Comments on "Local Free Convection, Similarity, and the Budgets of Shear Stress and Heat Flux"

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The authors (Wyngaard, Coté and Izumi, 1971) are to be congratulated for their successful attempt in presenting, for the first time, the complete budgets of shear stress and heat fluxes from carefully taken observations in the surface layer of the atmosphere. From this and other reported studies using Kansas tower measurements has now emerged a more complete picture of the surface layer than ever before. It is comforting to see that the difficult-to-measure turbulent transport terms in the budgets are indeed small as compared to the production (mechanical and thermal), buoyancy and pressure interaction terms, and that the former could be estimated from some simple relations as in their Eq. (46). The authors have also shown that the molecular dissipation terms in these budgets are negligible, but that pressure interaction terms are quite large. Ellison (1957) had assumed quite the opposite in the simplified heat flux equation of his model, a serious error pointed out recently by Arya (1972).

The new data in support of the free convection similarity theory is illuminating. But, from their concluding remarks one gets the wrong impression that all the second and third moments behave according to the free-convection similarity predictions. This is not generally true except for the correlations involving only w and θ, which are less affected by shear. The dynamics of the u and v components, on the other hand, is strongly influenced by the presence of finite shear and these components may not be scaled on the basis of free convection similarity. Even in the hypothetical (so far as the atmosphere is concerned) case of zero mean wind or shear, there must always be some convection-induced circulation and hence shear-generated turbulence (Kraichnan, 1962). Laboratory measurements (Deardorff and Willis, 1967) indicate a basically different structure for the u and w components in the near-wall region, the difference mainly being caused by convective circulation in the chamber.

As the authors have pointed out, the quantities $\overline{u\theta}$, $\overline{uw^2}$, $\overline{uw\theta}$, etc., must vanish in true free-convection conditions. These are finite because of the finite shear. Hence, free convection scaling which essentially neglects the role of shear cannot be expected to be valid for these quantities. The hypotheses used in their Eqs. (23), (40), (42) and (44) are rather arbitrary as are their choices of u_f or $z\partial U/\partial z$ for the velocity scale, and T_f or $z\partial\theta/\partial z$ for the temperature scale in the various quantities. One could also add u_* and T_* to the above list, and arrive at different power laws for $\overline{u\theta}$, $\overline{uw\theta}$, etc., as functions of z/L. The use of scales other than u_f and T_f, however, is tantamount to recognizing that free-convection scaling is not valid for these quantities. Therefore, any hypothesis equating the scaled quantities to numerical constants cannot be valid either, because then one has another length scale (besides z) to worry about. In short, the authors' use of $\partial U/\partial z$ as a scaling parameter is equivalent to bringing back the deleted variable u_* in the similarity considerations, and it is no longer a free-convection type similarity.

Some comments on the measurements of $-\overline{u\theta}/\overline{w\theta}$ (Figs. 4 and 9) are also in order. The effect of 5-min running mean filtering may be small on $\overline{w\theta}$, but this might have removed some part of the contribution to $\overline{u\theta}$ by large eddies, and not just the trends. This may be the reason why the authors' values are too low when compared with the previously reported measurements in the atmosphere (Zubkovski and Tsvang, 1966; Wesely *et al.*, 1970), as well as with those in the laboratory (Webster, 1964: Arya, 1968). The decreasing trend of $-\overline{u\theta}/\overline{w\theta}$ with increasing z/L in stable conditions (Fig. 4) is also opposite to what has been observed in the above studies.

The ratio of the stress Richardson number ($\mathrm{R_s}$) to the usual flux Richardson number ($\mathrm{R_f}$) is an important parameter for determining the critical condition for the maintenance of turbulence in the stably stratified surface layer (Arya, 1972). This may be expressed as

$$\frac{R_s}{R_f}=\frac{r_{u\theta}}{r_{w\theta}}\frac{r_{uw}}{A_w^2}, \tag{1}$$

in which

$$r_{u\theta}=|\overline{u\theta}|/(\overline{u^2\theta^2})^{\frac{1}{2}},$$

$$r_{w\theta}=|\overline{w\theta}|/(\overline{w^2\theta^2})^{\frac{1}{2}},$$

$$r_{uw}=|\overline{uw}|/(\overline{u^2w^2})^{\frac{1}{2}},$$

are various correlation coefficients, and

$$A_w=(\overline{w^2})^{\frac{1}{2}}/(\overline{u^2})^{\frac{1}{2}}$$

is an anisotropy coefficient. In stable conditions, r_{uw}/A_w^2 is close to unity and does not appear to vary much with stability. Therefore, $\mathrm{R_s/R_f}$ is approximately equal to the ratio $r_{u\theta}/r_{w\theta}$. Laboratory measurements indicate this ratio to be increasing from a value of about 2 in slightly stable conditions to as large as 4 in very stable conditions. The results of Wesely *et al.* (1970) suggest somewhat lower values, but significantly higher than those indicated by the author's Fig. 9.

Because of their widely different Reynolds numbers, complete similarity between atmospheric and laboratory flows cannot be expected. But, in a fully developed turbulent flow, the Reynolds number effect is usually small except for its role in determining the range of eddy scales, and certain quantities, e.g., the normalized gradients of mean velocity and temperature, rms turbulent fluctuations, correlation coefficients, etc., can be fairly well simulated (Arya and Plate, 1969).

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