NOTES AND CORRESPONDENCE

A Note on the Effects of Pressure Gradients on Fluid Flow with Atmospheric Applications

NEIL B. WARD
National Severe Storms Laboratory, NOAA, Norman, Okla. 73069

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It has long been recognized that a decrease in fluid pressure downstream favors flow stability and maintenance of a laminar boundary layer. Contraction of the flow area in wind tunnels, which requires an accelerating pressure gradient, is a means used to damp air turbulence. An increase in pressure downstream, on the other hand, favors an unstable flow and a deeper, more turbulent boundary layer. In the absence of a pressure gradient, fluid flowing parallel to a flat plate develops a boundary layer, which, at some distance from the leading edge, becomes turbulent and increases in depth. As distance from the leading edge increases, the depth of the turbulent layer continues to increase. However, if a significant and favorable gradient of pressure is imposed in the direction of flow, the boundary layer decreases in depth and the turbulence is damped. If this is followed by the imposition of an adverse pressure gradient of sufficient magnitude, transition to turbulence occurs immediately. Most of the above concepts have been described by Schlichting (1960) and Scorer (1958).

Much of the work related to the principle involved here has been done in connection with boundary layers, but it applies to *any* fluid flow. The following simple illustration is offered to promote a better understanding of this important factor in fluid dynamics.

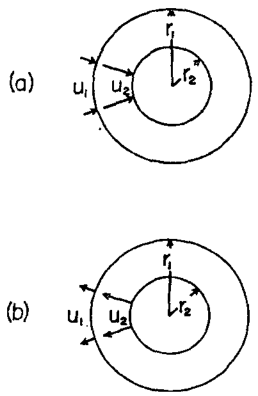

FIG. 1. Accelerating and decelerating flow.

In Fig. 1, consider a basic two-dimensional ideal fluid flow in the annular region between r_1 and r_2 where the radial speed u is the only component. In 1a there is sink flow indicated, where the velocity increases as the radius and pressure decrease. If ρ is density and p is fluid pressure, the equation of motion for this flow is

$$\rho du = \frac{\partial p}{\partial r} dt,$$

where the right side of the equation is the impulse required for the momentum change on the left. For uniform steady flow, a parcel at r_1 with a velocity u_1 will arrive at r_2 after a time interval dt with velocity u_2. During this traverse, the parcel has accelerated under the influence of the pressure gradient force $\partial p/\partial r$.

For sink flow, assume that a single parcel of fluid at r_1 has a non-uniform radial velocity, $u_1' < u_1$; also that ρ is a constant and $\partial p/\partial r$ affects the parcel the same as it does the ambient. Then dt', the time interval for the slow moving parcel to travel the distance dr, will be greater than dt for the ambient. (The condition where $dt' < dt$ would require the accelerating force $\partial p/\partial r$ to be greater for the parcel with non-uniform speed.) Since the time interval for the acting pressure gradient force is greater, the radial momentum gained by the slow parcel is greater than for the ambient, that is,

$$du' > du.$$

This indicates that the slow moving parcel accelerates more than the ambient through the distance dr, and thus approaches the ambient speed. If $u_1' > u_1$ initially, then dt' for the parcel will be less than dt for the ambient and

$$du' < du,$$

so that the fast parcel will accelerate less than the ambient and approach the uniform speed.

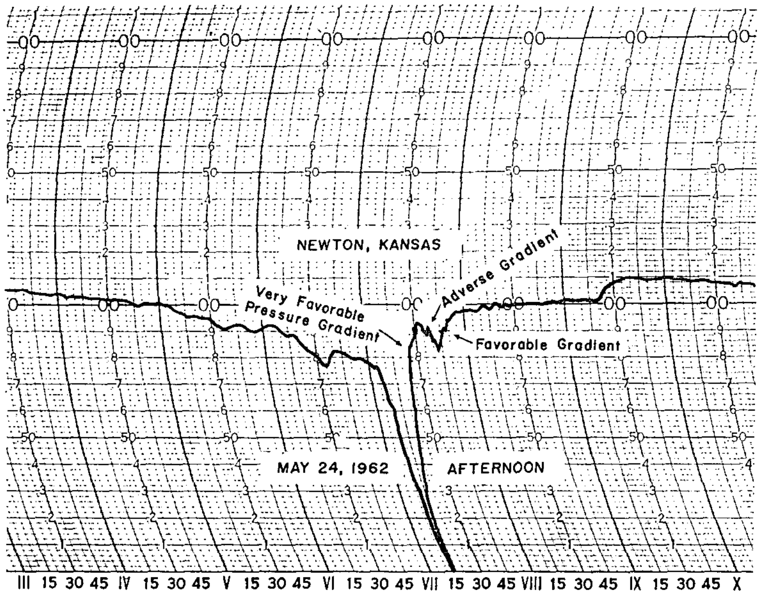

FIG. 2. Surface pressure trace through a rotating storm. The abscissa is time given in hours (Roman numerals) and minutes, and the ordinate is pressure given in inches of mercury. (The 00 constant pressure line in the center of the figure represents 29.00.)

