Free Convection Similarity and Measurements in Flows With and Without Shear

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ABSTRACT

The free convection similarity theory is examined in the light of recent observations in the atmosphere, convection chambers and wind tunnels. The theory describes the fluctuations of temperature and vertical velocity fairly well, but only in flows with finite shear. The horizontal components of velocity and mean temperature may not be scaled by the same in the range of stability ordinarily encountered in the surface layer of the atmosphere. Free convection similarity scaling of the outer layer is expected to be more successful, although sufficient atmospheric data are not available to test this assertion. Preliminary results of numerical calculations by Deardorff are very encouraging, and so are our limited observations in a wind tunnel boundary layer. Because of extremely variable conditions in the atmosphere under free convection, some aspects of this flow may be better studied in the laboratory under carefully controlled conditions. Both convection chamber (no shear) and wind tunnel (finite shear) flows have been used for this purpose; the latter is shown to give better similarity with the atmospheric boundary layer.

1. Introduction

The transfer of momentum and moisture at the earth's surface is intimately related to the rate of heating or cooling of the surface. To determine the interrelated nature of these transfer processes under a variety of meteorological conditions has been the main interest of micrometeorologists. The similarity hypothesis of Monin and Obukhov (1954) has provided a simple framework of determining the various relations empirically from carefully taken observations in the surface layer of the atmosphere. It is assumed that the height z, the buoyancy parameter g/T_a, the surface heat flux H_0, and the surface shear stress τ_0 are the only relevant variables in the similarity analysis, which yield the following velocity, temperature and length scales:

$$\left.\begin{aligned} u_* &= (\tau_0/\rho)^{\frac{1}{2}} \\ T_* &= -H_0/(\rho C_p u_*) \\ L &= -u_*^3 \Big/ \left(k\frac{g}{T_a}\frac{H_0}{\rho C_p}\right) \end{aligned}\right\}. \tag{1}$$

Here, ρ is the air density, C_p the specific heat at constant pressure and k is Von-Kármán's constant, which is introduced for the sake of convenience only.

The above similarity hypothesis predicts that dimensionless quantities $\phi_m=(kz/u_*)(\partial U/\partial z)$, $\phi_h=(kz/T_*)\times(\partial\theta/\partial z)$, σ_u/u_*, σ_θ/T_*, etc., must be universal functions of the stability parameter $\zeta=-z/L$, whose form would have to be determined from other theoretical or physical considerations, or empirically from direct observations (see, e.g., Dyer, 1965, 1967; Swinbank, 1969; Businger *et al.*, 1971).

For the case of large upward heat flux and light winds, it has been suggested by Obukhov (1946) and Priestley (1954) that τ_0 can be dropped from the list of independent variables in the similarity analysis. Then z, H_0 and g/T_a yield the following velocity and temperature scales for these so-called free convection conditions (Wyngaard *et al.*, 1971):

$$\left.\begin{aligned} u_f &= (H_0/\rho C_p)^{\frac{1}{3}}(gz/T_a)^{\frac{1}{3}} \\ T_f &= (H_0/\rho C_p)^{\frac{2}{3}}(gz/T_a)^{-\frac{1}{3}} \end{aligned}\right\}. \tag{2}$$

The free convection similarity theory predicts that the quantities $(z/T_f)(\partial\theta/\partial z)$, σ_θ/T_f, σ_w/u_f, σ_u/u_f, etc., must be universal constants. For the real atmospheric conditions with finite shear, the above scales can be related to those of Eq. (1) as $u_f\propto u_*\zeta^{\frac{1}{3}}$ and $T_f\propto T_*\zeta^{-\frac{1}{3}}$. The asymptotic forms of various similarity functions are then given as $\phi_h\propto\zeta^{-\frac{1}{3}}$, $\sigma_\theta/T_*\propto\zeta^{-\frac{1}{3}}$, $\sigma_w/u_*\propto\zeta^{\frac{1}{3}}$, $\sigma_u/u_*=\sigma_v/u_*\propto\zeta^{\frac{1}{3}}$, etc.

Tennekes (1970) has extended the above similarity approach to the case of free convection in the Ekman layer for which the characteristic velocity and temperature scales are given as

$$\left.\begin{aligned} u_{fe} &= (H_0/\rho C_p)^{\frac{1}{3}}(gh/T_a)^{\frac{1}{3}} \\ T_{fe} &= (H_0/\rho C_p)^{\frac{2}{3}}(gh/T_a)^{-\frac{1}{3}} \end{aligned}\right\}, \tag{3}$$

and the characteristic length scale is the height h of the boundary layer. There is a great deal of controversy about what this length scale should be.

It has not been clear under what conditions, if at all, the free convection similarity scaling would be generally valid. Priestley's (1954, 1959) original criterion of $\zeta \geqslant 0.03$ is certainly not correct (Elliot, 1966); at $\zeta = 0.03$, the buoyant production term in the turbulent energy equation is only about 5% of the mechanical generation term. For the latter to be negligible, one must have $\zeta \gg 1$, with the $\gg$ sign still undefined. To be more precise, free convection represents the asymptotic condition of $\zeta \rightarrow \infty$. But, the limit process may not be unique; it depends on whether u_* is finite, or $u_* = 0$. Some recently available atmospheric and laboratory observations will be discussed to distinguish between these different cases.

