On the Atmospheric Kinetic Energy Spectrum and Its Estimation at Some Selected Stations[1]

WAN-CHENG CHIU

Dept. of Meteorology, University of Hawaii, Honolulu 96822

(Manuscript received 11 August 1972, in revised form 8 January 1973)

ABSTRACT

The motion of the atmosphere, with its climatological trend and periodic components excluded, is considered to be a stationary random process, under the control of macroscopic factors such as the solar constant, the rate of earth's rotation, the distribution and physical natures of land and sea, etc. In an effort to develop understanding of the statistical features of this process, the Eulerian frequency energy spectra of the large-scale atmospheric motions were constructed at 12 North American and nearby island stations from lengthy wind data. Examination of these spectra reveals that: 1) they are red-noise in character, 2) there is no systematic relationship between the shape of the spectrum and the local rate of earth's rotation, 3) there is a cut-down of spectral energy at low frequencies at 950 mb at some stations, 4) the shape of the spectrum is very much the same at levels in the mid and high troposphere, and 5) a spectral peak at some stations may correspond to the frequency of cyclone occurrence.

1. Introduction

The atmosphere is a very complicated thermal and dynamical system. In constant motion, its temperature, pressure, wind and other fields change incessantly with time. When the minute features of these fields are taken into consideration, it is seemingly capable of taking on endless varieties of appearance. Because of the complexity of its minute details, it would be very difficult, or indeed impossible, for one to explain why the observed features of the atmosphere at any one particular moment appear as they are. What one can say is that the statistical features of the atmosphere are controlled by macroscopic factors such as the solar constant, the earth-sun geometrical relationship, the rate of earth's rotation, the earth's gravity, the distribution and physical characteristics of land and sea, and the composition, configuration and dimensions of the atmosphere, all of which may be taken as unchanging for our purpose.

Somehow, by its nature or whim, the atmosphere system has selected the prevailing statistical structures of its thermal and dynamical fields as its response to these controlling factors. We do not know whether this is the atmosphere's only possible and unique mode of response, or whether it could have selected a quite different statistical structure of its thermal and dynamical fields which would still constitute a perfectly satisfying and consistent response to the same conditions of the controlling factors. [Here by a mode of response, we mean a response with a certain statistical characteristic. Within a response there may be infinite possible realizations (appearances).]

Blackbody radiation is a case in which the response to the controlling factor is unique, for the distribution of the radiation intensity with frequency is uniquely determined by the temperature of the body. If the laboratory study of the fluid motion in a dishpan or in an annulus between two concentric cylinders under the influence of rotation and differential heating is any indication of the dynamical character of the atmospheric motion, then perhaps the response of the atmosphere to its controlling factors is also unique (Fultz *et al.*, 1959; Hide, 1969). However, in view of the fact that there are many differences between the laboratory model and the atmosphere (for example, the model has vertical boundary surfaces, while the atmosphere has none; and there exists a delicate nonlinear feedback mechanism between the atmosphere and its lower boundary, which is not modeled in the laboratory, etc.), we must still leave this question open. (We might add in passing that this is a very important question that must be considered in man's attempt to control weather. Should man someday be able to disturb weather to a considerable degree with nuclear or other devices, he must inquire into the possibility of causing the atmosphere to fall into a different mode of response, and the consequences of such a possibility.)

At its prevailing response mode, the atmosphere is considered to possess certain statistical characteristics "uniquely" representative of this mode. These statis-

---

[1] Contribution No. 72-9, Department of Meteorology, University of Hawaii.

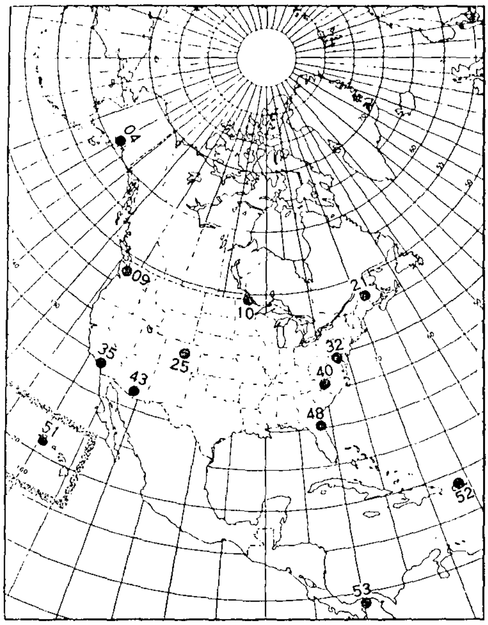

FIG. 1. Station locations.

