizable in natural clouds, the radius increase will be 0.5% at 600 sec and 1.2% at 800 sec over those predicted by the growth equation. Therefore, the effect of a fluctuating vertical acceleration on the accelerated growth of drizzle drops should be unimportant and negligible under realizable cloud conditions. Fig. 1 shows the logarithm of the probability density function of Y at $t=600$ sec with initial large drop size and cloud spectrum as mentioned before and with $\sigma=g/10$. For comparison, the drop would grow to 632 μ at $t=600$ sec under the same conditions but without the presence of vertical acceleration fluctuations.

TABLE 1. Growth history of drizzle drop with initial radius 200 μ in a cloud having a Khrgian-Mazin spectrum with $L=1$ gm m^{-3}, $y_{av}=7.5$ μ, and size range 3–29 μ.

t (sec)	0	100	200	300	400	500	600	700	800
r (μ)	200	241	291	352	427	519	632	771	942

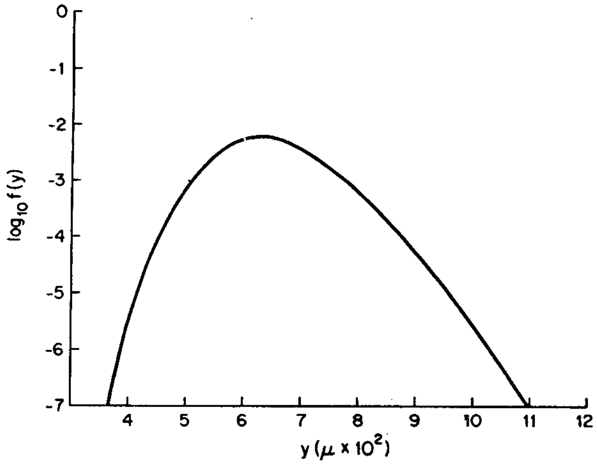

FIG. 1. $\text{Log}_{10}\ f(y)$ vs y at $t=600$ sec for vertical acceleration fluctuations normally distributed with $\sigma=g/10$, $y_0=200\ \mu$.

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