2. Atmospheric observations

a. The surface layer ($z \leqslant 0.1h$)

It would be impossible to visualize the atmospheric surface layer without its having some wind shear. Since its height is limited to about one-tenth of the total boundary layer thickness, the asymptotic condition $\zeta \rightarrow \infty$ is probably never realized, although ζ values of the order of 10 could be expected.

A basic limitation of the free convection similarity theory as it is applied to the atmosphere with finite shear is its inability to predict the wind profile. Obviously, u_f cannot be the proper scaling velocity for $\partial U/\partial z$, since its derivation basically assumes zero shear. Lumley and Panofsky (1964) have derived an expression for $\partial U/\partial z$ from that of $\partial \theta/\partial z$ in free convection, assuming that the ratio of the exchange coefficients of heat and momentum (K_h/K_m) is constant. On the other hand, recent observations (Swinbank, 1969; Businger *et al.*, 1971) have shown K_h/K_m to be a continuously increasing function of ζ, though nothing is known about its behavior for $\zeta \gg 1$. Without *a priori* assumptions about K_h/K_m and recognizing that the proper scaling velocity for $\partial U/\partial z$ is u_*, however small, all one can say from dimensional or similarity arguments is that ϕ_m is a universal function of ζ. This has been determined empirically in studies by Dyer and Hicks (1970) and Businger *et al.*, among others. The following simple relation has been found to fit the data up to ζ as large as 10:

$$\phi_m = (1+15\zeta)^{-\frac{1}{4}}. \qquad (4)$$

Regarding the behavior of turbulent velocity fluctuations, only the vertical component has been observed to approximately scale according to the free convection similarity for $\zeta > 0.5$ (Wyngaard *et al.*, 1971). In view of the observed lack of similarity for the horizontal components it is suspected that shear plays a significant role in the dynamics of these components even for large ζ. This can be illustrated simply by considering the balance of energy in each component.

Leaving aside the contribution due to the diffusion terms, which are likely to affect different components in the same sense, the energy in each component is partly due to mechanical generation and partly due to buoyancy. Actually, the former supplies energy directly to only the u component and the latter to the w component. The transfer between different components is then accomplished by pressure fluctuations; the direction of this transfer must be from the largest to smaller components. Under weak-to-moderate stratification ($\zeta < 1$), $\sigma_u > \sigma_w$, and the latter must receive energy from the former. As a result, although mechanical turbulence finds its way in σ_w, the buoyancy would not affect σ_u, at least directly (it may affect it indirectly through the shear production term). With increasing stratification, σ_w increases and so also the ratio σ_w/σ_u, so that an ever smaller part of mechanical turbulence will contribute to the former. But, σ_u and σ_v will still receive substantial contribution from shear-generated turbulence and, only if $\zeta \gg 1$, can buoyancy dominate the dynamics of these components. Even then it is doubtful that these would be scaled according to the free convection similarity.

Under conditions approaching free convection the flow field is characterized by many thermal plumes which probably have their origin in some local discontinuities in the surface temperature and move around in response to the mean wind or large-scale gusts. The return flow occurs in the form of weaker downdrafts which occur in the colder environment surrounding hot plumes. A major contribution to σ_u and σ_v in the surface layer probably comes from the large-scale lateral spreading of these downdrafts as they impinge on the surface. This lateral spreading or tangential flow, if seen on a scale which is smaller than that of the large-scale convective circulation, would produce its own local wind profile and consequently some mechanical turbulence, even though the mean wind may be zero or nearly so when averaged over a large (as compared to the scale of convection induced circulation) horizontal area.

In true free convection ($u_* = 0$), σ_u and σ_v may be related in some way to the large-scale convective motion. But for the atmospheric surface layer with finite mean wind and shear, there seems to be no alternative to the more general Monin-Obukhov similarity theory according to which σ_u/u_*, σ_v/u_*, etc., as functions of ζ, must be determined empirically.

Next, we examine the temperature structure in the surface layer vis-à-vis free convection similarity. It is somewhat puzzling that although the similarity behavior for σ_θ has been observed to set in for ζ as small as 0.05 (Wyngaard *et al.*, 1971), the same does not occur for the mean temperature gradient or ϕ_h, at least within the reported range ($\zeta < 5$) of observations (Dyer, 1967; Swinbank, 1969; Businger *et al.*, 1971). The slope of the ϕ_h vs ζ curve gradually changes from zero in forced convection conditions ($\zeta \ll 1$) to about $-1/2$ for large ζ; a slope of $-1/3$ appears to be only transitional which should not be mistaken for free convection. Whether the same would be more extensively observed for still