tical characteristics are more amenable to handling than the individual realizations. A complete statistical (or probabilistic) description of these characteristics would still be very complicated. Fortunately, one aspect of these characteristics that is very informative and yet simple, and has been widely studied, is the kinetic energy spectrum of the atmosphere.

However, not all the existing spectral studies of atmospheric kinetic energy are aimed at the same goal. Some, for example, deal with the spectrum of boundary layer turbulence. Others are concerned with the spectrum of large-scale atmospheric motions as a function of wavenumber (the Eulerian wavenumber spectrum),[2] or as a function of frequency (the Eulerian frequency spectrum).[2] Still others concentrate on the spectrum of atmospheric motion following "marked" particles (such as balloons) as a function of frequency (the Lagrangian frequency spectrum).[2] It would be necessary to state clearly what kind of spectrum is being dealt with.

In this study, we shall take the position that when the slow climatological trend, if any, and the periodic components associated with the earth and sun relationship are excluded, the remainder of the atmospheric motion (i.e., the aperiodic fluctuation part) at any instant may be considered as a realization of a stationary random process (Chiu, 1970). It is true that some have advocated that the atmospheric time series represents a random process with periodic structure rather than a stationary random process (Monin, 1963; Kolesnikova and Monin, 1965; Jones and Brelsford, 1967), and that this point of view is well taken. For example, the (ensemble) autocovariance of some meteorological parameter for a lag of few days may be different in winter from summer. In the present study, the autocovariances of various lags are constructed, through the time averaging method, from data for many consecutive years. By this method, their potential seasonal variation is deliberately smothered. When this kind of covariance is constructed from data for a sufficiently long period (over many years), they should reach a rather stable value, independent of the particular year the data started from. It is in this sense that we have considered the atmospheric fluctuations as a stationary process.

Although the atmospheric process[3] is temporarily stationary, it is not spatial homogeneous. Some of the macroscopic controlling factors mentioned earlier may take on local characteristics. For example, the physical character of the lower boundary varies with location, and at a single location its effect on atmospheric motion is expected to decrease with height. The local rate of the earth's rotation about the vertical is also different at different latitudes. Despite the fact that the Coriolis force is a virtual force that has no effect on the time rate of change of atmospheric kinetic energy, it is one of the factors that influence the shape of the energy spectrum of large-scale atmospheric motions (Chiu, 1970).

Therefore, we visualize that under its prevailing mode of response, the atmosphere possesses a unique Eulerian frequency energy spectrum at every location in the free atmosphere, which, nevertheless, may change from location to location due to local variation of the controlling factor or factors. The purpose of this study is to estimate this spectrum, to detect and to examine its variation with space, and to try to gain some understanding of it.[4]

In the case of blackbody radiation, the spectrum is completely explained by Planck's radiation law. With regard to the energy spectrum of atmospheric motion, we are now not unlike the physicists at the time before Planck's radiation law was introduced. We have some understanding of the atmospheric energy spectrum at the "inertial subrange" through Kolmogoroff's hy-

---

[2] What we refer to here as the wavenumber spectrum and frequency spectrum are often referred to in the literature as the space spectrum and time spectrum, respectively (Kao and Wendell, 1970; Julian, 1971). But the time spectrum may be presented as a frequency spectrum or a period spectrum and these two may have quite different shapes (Chiu, 1967). Similarly, the space spectrum may be presented as a wavenumber spectrum or a wavelength spectrum. To be more definite, I shall use frequency spectrum, period spectrum, wavenumber spectrum, etc., throughout this paper. I appeal to meteorologists to always employ these more precise terms.

