NOTES AND CORRESPONDENCE

A Note on Barotropic Instability and Predictability

D. K. LILLY

National Center for Atmospheric Research,[1] *Boulder, Colo. 80302*

27 September 1972

In a recent paper Lorenz (1972) has concluded that the barotropic instability of Rossby waves is sufficiently strong as to probably account for most of the growth of errors leading to loss of predictability of the large-scale atmosphere. The purpose of this note is to point out a result obtained in a quite different way which tends to confirm Lorenz's conclusion.

In a recent study (Lilly, 1972b) the predictability and stability of statistically steady-state, two-dimensional turbulence was investigated by means of numerical simulation. In the case most relevant to the present subject the turbulence was forced by the "negative viscosity" method, originally suggested by Kraichnan (1970), in which a part of the flow is positively fed back as a forcing function. The damping required to maintain statistical stationarity in these experiments consisted of both Newtonian drag and viscous terms, with the former introduced by analogy to atmospheric surface drag, as had also been proposed for the ocean by Bye (1970). Thus, the equation integrated was a vorticity equation in the form

$$\frac{\partial \omega}{\partial t} = -u\frac{\partial \omega}{\partial x} - v\frac{\partial \omega}{\partial y} + \frac{\omega_F}{\theta} - K\omega + \nu\nabla^2\omega, \qquad (1)$$

where u and v are the x and y components of a nondivergent flow and ω is its vorticity; K is the drag coefficient, with dimensions of $(\text{time})^{-1}$; and ν is the viscosity coefficient. The forcing term is directly proportional to ω_F, the portion of the vorticity field associated with the forcing wavenumber k_F, and inversely proportional to a time constant θ, which would be equal to the exponential growth time of ω_F in the absence of the nonlinear and damping terms. Statistically steady turbulence will generally develop if $\theta(K+\nu k_F^2)\ll 1$.

In spite of the various idealizations and limitations of this form of two-dimensional turbulence, including its Cartesian, nonrotating geometry and the artificial forcing mechanism, it bears a useful resemblance to large-scale atmospheric flow, as shown by Lilly (1972a). Charney (1971) has shown why quasi-geostrophic flow can be expected to have important features in common with two-dimensional turbulence.

The predictability experiments were performed by comparing a flow evolution predicted by numerical integration from a given initial state with the evolution obtained by integration from a slightly altered initial state. Fig. 1 shows the developing scalar energy spectrum of the difference between the two realizations for one such experiment. The forcing was confined to wavenumber 8, and other parameters, and the scaling procedures are discussed in the original paper (Lilly, 1972b). As can be seen, the initial difference spectrum has a $+3$ slope on log-log coordinates, corresponding to white noise in the streamfunction. After an early period of organization, the difference spectrum attains a self-preserving form, grows exponentially, and eventually equals the mean flow spectrum, shown by the heavy curve at the top of Fig. 1.

It may be noted that both the mean and difference spectra show peaks at the forcing scale, wavenumber 8, but that in both spectra most of the energy is associated with other scales, having been dispersed by the modal interaction effects of the nonlinear terms in the vorticity equation. The difference spectrum represents the contributions to loss of predictability which would occur if the initial difference between the two realizations were considered as an initial value error, with one of the realizations taken to be the predictor of the other. Thus, we see that the principal loss of predictability is not associated with the linear forcing term, but with the various wave-wave interactions produced by the nonlinear terms. These interactions are, in effect, the "barotropic instability" which Lorenz considered, while the negative viscosity forcing may be considered as analogous to baroclinic instability.

[1] The National Center for Atmospheric Research is sponsored by the National Science Foundation.