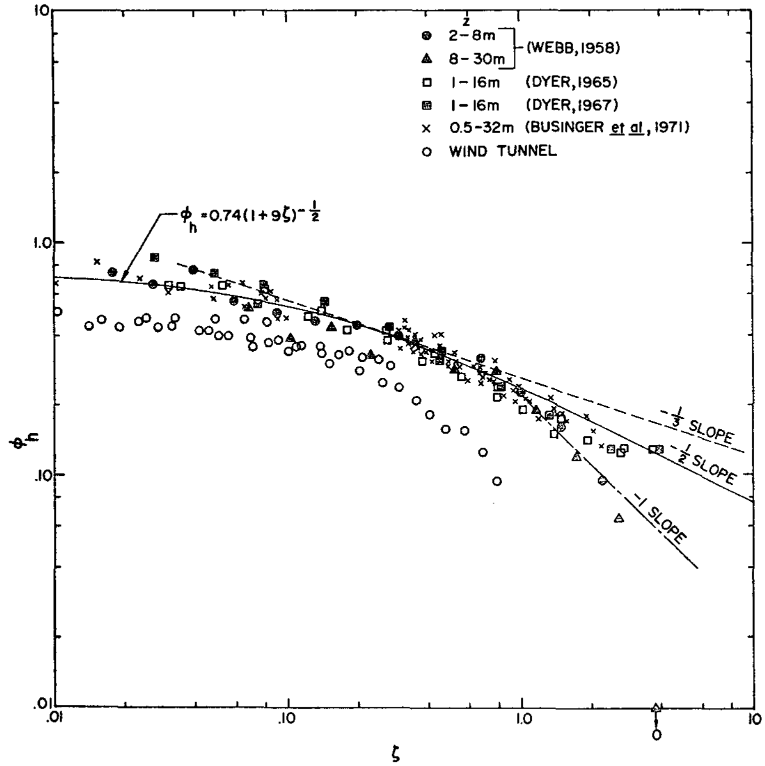

FIG. 1. The variation of ϕ_h with stability in the surface layers of the atmosphere and the wind tunnel.

larger values of ζ than covered by the above reported measurements, remains to be seen.

Townsend (1962), on the other hand, has stipulated a region of 'natural convection' in which $\partial\theta/\partial z \propto z^{-2}$, or $\phi_h \propto \zeta^{-1}$. Such a behavior has been observed in laboratory convection chambers, but the arguments for generalizing the same relation to the atmospheric boundary layer having finite shear are not very convincing. The particular observations of Webb (1958) which were used by Townsend in partial support of his hypothesis may not be representative of the atmospheric surface layer. This can be seen from Fig. 1, in which Webb's data are compared with those from more recent experiments (Dyer, 1965, 1967; Businger *et al.*, 1971). Also represented in the same figure are some wind tunnel observations which will be discussed later. In order to transform Webb's original data into similarity coordinates we have assumed, following Businger *et al.*, that $(\text{Ri})_{1.5\text{m}} = 1.5/L$, where L is in meters, and Ri is the Richardson number.

One notes that the atmospheric data of Fig. 1 are in good agreement with each other up to a value of $\zeta \approx 1.5$. For large ζ, however, Webb's data points deviate sharply away from what may be considered as a common similarity trend, indicating that these few observations are not typical of the surface layer (see also, Deardorff *et al.*, 1967b). Considering the rest of data points together, there is no clear-cut support for ϕ_h approaching either a -1 power law as suggested by Townsend (1962), or a $-1/3$ power law of free convection similarity, when $\zeta > 1$. On the other hand, a $-1/2$ slope fits much better in the range $0.2 < \zeta < 2$ (see also, Businger *et al.*). The failure of the mean temperature profile to follow free convection similarity indicates that it too is influenced by shear up to large values of ζ.

b. *Ekman layer*

Under conditions of large upward heat flux, the changes of mean wind and temperature with height above the surface layer have been observed to be rather small. Therefore, mean profiles themselves may not be of much interest in the Ekman layer, but the bulk transfer coefficients of momentum and heat and the structure of turbulence are of great significance.

A straightforward extension of the Monin-Obukhov similarity theory to the case of the Ekman layer leads to the relations (Zilitinkevich *et al.*, 1967):

$$\frac{u_*}{G} = f_1(\text{Ro}, \zeta_h), \tag{5}$$

$$\frac{T_*}{\Delta\theta} = f_2(\text{Ro}, \zeta_h). \tag{6}$$

Here, $\text{Ro} = G/fz_0$ is the surface Rossby number,

$\zeta_h = -h/L$ is the bulk stability parameter, and $\Delta\theta$ is the potential temperature difference across the boundary layer. For $\zeta_h \to 0$, f_1 and f_2 have been determined from similarity considerations (see Blackadar and Tennekes, 1968) to have the form

$$\left(\frac{u_*}{G}\right)^{-1} = \frac{1}{k}\left[\ln\left(\frac{u_*}{G}\text{Ro}\right) - A\right], \tag{7}$$

$$\left(\frac{T_*}{\Delta\theta}\right)^{-1} = \frac{1}{k_\theta}\left[\ln\left(\frac{u_*}{G}\text{Ro}\right) - C\right], \tag{8}$$

in which k_θ, A and C are universal constants. As a matter of convenience, the same relations can be used for stratified conditions provided A and C are considered as functions of the stability parameter ζ_h (Zilitinkevich *et al.*). Leovy (1969) has collected some measurements of $T_*/\Delta\theta$ as a function of the bulk stability parameter, $B = \alpha(g/T_a)(\Delta\theta/u_*^3)$, covering a very wide range of the latter ($1.7\times10^{-4} < B < 1.7$). A theoretical relation obtained by integrating the empirical temperature profile across the boundary layer represents the data fairly well.