[3] For simplicity, the atmospheric process from here on refers to the fluctuating part of atmospheric motion only, excluding its climatic mean and periodic part. If they are included it would be more appropriate to call it quasi-stationary (Chiu, 1970).

[4] Some of the existing studies of the Eulerian frequency spectrum employed once or twice daily data for a period of six months or less (Yanai *et al*, 1968; Wallace and Chang, 1969; Kao and Wendell, 1970; etc.) A spectrum constructed from such a length of data reflects the spectral characteristics of the particular period studied, and thus is subject to variation with season and year. The spectrum to be constructed in this study is, on the other hand, based on many years of data and purported to be an estimate of the stable spectrum of the stationary random process.

Table 1. Station list.

Designator	Station name	Latitude (N)	Longitude (W)	N	M	edf	b (cpd)
04	Anchorage, Alaska	61°10′	149°59′	2922	50	156	0.0267
09	Tatoosh Island, Wash.	48°23′	124°44′	2922	50	156	0.0267
10	International Falls, Minn.	48°34′	93°23′	2922	50	156	0.0267
21	Caribou, Me.	46°52′	68°01′	2922	50	156	0.0267
25	Denver, Colo.	39°46′	104°53′	2922	50	156	0.0267
32	Washington, D. C.	38°51′	77°02′	2922	50	156	0.0267
35	Santa Monica, Calif.	34°03′	118°27′	1064	50	57	0.0267
43	Tucson, Ariz.	32°07′	110°56′	2922	50	156	0.0267
48	Jacksonville, Fla.	30°25′	81°39′	2922	50	156	0.0267
51	Lihue, Hawaii	21°59′	159°21′	1095	50	58	0.0267
52	San Juan, Puerto Rico	18°26′	66°00′	2922	50	156	0.0267
53	Balboa, Canal Zone	8°58′	79°33′	2922	50	156	0.0267

pothesis, but very little understanding of it at low frequencies. It is unlikely, however, even were our understanding of the atmospheric energy spectrum to be greatly improved, that it would be as complete as that of blackbody radiation, due to the multiplicity and complexity of its controlling factors that we must take account of. However, every gain counts. If, in this study, through examination of the variation of the spectrum with space, we can make some connection (or de-connection) between the spectrum and some of its controlling factors, then we would have gained important understanding.

Therefore, we shall first construct the Eulerian frequency spectra from lengthy wind data observed over North American stations and adjacent island stations, discuss the reliability of the constructed spectra as estimates of the true spectra, and after satisfying ourselves as to their worthiness, go on to examine:

1) The shape of the spectrum.
2) Whether there is any systematic variation of the spectrum with latitude (related to the changing local rate of earth rotation).
3) Whether there is any systematic variation of the spectrum with height (related to the changing influence of the lower boundary).

Finally, we shall discuss the scientific meaning of the results and draw some conclusions.

2. Data

The wind data used for this study were obtained from the National Weather Records Center at Asheville, N. C. They consist of a set of once-daily wind observations for eleven North and Central America stations and one Pacific station, covering the period from March 1951 through February 1959 for all the stations except Santa Monica and Lihue, for which the periods are from April 1956 through February 1959, and from March 1956 through February 1959, respectively. These are the same data used in a previous study (Chiu and Crutcher, 1966), in which more information concerning these data may be found. Fig. 1 shows the locations of the stations, designated by numbers. Table 1 gives the designators, names, and the latitudes and longitudes of the stations, the total number N of daily data, the maximum number M of lag used in covariance calculations, the equivalent degrees of freedom for the spectral estimates (edf), and the bandwidth b of the spectral window, in cycles per day (cpd).

3. Construction of spectra

At each station and level, wind observations were resolved to form one time series of west-east wind component u and one time series of south-north wind component v. The annual variation was extracted from each series. This was done by first calculating the monthly mean values for the entire period of each series (e.g., mean January value for all Januarys within the period). From these 12 monthly mean values the first harmonic representing the annual variation was obtained. The daily values of this first harmonic were interpolated and subtracted from the corresponding daily values of the series in each year of the period. The resulting series were used to construct the spectra.

The spectra were constructed by the Blackman-Tukey's method, and smoothed by hanning (Blackman and Tukey, 1958).

