Observations of Helmholtz Waves in the Lower Atmosphere with an Acoustic Sounder

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ABSTRACT

Acoustic probing of the lower atmosphere (<150 m) reveals structures that appear similar to those of instability waves produced by wind shear at the stable interface of a temperature inversion. The acoustic sounder was located in the vicinity of a meteorological tower 152 m in height. Profiles of wind velocity and temperature were taken during the acoustic sounder operation. Regions of enhanced thermal stability and wind shear produce strong echoes which the acoustic sounder maps on a height vs time facsimile record. In this paper we limit our discussion of those echo returns that have the characteristic appearance of Helmholtz waves. Richardson numbers calculated from the tower measurements over the layer thickness as determined from the acoustic sounder returns appear to be of the order of $\frac{1}{2}$, while sub-strata embedded within the layer thickness exist where the Richardson number drops near the theoretically predicted critical value of $\frac{1}{4}$. In addition, measurements of the wavelengths associated with the "breaking" phenomenon conform to the theoretically predicted range of values.

1. Introduction

The advent of remote sensing techniques during the last few years and the subsequent observation of wave instability phenomena in the clear atmosphere have given new impetus to one of the oldest and most fundamental studies of hydrodynamics, that of internal waves in the atmosphere. These waves are now recognized to be an important link between meso- and micro-meteorological phenomena. Indeed, such waves are the subject of intense research, both in the clear atmosphere and laboratory, as the possible origin of clear air turbulence.

Theoretical considerations, laboratory observations, as well as observations in the clear atmosphere, suggest that clear air turbulence may be attributed to a form of dynamic instability commonly referred to as Helmholtz (or Kelvin-Helmholtz) instability. This problem, however, is far from being completely, or even satisfactorily, resolved.

In the clear atmosphere, Helmholtz instability occurs within a layer in which both the thermal stability and wind shear assume large values. The instability initially appears as amplifying waves which, in time, "roll" and "break" into turbulent flow on a range of smaller scales. The onset of Helmholtz instability within a hydrostatically stable layer of thickness Δz is determined by the value of the corresponding layer Richardson number

$$\mathrm{Ri}=\frac{g}{\bar{T}}\left(\frac{\Delta T}{\Delta z}+\Gamma\right)\bigg/\left(\frac{\Delta V}{\Delta z}\right)^2, \qquad (1)$$

where Γ is the adiabatic lapse rate of temperature and V the magnitude of the vector wind.

Considerable theoretical work has been done on the critical value of the Richardson number for the onset of Helmholtz instability. Taylor (1931) and Goldstein (1931) attacked this problem and found that $\mathrm{Ri}_c \approx 0.25$. Some years later, Drazin (1958) and Drazin and Howard (1961), among others, also studied this problem and found that Ri_c is indeed 0.25. More recently, Miles and Howard (1964) have found that $\mathrm{Ri}_c \leqslant 0.25$ is a necessary but not sufficient condition for the onset of Helmholtz instability.

From laboratory experiments, Thorpe (1968) was able to obtain a critical Richardson number value of 0.25. Because of observational limitations, however, it has been difficult to confirm this for the free atmosphere. In a recent and quite complementary study, Browning (1971) was able to detect several cases of Kelvin-Helmholtz billows in the tropospheric layer with a high-power radar. These billow events had lifetimes of the order of 2–18 min, with only one event having a remarkably long lifetime of ~240 min. Profiles of wind and temperature were obtained from balloon-borne radiosondes which were released hourly from a site which was located 8 km away from the radar site. On the average, the radiosondes reached the altitude of the billows (5.6–11.0 km) in approximately 20 min during which time they also traveled 10–75 km downwind. By calculating Richardson number profiles over a layer depth of 200 m, Browning was able to correlate minimum Ri values (typically 0.15–0.30) with the region of Kelvin-Helmholtz billows. In fact, on a few occasions the height of the billow events had to be adjusted by as much as 250 m so as to give the best correlation with the low Richardson number layer.