For the turbulence structure, the similarity theory predicts that σ_u/u_*, σ_w/u_*, σ_θ/θ_*, etc., must be universal functions of z/h and ζ_h. The available atmospheric data on these quantities is far too less and too unsystematic to determine these functions completely. For $\zeta_h \gg 1$, free convection similarity ideas could simplify the problem greatly.

For unstable conditions in the atmosphere, ζ_h often exceeds unity. Therefore, free convection is more likely to be encountered in the outer layer ($z/h > 0.1$) than it is in the surface layer. Tennekes (1970) has discussed its implications for the former, giving some useful relations such as, $\langle\sigma_\theta\rangle/\theta_* \propto \zeta_h^{-\frac{1}{3}}$ and $\langle\sigma_w\rangle/u_* \propto \zeta_h^{\frac{1}{3}}$, where $\langle\ \rangle$ represents averaging over the whole Ekman layer. His interpretation of the parameter ζ_h in terms of the ratio of the time scales of buoyant and shear motions leads to a criterion for determining whether free convection would occur in a particular situation. This is based on the assertion that if the time scales of buoyant and shear turbulence differ by at least an order of magnitude, the interaction between the two will be relatively weak, irrespective of the ratio of their production terms. Taking $t_s = 1/f$ and $t_b = [(g/T_a)(\sigma_\theta/h)]^{-\frac{1}{2}}$, Tennekes has estimated a minimum value of ζ_h of about unity (for free convection to occur), which is likely to be exceeded for almost all cases of unstable conditions in the planetary boundary layer.

Businger (1971) has raised the question of the uniqueness of characteristic length and time scales for the Ekman layer. Tennekes' assumption that $h_t = h_m \propto u_*/f$, where h_t and h_m are thermal and momentum boundary layer thicknesses, cannot be valid for conditions approaching true free convection ($u_* \to 0$) and also for penetrative convection. The above assumption is also implied in the recent Russian literature (Zilitinkevich *et al.*, 1967).

Deardorff (1970a,b), on the other hand, finds the height z_i of the lowest inversion base to be the most important length scale, and u_*/f to be essentially irrelevant for unstable conditions. For $z > -L$, velocity and temperature are scaled by u_{fe} and T_{fe} as given in Eq. (3), in which $h = z_i$. Deardorff's preliminary results of numerical integration indicate that σ_w/u_{fe} and σ_θ/T_{fe} as functions of the dimensionless height z/h would attain a similarity shape in bulk of the boundary layer for $\zeta_h \gtrsim 4.5$. Similarity scaling of the horizontal components of velocity would require somewhat larger values of ζ_h. So long as there is some mean wind (finite shear), the momentum flux is scaled by only u_*^2. The numerical model also reveals some other interesting features of the free convection flow (Deardorff, 1970a).

3. Laboratory measurements without shear

The absence of true free convection ($u_* = 0$) in the atmosphere has led to its study in the laboratory under carefully controlled conditions. Of special interest are the experiments on fully developed turbulent convection at large Rayleigh numbers in which detailed measurements of mean temperature and of the fluctuating motion have been made in addition to those of heat transfer.

Thomas and Townsend (1957) and Townsend (1959) have reported heat flux and temperature measurements from both the closed and open top convection chambers. In spite of the comparatively small Rayleigh numbers in the former, the structure of the surface layer and heat transfer are found to be similar in the two cases. The results are shown to favor non-dimensionalizing based on the following scales of velocity, temperature and height (Townsend, 1959):

$$\left.\begin{aligned} u_0 &= \left(\frac{H_0}{\rho c_p}\frac{g}{T_a}\alpha\right)^{\frac{1}{3}} \\ \theta_0 &= \left(\frac{H_0}{\rho c_p}\right)^{\frac{3}{4}}\left(\frac{g}{T_a}\alpha\right)^{-\frac{1}{4}} \\ z_0 &= \alpha^{\frac{3}{4}}\left(\frac{H_0}{\rho c_p}\right)^{-\frac{1}{4}}\left(\frac{g}{T_a}\right)^{-\frac{1}{4}} \end{aligned}\right\}, \tag{9}$$

where α is the molecular thermal diffusivity. Some plausible reasons for including α in the similarity formulation have been given by Townsend (1959). These are valid only for the windless (natural) convection in laboratory convection chambers, for which distributions of the mean and the rms temperature are found to be at large variance with the predictions of the Obukhov-Priestley similarity theory. On the other hand, these measurements indicate that well above the conduction layer ($z/z_0 \gg 1$), $\partial\theta/\partial z \propto z^{-2}$ and $\sigma_\theta \propto z^{-0.6}$.