4. Reliability of the estimated spectra

Spectral estimates include numerous uncertainties and artificial modifications peculiar to the nature of available data and to the method of spectral construction. It is therefore pertinent to review the reliability (or lack of it) of the estimates before an attempt is made to assess their scientific significances. In what follows we shall discuss the major sources of uncertainties and modifications in our estimates and how much the estimates are affected by them.

a. Missing observations

Such gaps in our data were made up and filled in by scaling, interpolation and extrapolation techniques. The percentage of missing observations is small at lower levels, increases to about 25% at 200 mb, continues to increase slowly to 50 mb, and then rapidly

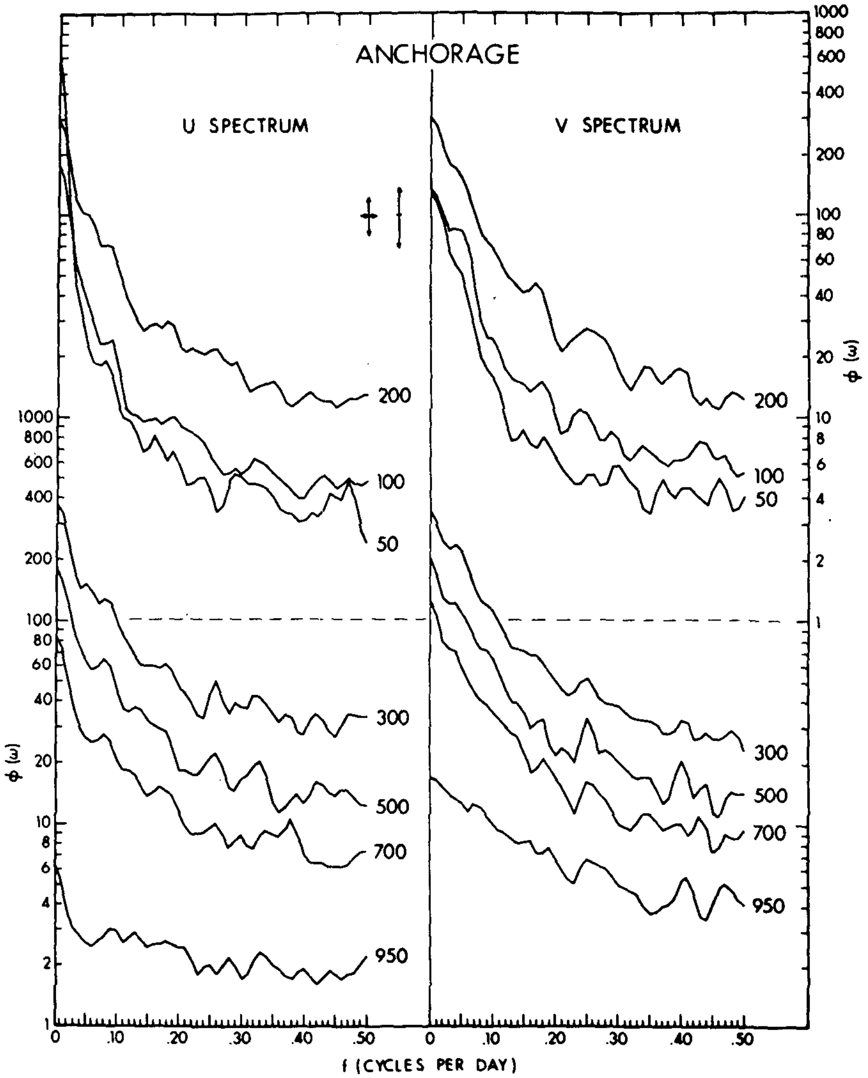

FIG. 2. Kinetic energy spectra of the u and v components at various pressure levels (mb) over Anchorage, Alaska. The abscissa is circular frequency $\omega(=2\pi f)$, in radians per day, with f indicated. The ordinate is the spectral density in $m^2 sec^{-2}$ (radians per day)$^{-1}$; with the left-side scale for the 950-, 700-, 500- and 300-mb levels, and the right side (starting from the horizontal dashed line) for the 200-, 100- and 50-mb levels. The 95% and 99.8% confidence intervals are indicated by the shorter and longer vertical arrows, respectively. The bandwidth is indicated by the horizontal arrows.