The acoustic sounder techniques, as advanced by McAllister (1968), have been used in the experimental study we report on to probe the lower region of the planetary boundary layer. Turbulent fluctuations in both the temperature and velocity fields produce inhomogeneities in the acoustic refractive index and, as a result, it is possible to map the turbulent field from the scattered acoustic waves. The details of such procedures have already been published elsewhere (McAllister *et al.*, 1969; Little, 1969). During the present study the acoustic sounder was operating in the monostatic, or backscatter, mode. Consequently, only temperature fluctuations contributed to the received scattered power.

In this study we have, in addition to the acoustic sounder returns, concurrent measurements of wind velocity and temperature taken on a 152 m meteorological tower in the vicinity of the acoustic sounder.

2. The experimental work

The experimental work was carried out on the high plains of southeastern Colorado during the latter part of 1969. The 152 m tower is centrally located in a "bowl-like" depression measuring approximately 20 km across with a maximum depth of 55 m. The site is at an elevation of 1307 m and displays a characteristic high-plain meteorological regime with strong solar heating during the day and radiative cooling at night. In the vicinity of the tower the terrain is flat, free from natural and man-made obstacles, and is sparsely covered with 15-cm high clumps of buffalo grass for a minimum of 3 km in any direction from the tower.

The meteorological tower was instrumented to measure the vector wind, azimuth and elevation, temperature, as well as humidity and refractivity at three fixed levels: 39, 93 and 149 m. In addition to the fixed levels, a carriage capable of moving along the outside of the tower was instrumented to measure the same physical parameters as the fixed tower levels. All sensors were mounted on booms which were secured on platforms protruding 2 m out from the tower in the direction of the prevailing wind.

The acoustic sounder, an array of forty-nine, 20-cm speakers, was located 240 m away from the meteorological tower along a line to the northwest. It operated at a frequency of 950 Hz, transmitting 20-msec pulses at 2-sec intervals with a power output of 8 W (acoustic).

3. Analysis of records

The theoretical treatment of the stability of an infinitesimal wave disturbance of the form $e^{i(\alpha x+\beta z)}$ introduced at an internal surface of density as well as velocity discontinuity was first done by Helmholtz (1868). The results of this formulation, however, are strictly applicable to a vortex sheet. Recently, Scorer (1969) was able to show that for the case when the shear layer thickness is small compared to the wavelength, the Helmholtz result still holds. Thus, from either formulation, one finds that the waves will grow if

$$k\Delta z \geqslant 2\,\mathrm{Ri}, \tag{2}$$

where k is the wavenumber and Δz the layer thickness.

One of the most fundamental studies performed on the stability of a shear layer characterized by a velocity distribution $U\approx\tanh(z/d)$ and density distribution $\exp(-\beta z)$ must be attributed to Drazin (1958). In this formulation, z is the vertical coordinate and d is approximately half the shear layer thickness. The introduction of a perturbation streamfunction of the form

$$\psi' = f(z)e^{i\alpha(x-ct)},$$

where α is the wavenumber and $c=c_r+ic_i$ is the complex wave velocity, into the equations governing the flow of the fluid, allowed Drazin to solve for the critical Richardson number. The analysis showed that $\mathrm{Ri}_c=0.25$, and since the perturbations will neither amplify nor decay when $c_i=0$, Drazin was able to solve for the neutral stability equation on the αd, Ri plane, i.e.,

$$\alpha d=[0.50\pm(0.25-\mathrm{Ri})^{\frac{1}{2}}]^{\frac{1}{2}}. \tag{3}$$

This particular stability criterion indicates that for $\mathrm{Ri}>0.25$ all disturbances of dimensionless wavenumber αd will be stable, while for $\mathrm{Ri}<0.25$ only those disturbances will be unstable whose dimensionless wavenumbers which lie inside the boundary specified by (3). As a result, the instability regime of Drazin's model may be written as

$$[0.50-(0.25-\mathrm{Ri})^{\frac{1}{2}}]^{\frac{1}{2}}<\alpha d<[0.50+(0.25-\mathrm{Ri})^{\frac{1}{2}}]^{\frac{1}{2}}. \tag{4}$$