Croft (1958) has made visual as well as quantitative measurements using optical methods in both the open and closed chambers. Visual observations suggest the existence of cellular structure close to the surface, followed by a region of penetrative or columnar elements. Temperature profiles are shown to be scaled by θ_0 and z_0. But the exponent δ in the relation $\partial\theta/\partial z \propto z^{-\delta}$ varied from 1.4–1.6, its value increasing with a decrease in heat flux. The lower values of δ then observed in Townsend's experiments may be attributed in part to the effect of some mean circulation in Croft's apparatus. His observations were also limited to a much narrower region ($z/z_0 < 16$) and might have been affected by the close proximity of the conduction layer.

More recently, Deardorff *et al.* (1967a) have made detailed observations of convection between parallel plates covering Rayleigh numbers up to 1.0×10^7. Their observations are also consistent with a thermal structure dominated by plumes. The temperature profiles show characteristic boundary layers near the top and bottom surfaces and a nearly isothermal region in the center. Vertical distributions of σ_θ, σ_u and σ_w show some very interesting features. There is a close similarity between σ_θ and σ_u profiles, each showing a sharp maximum near the surface and thereafter decreasing and flattening out in the central region. In contrast, σ_w gradually increases from zero at the surface and gives a rather broad maximum in the central region. The central region is characterized by nearly constant intensities, and here σ_w is greater than σ_u by a factor of 1.5–2 depending upon Ra. On the other hand, $\sigma_u > \sigma_w$ in the surface layer in which most of the temperature change occurs.

A particularly disturbing conclusion from the above-mentioned laboratory experiments is that even though the similarity theory assumption of $u_* = 0$ is more closely satisfied than in the atmosphere, the laboratory observations in the surface layer are at much greater variance with the predictions of the Obukhov-Priestley theory. According to Townsend (1962), the main difference between laboratory and atmospheric convection is the presence of finite wind shear in the latter, which smears out the effect of the conduction layer and of hot penetrative plumes originating therefrom. Apparently, the similarity theory describes some of the characteristics, e.g., σ_θ and σ_w, more closely in flows in which substantial interaction of buoyancy and inertia forces occurs rather than in the case of "natural" convection in which there is very little or no such interaction. This view was accepted by Priestley (1962), but has been opposed by Deardorff and Willis (1967b) who have contended that $\partial\theta/\partial z \propto z^{-4/3}$ and that other similarity relations would also be observed in laboratory chambers at much larger Rayleigh numbers. This could not be proven, however, by their parallel plate experiments, which despite their different interpretation, showed the region of -2, and not $-4/3$, power to increase with increasing Ra, more in line with Townsend's open box experiments.

It can be argued that even though the surface layers of the two flows, one with shear and the other without it, may have distinctly different structures, their outer layers are more likely to be similar. This, of course, assumes that the effect of the forced convection region in the former and that of the conduction layer in the latter would not extend too far above their respective surface layers, and that the outer region in both cases will be dominated by similar thermal plumes. Deardorff (1970b) has demonstrated this similarity in a rough sense, by comparing the values of σ_w/u_{fe} and σ_θ/u_{fe} at the level where σ_w is maximum, for the atmosphere and a convection chamber. Because of the basically different distribution of heat flux and different boundary conditions, however, the similarity of flow in these two cases cannot be stressed too far. Laboratory modeling with some finite shear, e.g., in a wind tunnel, is expected to give better similarity with the atmospheric flow. This has been demonstrated earlier for the case of stably stratified boundary layer (see, Arya and Plate, 1969). Some wind tunnel observations in unstable conditions are discussed in the following section.

4. Laboratory measurements with shear

Experiments were made in a closed circuit wind tunnel having a 1.8×1.8 m cross section over a test section length of 29 m. After the initial 12 m of the test section, the floor consists of a 12 m long aluminum plate, which was electrically heated at a uniform temperature. The air entering the test section was cooled by passing it over the refrigeration coils in the widest part of the return duct. For complete details of the wind tunnel and its heating and refrigeration system, see Plate and Cermak (1963).

The power supply to the tubular heaters, which are fastened to the bottom of the plate at intervals of 15 cm, was adjusted so as to obtain a fairly uniform temperature (to within ±2C) of the plate surface when the tunnel was run at an ambient air speed of 6.1 m $\sec^{-1}$ and an ambient temperature of about 2.5C in the test section. With fixed settings of the fan speed and pitch and of the heating and refrigeration system, steady state could be attained in approximately 10 hr.

Mean velocities were measured using a standard pitot-static tube whose read-out on a differential pressure transducer (Transonics Model 120B) was integrated and averaged over a period of 4 min. Air temperature was simultaneously measured point by point with a copper-constantan thermocouple whose output was averaged and read out on a sensitive potentiometer. Plate temperatures were monitored by thermocouples embedded into the plate at 30-cm intervals along the centerline, whose outputs were read out and recorded on a multi-point strip chart recorder. Temperature fluctuations in the air were measured by a platinum-rhodium resistance thermometer having a 0.5 μm sensor, which was used in a Wheatstone bridge specially designed to pass small currents $\leqslant 0.1$ mA. This ensured essentially cold operation of the wire, sensitive to only temperature fluctuations. The output was amplified and read out on a true rms meter.