to 10 mb. A previous study by Chiu (1960) showed that the effect of 10% filled-in data on spectral estimates is negligible. At higher levels, due to the existence of strong persistence of motion, even a larger percentage of filled-in data is tolerable. In this study we may consider that the effect of missing data on spectra is not important up to about 200 mb. For higher levels, the spectra should be viewed with reservation.

b. Aliasing

This effect, if any, is expected to be insignificant in our spectra. The largest possible source of energy at frequencies larger than 0.5 cpd (our Nyquist frequency) would be that associated with diurnal variation. Our data include only those observations made once daily at a fixed hour and therefore contain very little diurnal variation.

c. Removal of annual cycle

The first harmonic of the mean monthly values was removed from the data, as described previously, in order to eliminate the possible smearing effect of the periodic annual variation on the shape of spectrum. However, with a finite length of data, this subtraction of the periodic annual variation is never perfectly executed, and may induce a depression in the spectrum at and near the annual period. The result is to bias the

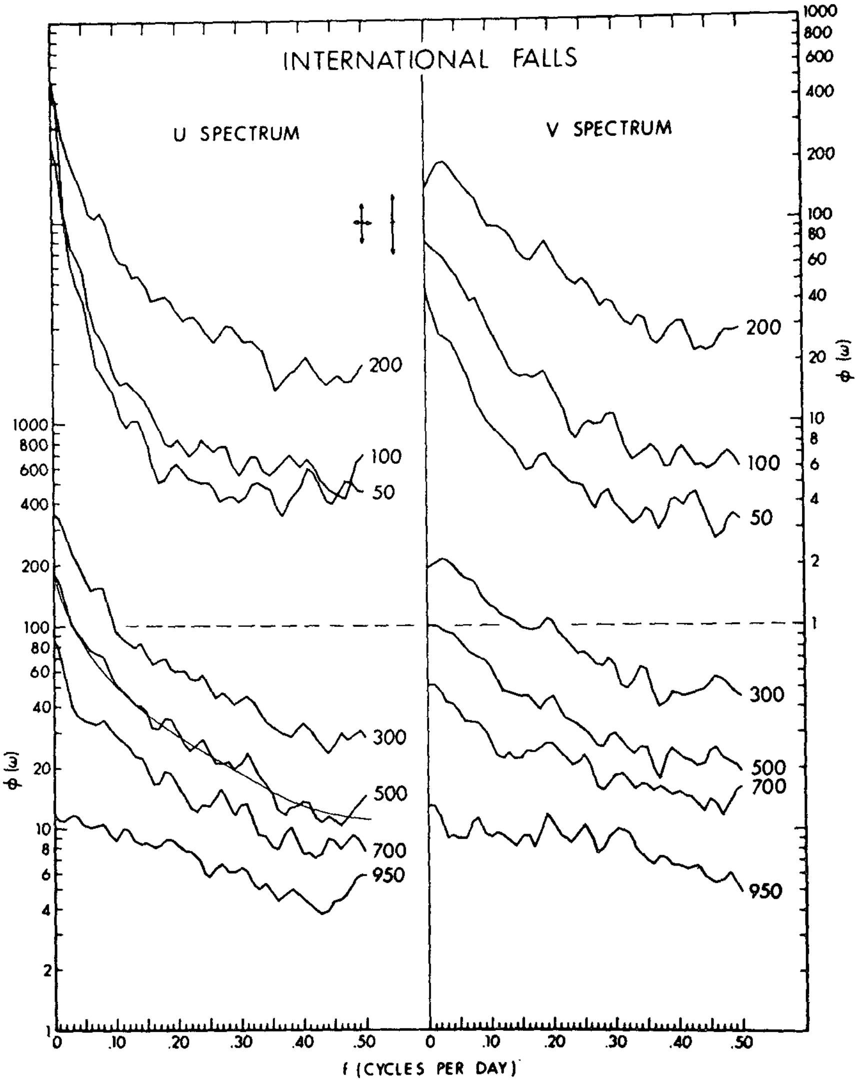

Fig. 3. Same as Fig. 2 except for International Falls, Minn. The smooth curve running through the u spectrum at 500 mb portrays the general trend of decrease of the spectral density with frequency.

estimated spectrum at the frequency of 1 cycle per year toward zero density by a factor (edf−2)/edf [Jones (1971)]. For most of the spectral estimates of this study the edf is 156, which makes (edf−2)/edf practically equal to 1. Therefore, the distortion of our spectra due to this source is, for all practical purpose, entirely inconsequential.