Thus, the wavelength of most rapid growth is found to be (since $d\approx\frac{1}{2}\Delta z$)

$$\lambda_D\approx 4.4\Delta z. \tag{5}$$

For a shear layer of finite thickness Δz, and within which both the velocity and density vary linearly and are constant outside, Miles and Howard (1964) have shown that the fastest growing wavelength is given by

$$\lambda_{MH}=7.5\Delta z. \tag{6}$$

With this brief theoretical discussion, we turn our attention now to the acoustic sounder records. From the many acoustic sounder returns available we utilize the few time periods for which there are carriage traverses through the layered structures. Furthermore, care was taken to select only those few cases for which the wind direction at all levels was normal to or along the tower-acoustic sounder direction. Consequently, in the calculation of the Richardson number profiles only the gradient of the horizontal wind was used. The carriage records were also checked for consistency against the fixed level measurements and our attention in this study shall be confined to the carriage data. The time periods chosen for analysis are: 0513–0533, 0717–0734, 7 October 1969; 0104–0134, 8 October 1969; and 0812–0832, 9 October 1969.

For each carriage traverse the following parameters were obtained: (i) the mean wind profile, (ii) the mean temperature profile, and (iii) the layer Richardson number. The carriage profiles were subjected to a low-pass digital filter (Ormsby, 1961) to separate the mean profile from the fluctuations having dimensions <3 m. The resulting low-pass profiles were used to determine $\bar{u}$, $\bar{T}$, and the Richardson number profiles.

The carriage measurements have a vertical resolution of 1 m. However, under existing operating conditions, the acoustic sounder had a height range resolution of the order of 3 m. As a result, the Richardson number profiles shown for each case have been obtained over a height interval of 3 m using (1).

The spurious appearing echo at 95 m on Figs. 1 and 2, and those at 45, 65 and 95 m on Fig. 3 represent reflections from constant sources (meteorological tower, instrumentation vans, etc.) which the antenna side lobe pattern picks up as the acoustic refractive index field changes. Such echoes are prevalent during the nighttime hours and do not represent any physical structure of the atmosphere.

Fig. 1 (Case I) presents the acoustic sounder record for the period 0513–0533, 7 October 1969, as well as the mean horizontal wind and temperature profiles measured at the tower. Below the layer as indicated by the strong acoustic returns, the wind speed is constant and approximately zero. Through the layer in the region of instability waves as indicated by the acoustic sounder record, the wind speed increases with height to a maximum of approximately 3 m sec^{-1}. This is the layer confined between 35 and 65 m at about 0520. Note, however, that both below and above this layer, there appears to be considerable echo return from smaller regions as evidenced by the specular structure. Also note the existence of a secondary layer of return at about 80 m which follows the main layer quite faithfully. These latter returns represent turbulent regions on a smaller scale than the main layer.

Immediately above the layer the wind speed decreases, then attains a constant value of $\sim$2.75 m sec^{-1} at about 120 m. A temperature inversion is indicated at the base of the layer, an increase of 1C through the layer (0.03C m^{-1}), i.e., a second inversion is indicated at approximately the top of the layer. The temperature then becomes constant above 120 m.

Fig. 2 (Case II) presents the same information as Case I but for the period 0717–0734, 7 October 1969. A wind speed of approximately 0.4 m sec^{-1} prevails below the layer. Through the layer which is confined between 48 and 75 m the wind gradient is $\sim$0.08 sec^{-1}, then the wind attains a constant value with height. The temperature shows an increase with height below 30 m, then remains approximately constant between 30 and 57 m. Through the layer the temperature shows an increase of 1.65C, then undergoes a slight increase of $\sim$0.5C to about 115 m.

The acoustic sounder record and the prevailing wind and temperature profiles for the time period 0104–0134, 8 October 1969 (Case III) are shown in Fig. 3. In this case sharp gradients in both wind and temperature are evident in the tower profiles. The thickness of the layer