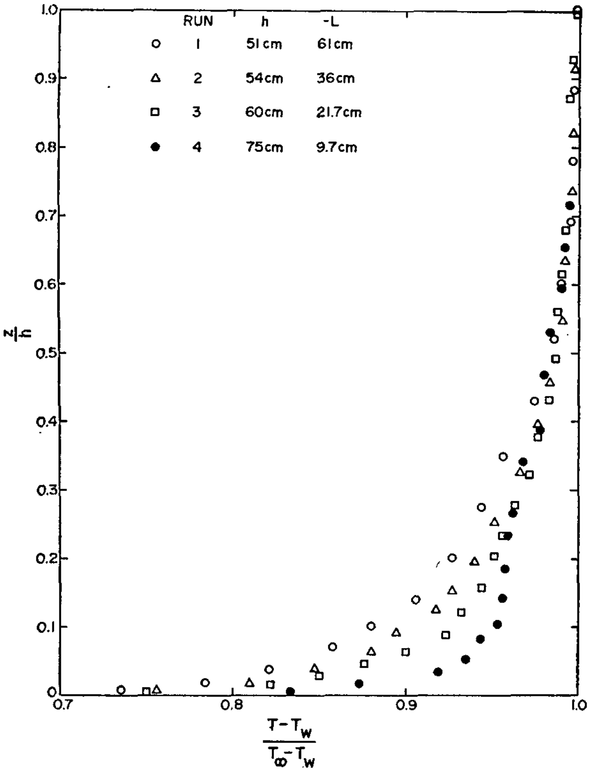

FIG. 2. The normalized mean temperature profiles in the wind tunnel.

As the cold air enters the test section, a constant-pressure boundary layer develops along the floor, which remains isothermal for the initial 12 m, after which a thermal boundary layer also begins to develop over the heated section. The measurements to be reported here were made at a station near the downstream end of the heated plate, where both the momentum and thermal boundary layers were fairly thick ($\geqslant 40$ cm) and well developed. Two-dimensionality of the mean flow was checked by taking lateral traverses of velocity and temperature at different heights. Observations were made at ambient velocities ranging between 2–6 m sec^{-1}.

For all the runs the power supply to the plate heaters was kept constant. Therefore, each reduction in the wind speed was followed by a gradual increase in the plate temperature until a new steady state was reached. Because of the uncertainty of heat losses due to radiation, conduction through metal supports, etc., the heat flux could not be determined very precisely. From measurements of electric power and after accounting for the estimated losses, values ranging between 150–170 mW cm^{-2} are obtained for the same with an uncertainty of $\pm 20\%$. These are almost an order of magnitude greater than the values encountered in the atmosphere, and 2–3 times the heat flux used in the experiments by Townsend (1959) and Deardorff and Willis (1967a). Such high fluxes were necessitated by our desire to simulate Richardson numbers or ζ of the same magnitude as found in the atmospheric surface layer under unstable conditions. The same could have been accomplished at lower heating rates by using lower ambient velocities. But, for $U_\infty < 2$ m sec^{-1}, evidence of secondary circulation in the tunnel was considered to cause more serious problems than the alternative of using large heating rates. Our measurement techniques would also be suspect at very low wind speeds.

Mean temperature distributions in the boundary layer for different conditions of thermal stratification are shown in Fig. 2. The corresponding mean velocity profiles are presented in Fig. 3. The various parameters and the inner and outer scales for the same runs are given in Table 1.

The length scale h has been chosen to represent the thermal boundary layer thickness defined such that $\partial T/\partial z = 0$, when $z \geqslant h$. This is also found to be the level where σ_θ is close to zero (see Fig. 4), although some extremely intermittent temperature fluctuations could be observed even at larger heights. Because of very small temperature changes in the outermost part of the boundary layer, h could not be determined with an accuracy better than $\pm 10\%$. Determination of the momentum boundary layer thickness from mean velocity profiles (Fig. 3) would be even more difficult and uncertain in view of some observed anomalies in these profiles in the outer region, which appear to increase with decreasing tunnel speed. These anomalies, which are less than 2% of the ambient velocity, may be