On the other hand, in order to get an idea how much the spectrum of the fluctuating motion would have been distorted by the presence of the first harmonic, we also constructed the u and v spectra at the 300- and 200-mb levels over Jacksonville, Fla., without removing the first harmonic. These spectra and those with the first harmonic removed differ only for the first two or three frequencies at the low-frequency end and they practically coincide for other frequencies.

d. Removal of sample mean

In the same vein, the subtraction of the sample means from the data, as employed in the present study for the calculation of covariances in accordance with the Blackman-Tukey method, tends to induce a spectral depression at and near zero frequency. This may occasionally give an erroneous impression of a spectral peak in the neighborhood of zero frequency. The larger the total number N of data employed in the spectral estimate, the closer to zero frequency the artificial peak appears (i.e., the narrower the spectral band next to zero frequency that is distorted by this operation). In the present study, N is very large, and so the artificial spectral peaks of the "unsmoothed" spectra, if created, should be very close to the zero frequency, and are likely eliminated by the hanning operation. This explains why in all but a few of our spectra there are no spectral peaks appearing at near-zero frequencies. Thus, the distortion of our spectra from this source is insignificant and confined to the immediate neighborhood of zero frequency.

e. Smoothing

Construction of the spectra through the Blackman-Tukey method means that our spectra have been smoothed with Tukey's spectral window. The amount of smoothing is indicated by the bandwidth of the window, which we have included in Figs. 2–8. It is seen that the bandwidth is rather small and so the distortion of spectrum due to this smoothing should be small, especially for the spectra that are essentially smooth, like those obtained in this study.

The above discussions indicate that the distortion in our spectra should be small throughout the entire (or nearly entire) range of frequencies under investigation (i.e., the bias of the expected values of our spectral estimates should be small). The degree of sampling fluctuation of our estimates may be indicated by the 95% confidence interval which we have included in each of Figs. 2–8. The way these figures are plotted makes the same confidence interval applicable to the entire spectrum. For most of our spectral estimates, this interval is small. Therefore, it would seem safe to say that our spectra should represent a rather reliable estimate of the stationary kinetic energy spectra of the large-scale atmospheric motions, especially those for levels at and below 200 mb.

5. Discussions of results

Since most of the spectra have similar shape, we shall not present all the spectra that we have constructed,