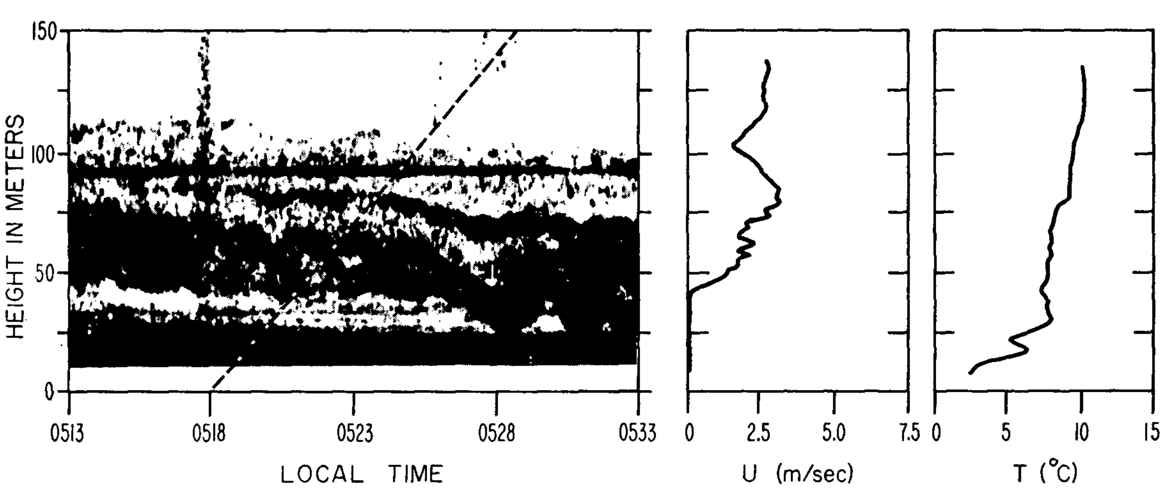

FIG. 1. Acoustic sounder record for 0513-0533, 7 October 1969, and the wind and temperature profiles as measured at the meteorological tower. The dashed line on the acoustic sounder record indicates the carriage traverse.

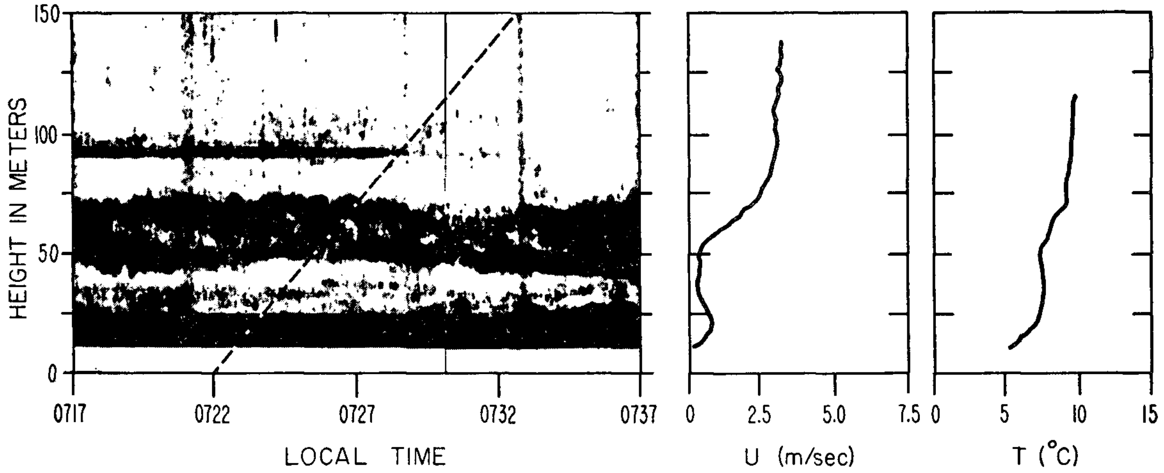

Fig. 2. Same as Fig. 1 except for 0717-0734, 7 October 1969.

at the height where the carriage passes through is only about 10 m. The thickness, however, is not constant with time; the layer thickness appears to more than double shortly after 0111 and indications are present of the existence of two layers from about 0116 onward. During and immediately prior to the carriage traverse through the layer itself, small-scale instabilities of ~30 m in wavelength are shown. In this case, therefore, we have small-scale wave instabilities, as shown prior to 0109, embedded in a large-scale wave motion (wavelength ~350 m) which, since the wave dissipates, appears to be stable. However, we do not exclude the possibility of the existence of very small Helmholtz instabilities after 0109. Indeed, such waves could very well be present, but they are of such small wavelength as to be smeared out by the integrated nature of the sounder system.