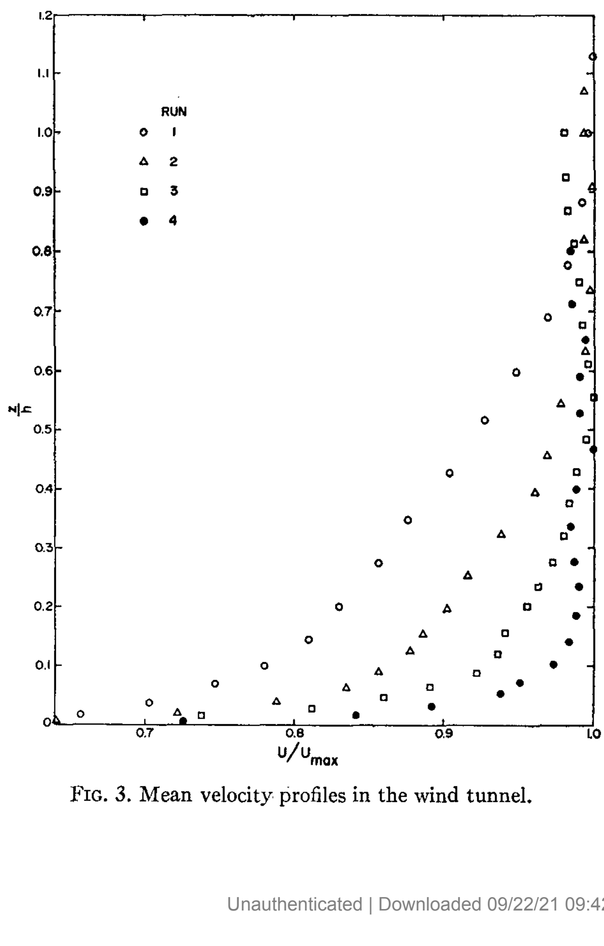

FIG. 3. Mean velocity profiles in the wind tunnel.

TABLE 1. Boundary layer parameters and scales.

Run no.	U_{max} (cm sec^{-1})	$T_w - T_\infty$ (°C)	$H_0/(\rho C_p)$ [cm (°C) sec^{-1}]	u_* (cm sec^{-1})	T_* (°C)	$-L$ (cm)	h (cm)	T_{fe} (°C)
1	612	123	169	24.0	7.0	61	51	5.5
2	456	133	166	20.0	8.3	36	54	5.4
3	323	143	154	16.5	9.3	22	60	4.9
4	210	159	150	12.5	12.0	9.7	75	4.5

partly due to the increasing error in mean velocity measurements at low speeds, specially in the presence of highly intermittent but intense fluctuations. For Run No. 4, there is also the possibility of some secondary flow which might have also caused the peculiar temperature distribution (Fig. 2) above the surface layer.

The normalized temperature and velocity profiles in the wall region show their steepness increasing with stratification. This trend is similar to what has been observed in the atmosphere (Clarke, 1970). Nearly logarithmic variations of temperature with height in the surface layers of both the wind tunnel and atmospheric flows (Fig. 1) are in marked contrast with the observed temperature distributions in convection chambers (Deardorff *et al.*, 1970a). Distributions of σ_θ in the two cases are also quite different. Fig. 4 shows that the similarity scaling of σ_θ/T_{fe} vs z/h is fairly successful for the wind tunnel flow. Data points for the

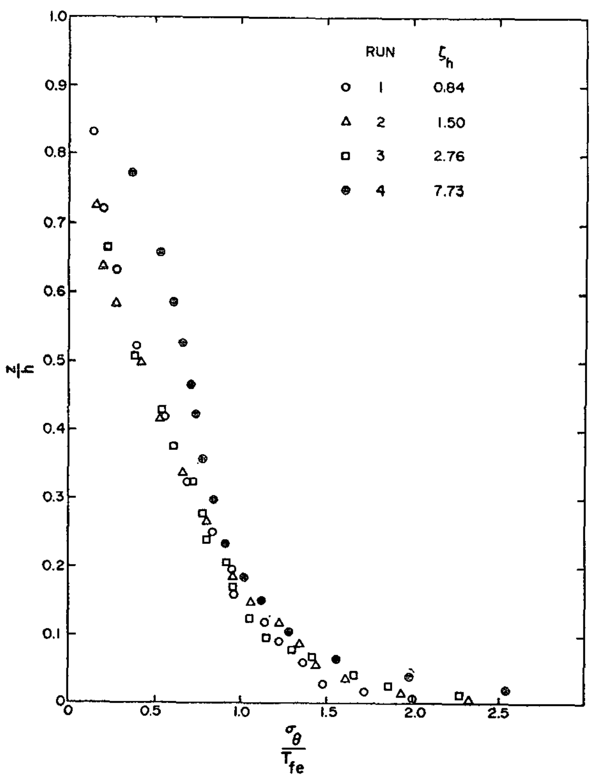

FIG. 4. The distribution of rms temperature fluctuations.

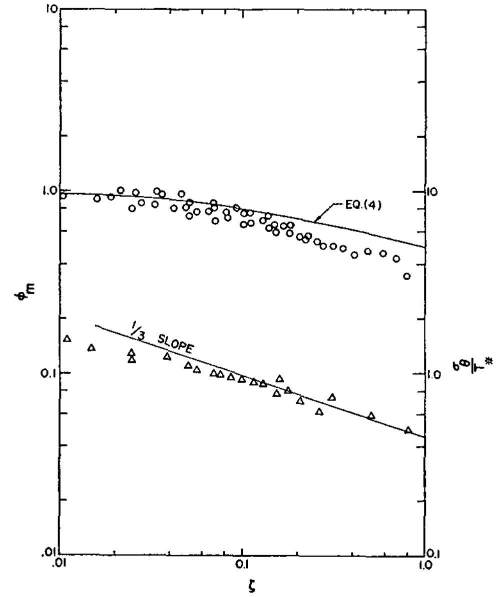

FIG. 5. Variation of ϕ_m and σ_θ/T_* with stability in the wall region ($z/h \leqslant 0.1$).

runs 1, 2 and 3 fall very close together; large deviation of some points for the run 4 is probably caused by the aforementioned secondary flow. The temperature vs time records showed increasing intermittency with the increase in height (z/h) as well as with the increase in stratification (ζ_h). Plume-like structures could also be easily identified.