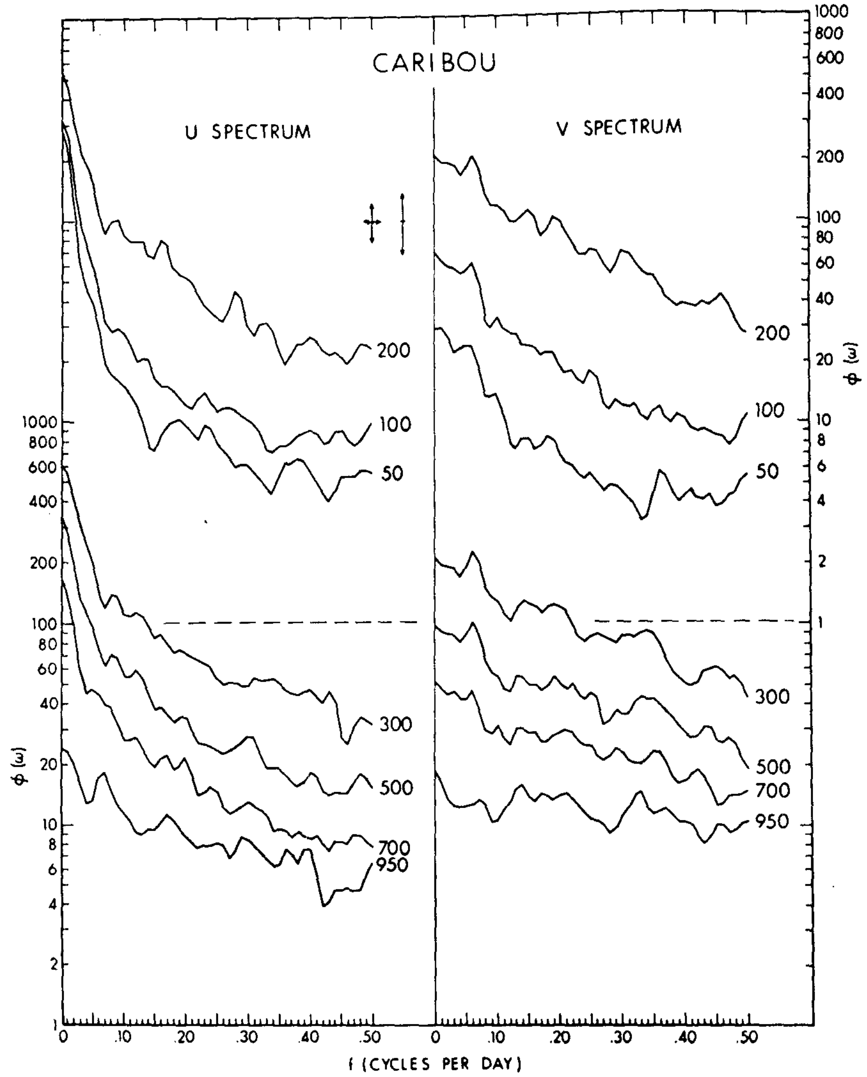

FIG. 4. Same as Fig. 2 except for Caribou, Me.