Fig. 4 (Case IV) presents the pertinent information for the time period 0812–0832, 9 October 1969. The marked difference between this record and those shown in the previous figures is evident. Although this "jagged" structure might initially be interpreted as an extremely violent event, it merely represents an undulating layer. Superposed on this layer small-scale instabilities may be seen on the upward side of some of the undulations. These instabilities have an average wavelength of 30 m. The undulating layer itself as depicted by the acoustic sounder echoes has an average wavelength of 120 m.

In addition to the Richardson number profiles which were obtained for a vertical separation of 3 m, we also calculated the "layer" Richardson number over a vertical spacing equivalent to the depth occupied by the acoustic sounder echo returns for each case. This

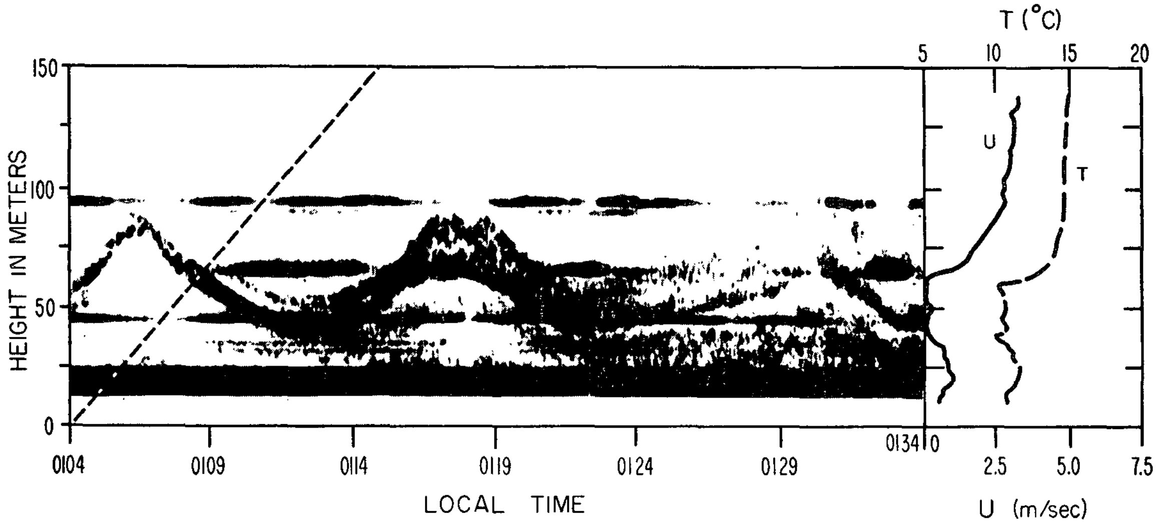

Fig. 3. Same as Fig. 1 except for 0104-0134, 8 October 1969.

