Comments on "Stratified Flow Over Extended Obstacles and its Application to Topographical Effect on Vertical Wind Shear"

JOHN W. MILES

Institute of Geophysics and Planetary Physics, University of California, La Jolla 92037

9 November 1970 and 18 January 1972

Wong and Kao (1970) consider the initial-value problem for a source in a two-dimensional stratified flow between two parallel planes on the basis of the linearized equations of motion and the Boussinesq approximation. They conclude "that upstream disturbances are present whenever lee waves are present," and compare their calculated flow pattern for the *semi-infinite* obstacle that is equivalent to the source with those flow patterns observed by Long (1955) for obstacles of *finite* length.

In fact, Wong and Kao's conclusion does not apply for *unseparated* flow over an obstacle of finite length, for which the corresponding initial-value problem yields a solution that exhibits a dipole behavior far upstream of the body and tends asymptotically to the steady-flow solution implied by Long's hypothesis of no upstream influence, with a transient falling off like $1/t$ (Miles, 1969). [The implicit assumption that the steady flow exterior to any closed stream surface may be regarded as due to an equivalent dipole distribution interior to that surface may be established by constructing the flow in terms of the velocity and vorticity distributions on that surface (Miles, 1970a).] This result depends on the linearization of the initial-value problem, to be sure, but so does the analysis of Wong and Kao. Including second-order interactions in the solution of the initial-value problem does yield a second-order disturbance far upstream of a finite obstacle in a stratified flow between parallel planes if more than two modes are propagated (McIntyre, 1972), but this disturbance is quite different from that obtained by Wong and Kao.

It is true, on the other hand, that Long's hypothesis of no upstream disturbance breaks down at first order for a separated flow (as Long recognized) and that the representation of such a flow within the context of an inviscid model may require the introduction of a source singularity (Stewartson, 1968). However, it seems unlikely that viscous effects can be neglected in any realistic representation of a separated flow. It may be that these effects can be at least partially represented for bluff bodies by an appropriate modification of the boundary conditions for the inviscid equations, as proposed by Stewartson (see also Trustrum, 1971), but it appears that an adequate treatment of the upstream flow requires an explicit incorporation of the viscous stresses in the equations of motion, as in Long's (1959, 1962) studies of jets. It also appears that the source (mass-flux) singularity for the inviscid equations must be replaced by a force singularity (Miles, 1970b).

In brief, Wong and Kao's analysis is relevant for stratified flow over a semi-infinite obstacle, but not for unseparated flow over a finite obstacle. It may offer a qualitative description of some features of separated flow over a finite obstacle, but any adequate description of such a flow appears to demand the incorporation of viscous effects.

Acknowledgments. This work was partially supported by the National Science Foundation and by the Office of Naval Research.

REFERENCES

Long, R. R., 1955: Some aspects of the flow of stratified fluids. Part III. *Tellus*, **7**, 341–357.

——, 1959: The motion of fluids with density stratification. *J. Geophys. Res.*, **64**, 2151–2163.

——, 1962: Velocity concentrations in stratified flow. *J. Hydraulics Div., Proc. ASCE*, **88**, 9–26.

McIntyre, M. E., 1972: On Long's hypothesis of no upstream influence in uniformly stratified or rotating flow. *J. Fluid Mech.*, **52**, 209–244.

Miles, J. W., 1969: Transient motion of a dipole in a rotating flow. *J. Fluid Mech.*, **39**, 433–442.

——, 1970a: The lee-wave regime for a slender body in a rotating flow. Part 2. *J. Fluid Mech.*, **42**, 201–206.

——, 1970b: The oseenlet as a model for separated flow in a rotating viscous liquid. *J. Fluid Mech.*, **42**, 207–217.

Stewartson, K., 1968: On inviscid flow of a rotating fluid past an axially-symmetric body using Oséen's equations. *Quart. J. Mech. Appl. Math.*, **21**, 353–373.

Trustrum, K., 1971: An Oséen model of the two-dimensional flow of a stratified fluid over an obstacle. *J. Fluid Mech.*, **50**, 177–188.

Wong, K. K., and T. W. Kao, 1970: Stratified flow over extended obstacles and its application to topographical effect on vertical wind shear. *J. Atmos. Sci.*, **27**, 884–889.