Accelerating conditions then provide dynamic stability of flow and a favorable pressure gradient tends to damp out non-uniformities and turbulence, favoring laminar flow. This type flow is characteristic of fluid motion observed in a converging nozzle or channel.

Consider now Fig. 1b where source flow is indicated, with velocity decreasing as the radius and the pressure increase. Assume a single parcel at r_2 with non-uniform radial velocity $u_2'<u_2$. The time interval dt' for the slow moving parcel will be greater than dt for the ambient. Therefore, again

$$du'>du,$$

but here the acceleration is opposite the flow direction (deceleration) so that the speed deficit for the slow moving parcel will be *greater* when it reaches r_1. Also, if a single parcel initially at r_2 has a radial velocity $u_2'>u_2$, then the time interval dt' for the fast moving parcel will be less than for the ambient, and

$$du'<du.$$

Thus, the fast moving parcel will decelerate *less* than the ambient and therefore arrive at r_1 with a *greater* excess of speed than it had at r_2.

Deceleration then provides conditions for dynamic instability, where a small disturbance in a flow pattern tends to intensify with time. An adverse pressure gradient favors instability and fluid turbulence, a condition characteristic of the observed fluid motion in a diverging channel or nozzle (Prandtl and Tietjens, 1957).

In summary, these arguments, of course, are not rigorous; they are meant to be qualitatively descriptive. Instability and turbulence in a real fluid, associated with inertial effects and eddy and viscous stresses, may be considered as passive or unchanging effects depending upon velocity differences in the flow field. The principle discussed here states that there is an active force field present in accelerating and decelerating flows which tends to either increase or decrease the inertial effects and shear stresses, thereby changing the tendency toward instability and turbulence. Two elements of fluid physics are involved which, in general, can be described as follows:

1) The activating force associated with the principle is the pressure gradient parallel to the direction of flow. The change in kinetic energy due to a given pressure gradient may be considered the same for all fluid parcels moving through the field, but the magnitude of the change in velocity (du) is always greater for the slower moving parcels. This is due to the change in kinetic energy being proportional to the square of its velocity.

2) With some limitations, it can be assumed that a parcel of fluid is primarily affected by the pressure field of the ambient fluid, due to the continuity of pressure. Any non-uniformity of flow, of course, results in a perturbation of the local pressure field, but over some limited range of disturbances, these can be considered as secondary effects.

In the atmosphere, much of the large-scale flow is along lines of equal pressure in nearly geostrophic balance. However, on the mesoscale, such as in the vicinity of thunderstorms, the air flow can be largely across the isobars, in accelerating or decelerating motion. Therefore, the effects of pressure gradients upon these flows can be very significant. In cumulus (thunderstorm) convection, the magnitudes and organization of updrafts, turbulence and entrainment should be significantly affected by the presence of accelerating and decelerating vertical pressure gradients. One should expect that the updraft in the lower portion of a thunderstorm may be well organized, with smooth, accelerating upward motion accompanied by little turbulence and entrainment [as found by Auer and Sand (1966) below cloud base]. The upper portion of updrafts, where the vertical motion is being decelerated, should be accompanied by strong turbulence and much entrainment. In this respect atmospheric convection is much different from that observed in a pure jet in a fluid, where there is only decelerating motion in the primary fluid.

Boundary layer characteristics of many local winds are significantly affected by accelerating or decelerating cross-isobar flow. Examples are the inflow and low-level outflow associated with thunderstorms, sea breezes and mountain-valley winds. The rate of vertical diffusion of atmospheric pollutants is an important topic in those considerations.

One of the most important features of tornadic activity is the flow characteristics of the surface boundary layer. The strong radial pressure gradient associated with the highly rotational vortex is imposed upon a surface-based layer which is less rotational due to frictional effects. Therefore, there is a strong radial inflow component into an area of rapidly decreasing pressure. The surface boundary layer is then greatly modified in the immediate vicinity of the vortex core, becoming very thin and much less turbulent. At a greater distance from the core, there is frequently observed an adverse pressure gradient (Fujita, 1960) probably associated with loss of inflow momentum, where the boundary layer is relatively deep. An example of this is suggested by the barograph trace of the Newton, Kansas, tornado, reproduced in Fig. 2. Here the two low pressure minima on either side of the extreme low pressure (assumed to be the vortex axis) are believed to be associated with strong radial inflow at the base of the thunderstorm updraft. The annular ring of relatively high pressure is associated with strong vertical acceleration in the updraft. The inflow speed near the ground may be very light, or even reversed to outflow, due to the outward directed pressure gradient, and could explain the frequent observations of light surface winds reported just prior to the occurrence of a tornado.

The purpose of this note is to emphasize the importance of pressure gradients on the characteristics of fluid flow with reference to the atmosphere. It appears that much additional work needs to be done in this area, with emphasis on meteorological effects.

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