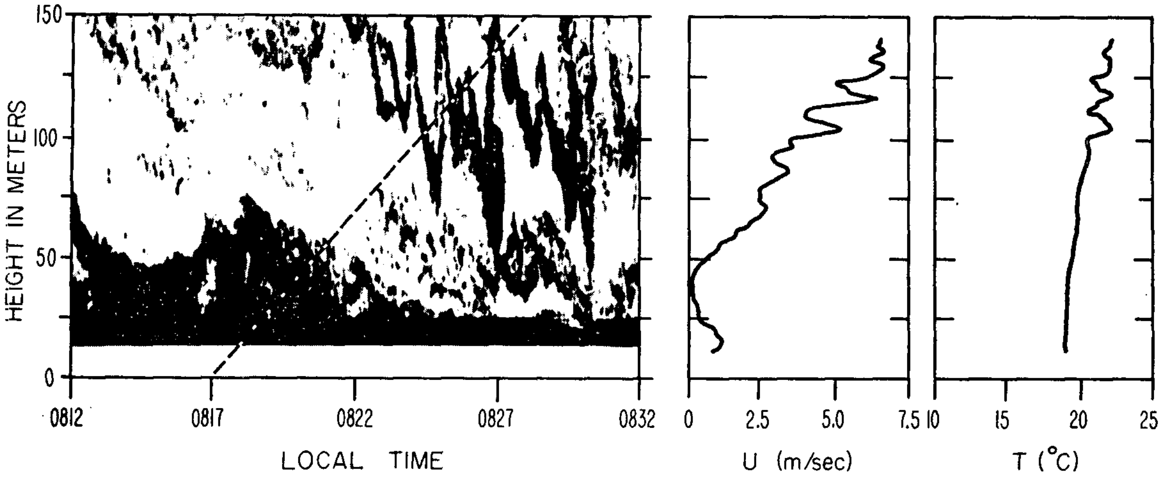

FIG. 4. Same as Fig. 1 except for 0812-0832, 9 October 1969.

latter Richardson number characteristic of the entire echo return layer, is denoted as Ri_L and is given in Table 1.

In addition, the wavelengths predicted by (2), (4) and (5) are also shown in Table 1. Since the phase velocity of the Helmholtz instabilities is equal to the mean wind speed of the sheared layer [that this is indeed the case has been demonstrated by some recent results using pressure transducers on the tower (Hooke, 1972)[1]], we were able to calculate the wavelengths of the "breaking" waves from the discernible "rolls" as indicated by the acoustic sounder returns. These are also given in Table 1 as $\bar{\lambda}$(m): acoustic sounder. Table 1 summarizes the pertinent results of the present study.

The unstable regime, as given by Drazin's analysis, was also calculated for each case. The Richardson number values were obtained from Fig. 5 and were used in the evaluation of (3). In addition, we have calculated αd from the acoustic sounder returns, as described above, and tabulated the results in Table 1. In each case, the wave instability phenomena that we have described fall well within the instability regime of the Drazin model.

4. Discussion of results

The results presented for all cases represent the "breaking" phenomenon of the Helmholtz instability. The wavelengths $\bar{\lambda}$ given in Table 1 as determined from the acoustic sounder records do fall within the instability regime as predicted by the Drazin model.

The Richardson number of the layer for Cases I and II is of the order of $\frac{1}{2}$. The layer thickness was determined from the acoustic sounder records, although this determination could well have been made from the carriage profiles as well.

The acoustic sounder records presented for Cases III and IV show instability wave structures upon which smaller scale instabilities may be seen. For these smaller scale instabilities the layer Richardson number is found to be less than $\frac{1}{4}$. Strata of $\mathrm{Ri}\leqslant 0.25$ are interpreted as regions capable of generating Helmholtz waves; these grow in time until finally they "roll" and "break" as clearly shown in the first two cases.

In addition to the layer Richardson number calculated for the events presented, the Richardson number profile was also obtained for each carriage traverse.

TABLE 1. Unstable regime according to Drazin's analysis and that obtained from acoustic sounder records.

Case	Δz (m)	Ri_L	$\bar{\lambda}$(m) acoustic sounder	λ_H Eq. (2)	λ_D Eq. (5)	λ_{MH} Eq. (6)	Drazin criterion (5)	αd acoustic sounder	$\dfrac{\Delta z}{\bar{\lambda}(\mathrm{m})}$
I	30	0.50	155	377	133	225	$0.53<\alpha d<0.85$	0.61	0.19
II	27	0.51	100	339	120	202	$0.23<\alpha d<0.98$	0.87	0.27
III	5	0.17	30	63	22	38	$0.47<\alpha d<0.88$	0.53	0.17
IV	6	0.10	30	75	27	45	$0.34<\alpha d<0.94$	0.63	0.20

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[1] Private communication.