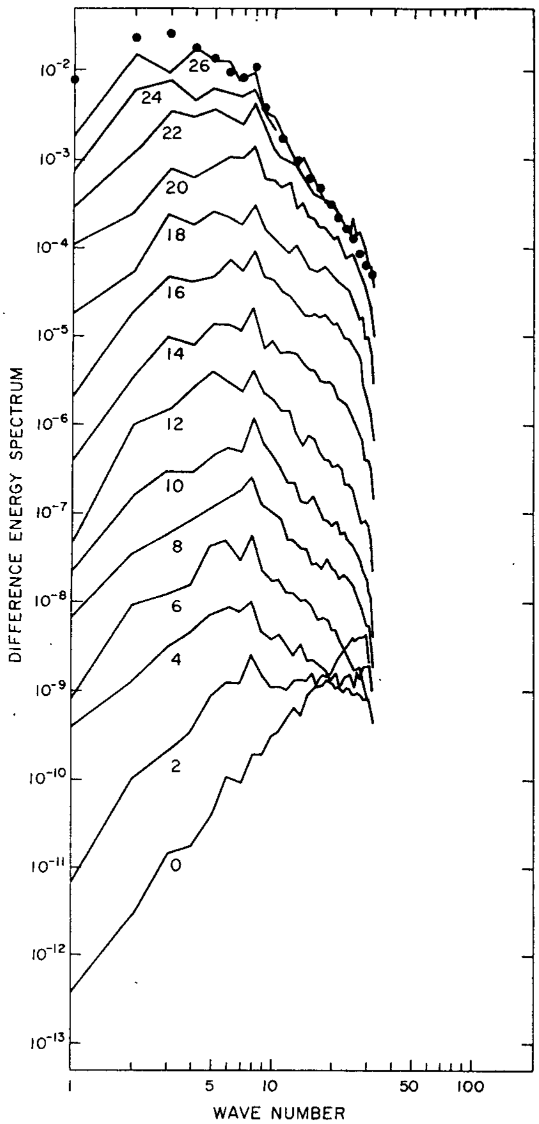

FIG. 1. Time-averaged mean flow (unconnected points) and evolving difference (continuous lines) spectra for a negative viscosity forcing predictability experiment. The difference spectra are labeled by the time from start of the experiment. All units are dimensionless (from Lilly, 1972b).

A further demonstration of the importance of barotropic instability comes from the results of a similar experiment conducted using a forcing function chosen again from wavenumber 8 modes, but with random time changes uncorrelated with the total flow. In the predictability experiment this forcing is kept the same for both realizations, so that there is *no* linear forcing of the *difference* vorticity. Fig. 2 shows the development of the corresponding difference energy spectrum, for a case in which the other parameters and the energy input rate are similar to those for the negative viscosity forcing experiment described above. The difference spectrum is similar to that of Fig. 1 except for the lack of a peak at the forcing scale, and the growth rate is only 30% smaller, indicating the primary importance of the barotropic instability in the loss of predictability.

The ultimate result of the predictability experiment is, for this case, the loss of all predictability except for that associated with the peak at the forcing function, i.e., about half of the energy of wavenumber 8. Since the uncorrelated forcing mechanism can probably be considered as analogous to topographic forcing in the atmosphere, this result suggests the limits within which such forcing can produce predictable response. For the real atmosphere, however, the variation of the Coriolis parameter with latitude produces dispersive β waves on the planetary scale which are less subject to nonlinear interaction than the smaller scale modes and probably more strongly maintain the effects of their forcing.

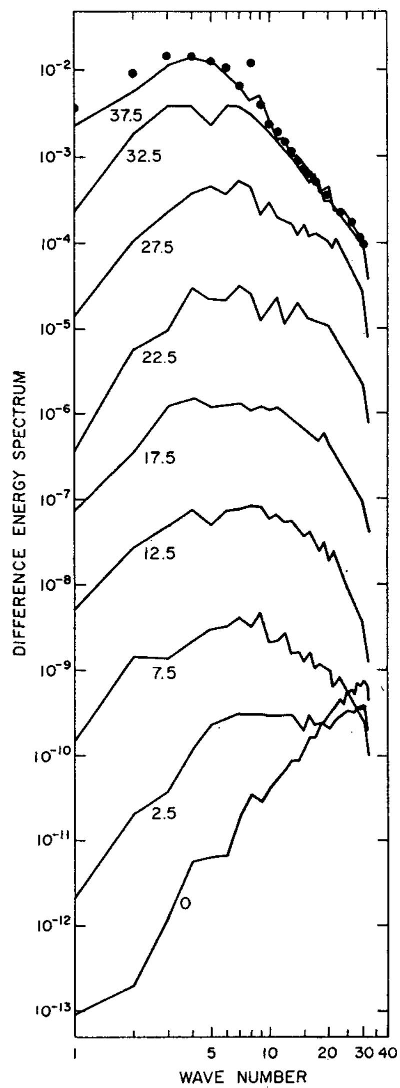

FIG. 2. As in Fig. 1 but for an uncorrelated forcing experiment.

REFERENCES

Bye, John A. T., 1970: Eddy friction in the ocean. *J. Marine Res.*, **28**, 124–134.

Charney, Jule G., 1971: Geostrophic turbulence. *J. Atmos. Sci.*, **28**, 1087–1095.

Kraichnan, R., 1970: Instability in fully developed turbulence. *Phys. Fluids*, **13**, 569–575.

Lilly, Douglas K., 1972a: Numerical simulation studies of two-dimensional turbulence: I. Models of statistically steady turbulence. *Geophys. Fluid Dyn.*, **3**, 289–319.

——, 1972b: Numerical simulation studies of two-dimensional turbulence: II. Stability and predictability studies. *Geophys. Fluid Dyn.*, **4**, 1–28.

Lorenz, Edward N., 1972: Barotropic instability of Rossby wave motion. *J. Atmos. Sci.*, **29**, 258–264.