It can be noted from Table 1 that values of u_*/U_{max} range between 0.039–0.060 and those of $T_*/(T_w - T_\infty)$ between 0.056–0.076, both increasing with ζ_h. For a flat plate boundary layer these are also expected to depend on the Reynolds number, but within the limited range of Reynolds number in our experiments this effect would not account for more than 5% variation of these ratios.

One can compare the nondimensional functions ϕ_h, ϕ_m and σ_θ/T_* for the wall region of our wind tunnel boundary layer with those for the atmospheric surface layer. Figs. 1 and 5 indicate that the wind tunnel values are too low. The trend $\sigma_\theta/T_* \propto \zeta^{-\frac{1}{3}}$ agrees with the prediction of free convection similarity, but the proportionality constant is only about half of the atmospheric value. The main reason for this lack of similarity between the atmospheric and the present wind tunnel observations seems to be the use of too large heating rates in the latter, which resulted in too large values of $(T_w - T_\infty)$. Far better similarity was observed in our previously reported (Arya and Plate, 1969) wind tunnel observations under stable conditions for which $(T_w - T_\infty)$ was kept reasonably small ($<$45C). Improved simulation can also be expected for unstable conditions if one restricts the temperature difference to about the same but uses lower wind speeds to produce strong stratification. This would, of course, require greater care to eliminate secondary flows, and possibly, spatial averaging of the flow characteristics to be measured.

Some measurements of mean velocity and temperature in the same tunnel have been reported earlier by Plate and Lin (1966), and Chuang and Cermak (1967). Reasonably small values of $(T_w - T_\infty)$ and U_∞ were used in these experiments. In spite of the suspected secondary flow currents at low speeds, a good similarity with atmospheric observations has been indicated.

Laboratory measurements in unstable boundary layers have also been reported by Nicholl (1970), who used free stream velocities of 150 and 240 cm sec^{-1} and temperature differentials of 20 and 80C in a small open-circuit wind tunnel. Due to the small length (2.2 m) of the test section, however, momentum and thermal boundary layers are not as thick and as well developed as would be necessary for good similarity with the atmospheric flow. Nicholl's observation stations ($h \leqslant 74$ cm) were essentially located in the transition region following the step change in the floor temperature, and as such, indicated large variations of the various characteristics in the direction of flow.

5. Conclusions

The free convection similarity theory is critically examined in the light of recent atmospheric and laboratory observations. The partial agreement of the same with observations in the surface layer can only be termed as fair. The largest discrepancy lies in the predictions of the horizontal components of velocity whose dynamics is shown to be influenced by shear even at large values of the stability parameter ζ. Shear apparently also plays a significant role in determining the mean temperature profile, which neither obeys the similarity law, nor seems to approach the behavior stipulated by Townsend (1962).

Although the free convection condition of $u_* \rightarrow 0$ is almost exactly satisfied in laboratory convection chambers, measurements from the same are at much greater variance with the predictions of the similarity theory. The basic difference between this flow and that in the atmosphere with finite shear is the presence of a region of effectively forced convection in the latter, which helps detach the regime of quasi-free convection above from the extremely thin conduction layer below. In a convection chamber, on the other hand, the plume structure remains essentially tied to the conduction layer and thermometric conductivity is a pertinent variable in the similarity analysis up to heights far above the conduction thickness. Although this effect may not extend too far into the outer part of the boundary layer and into the essentially isothermal region in the center, too close a similarity between this flow and the atmospheric convection is precluded by basically different heat flux distributions and different boundary conditions in the two cases. Laboratory studies of "natural convection" are nevertheless quite helpful in that detailed measurements of turbulence structure and other aspects of the flow can be made under carefully controlled conditions. It is proposed that modeling of the atmospheric convection would be far more successful in the presence of small but finite shear, as in a wind tunnel or duct, than it has been for flows without shear, e.g., in convection chambers.

Some measurements of boundary layer above a long heated plate in a closed circuit tunnel are presented to show that some basic features of the atmospheric flow are indeed simulated. The similarity is nevertheless far from perfect, but one can make further improvements by using lower heating rates and lower wind speed than were used in the present experiments. Of course, one would have to take great care to eliminate secondary currents which are likely to appear under these conditions. Horizontal averaging in the lateral direction may also be desirable. Test section lengths of several tens of meters are required to obtain fairly thick and self-preserving boundary layers.

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