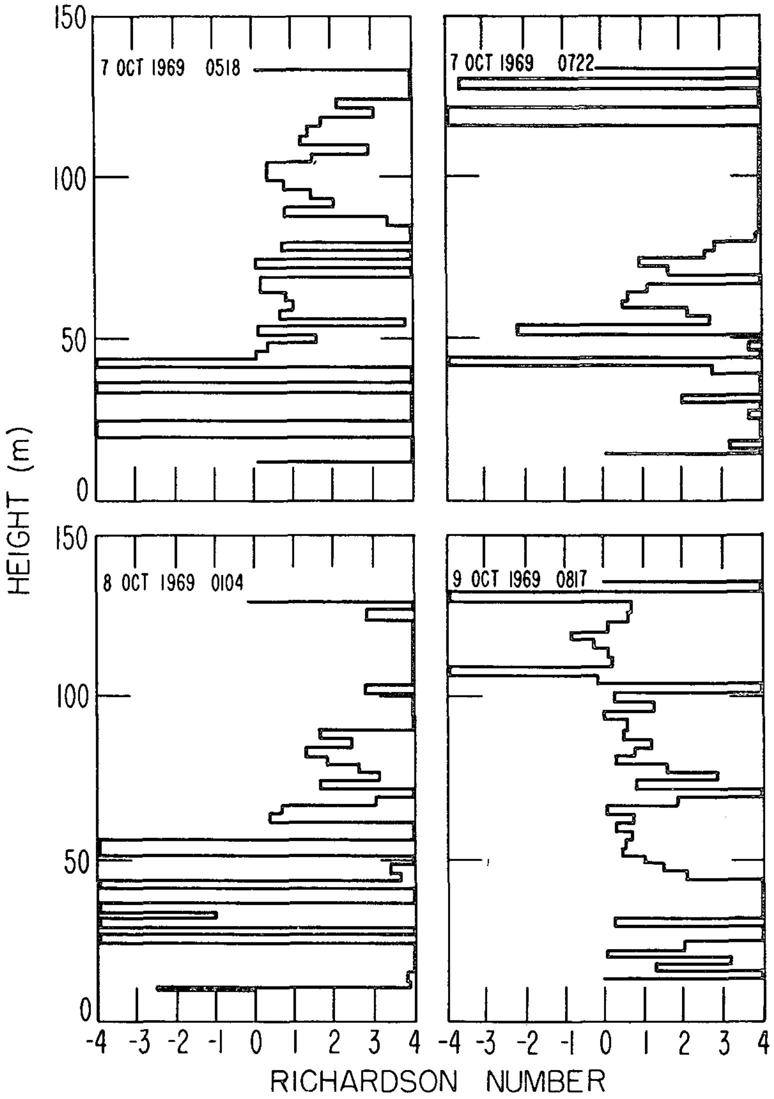

Fig. 5. Richardson number profiles.

The vertical separation was taken to be 3 m, this being the height range resolution of the acoustic sounder. Profiles so obtained are shown in Fig. 5. It is quite evident from these profiles that embedded within the layer thickness defined by the "breaking" waves, regions having a Richardson number of ~0.25 may be found. These regions represent sub-strata where the Helmholtz instability is generated on a small scale. In time, the instabilities undergo a growth resulting ultimately in the complete "rolling" that we see on the acoustic sounder records. The maximum depth of the layer over which this "rolling" takes place is determined by the local Richardson number of the layer. As we have seen, this occurs for a layer Richardson number of 0.50.

In addition, we have also calculated the ratio between the crest-to-trough amplitude and wavelength for each case (Table 1), which, according to a study done by Rosenhead (1931) on billows forming on a surface of discontinuity, should be less than 0.50. Values so obtained in this study vary from 0.17 to 0.27 with an average value of 0.24. These values agree well with those obtained by Browning (1971).

5. Concluding remarks

The independent and concurrent meteorological measurements taken at the tower support well the acoustic sounder records, thus lending credit to the use of acoustic sounder techniques in low-level atmospheric studies.

The acoustic sounder records obtained in this experimental study point to the fact that the sounder can not only map low-level regions of thermal turbulence, internal and breaking waves, but can also delineate such time periods most suitable for analysis of particular atmospheric phenomena. In addition, one notes from the accompanying figures that the acoustic sounder returns are largely confined to the lowest portion of the transition region between the two air masses as indicated by the temperature inversion.

The Richardson number calculated from the tower wind and temperature profiles for the layer thickness as indicated by the acoustic sounder records is found to be approximately 0.5. Consequently, we interpret this to be the critical layer Richardson number attained when all Helmholtz waves "roll" and "break." Within this layer thickness, however, we observe substrata having $\mathrm{Ri} \leqslant \frac{1}{4}$ which we interpret as regions of generation of such instability waves.

Finally, from the results presented in Table 1 it appears that the Drazin model fits best to the description of the dynamic instabilities observed.

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