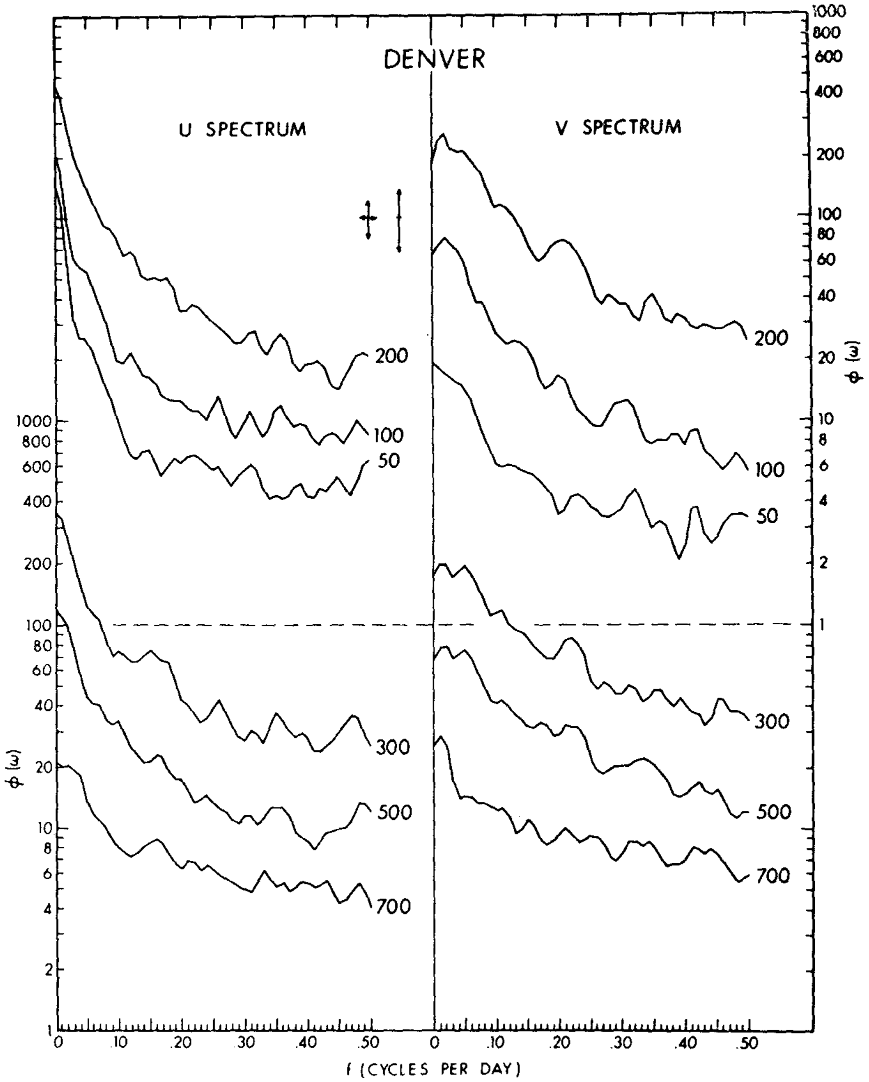

Fig. 5. Same as Fig. 2 except for Denver, Colo.

but only some of them that are deemed necessary for the following discussions. The spectra, $\phi(\omega)$, as a function of circular frequency, $\omega=2\pi f$, are shown in Figs. 2–8. The abscissa is ω, but for simplicity only f (frequency, cpd) is shown. It should be noted that the energy within a certain frequency band Δf is $2\pi\phi(\omega)\Delta f$, not $\phi(\omega)\Delta f$.

a. *The general trend of the spectra*

The spectral density in general decreases with increasing frequency for both the u and v spectra at all stations and levels, i.e., it has the character of a red-noise spectrum. There are only a few cases, all at 950 mb, in which the u and v spectra show little change of spectral density with frequency.

A process that exhibits some persistence possesses a red-noise spectrum (Gilman *et al.*, 1963; Ward and Shapiro, 1961); and the stronger the persistence, the "redder" is the spectrum (i.e., the faster the decrease of the spectral density with frequency). The less redness of some of the spectra at 950 mb means that for these exceptional cases the atmospheric motion at 950 mb is not as persistent as at higher levels. Fig. 9 shows the autocorrelation coefficient, $R(\tau)/R(0)$, of the u component at the lower three levels over four selected stations for various lags τ up to the 8th lag (one lag equals a separation of one day). Curves A and B of this figure are for Anchorage and Santa Monica, respectively, where the u spectrum at 950 mb is not as red as that at upper levels; while C and D are for Lihue and San Juan, respectively, where the u spectrum at 950 mb is as red as that at upper levels. It can be seen at Anchorage and Santa Monica that the correlation coefficient drops off much more rapidly with increasing lag (i.e., it has less persistence) at 950 mb than at higher levels; while at Lihue and San Juan, this is not the case.

This is likely a reflection of the fact that at low levels the proximity of the ground tends to inhibit the growth of larger (space) scale motions; thus, over the long run the motions there may tend to have smaller space scales, shorter life spans, and less persistence than the motions at higher levels. At Anchorage and Santa Monica, the 950-mb level seems to possess such characteristics. At Lihue and San Juan, where the average roughness of the surrounding water surface may be smaller than that of land surface, the boundary's prohibition of atmospheric motions may have already so weakened at 950 mb that the motions at this level have the same characteristics as those at higher levels.

Oort and Taylor (1969), using 10 years of surface wind (at 33 m above the ground) data at Caribou, found in the Eulerian frequency spectrum a strong spectral maximum of horizontal wind speed at frequencies corresponding to periods of 3–6 days, and that there is a marked decrease of spectral density toward lower frequency (see Fig. 4 of their paper). It indicates that the atmospheric motion is limited to still smaller scales at the ground level.

b. Superimposed fluctuations

Superimposed on the general trend of decreasing spectral density with frequency are many peaks and valleys. From sampling theory alone, peaks and valleys of this sort can be expected to appear in any spectrum constructed from a finite amount of data. Some of the peaks (or valleys) in our spectra, however, are rather prominent, and some of them appear repeatedly at the same frequency at many levels of a station. Their significance should be assessed.

When there is no *a priori* reason to expect a peak (or valley) at a certain frequency, the significance of a peak (or valley) in the estimated spectrum that happens to appear at that frequency should be judged

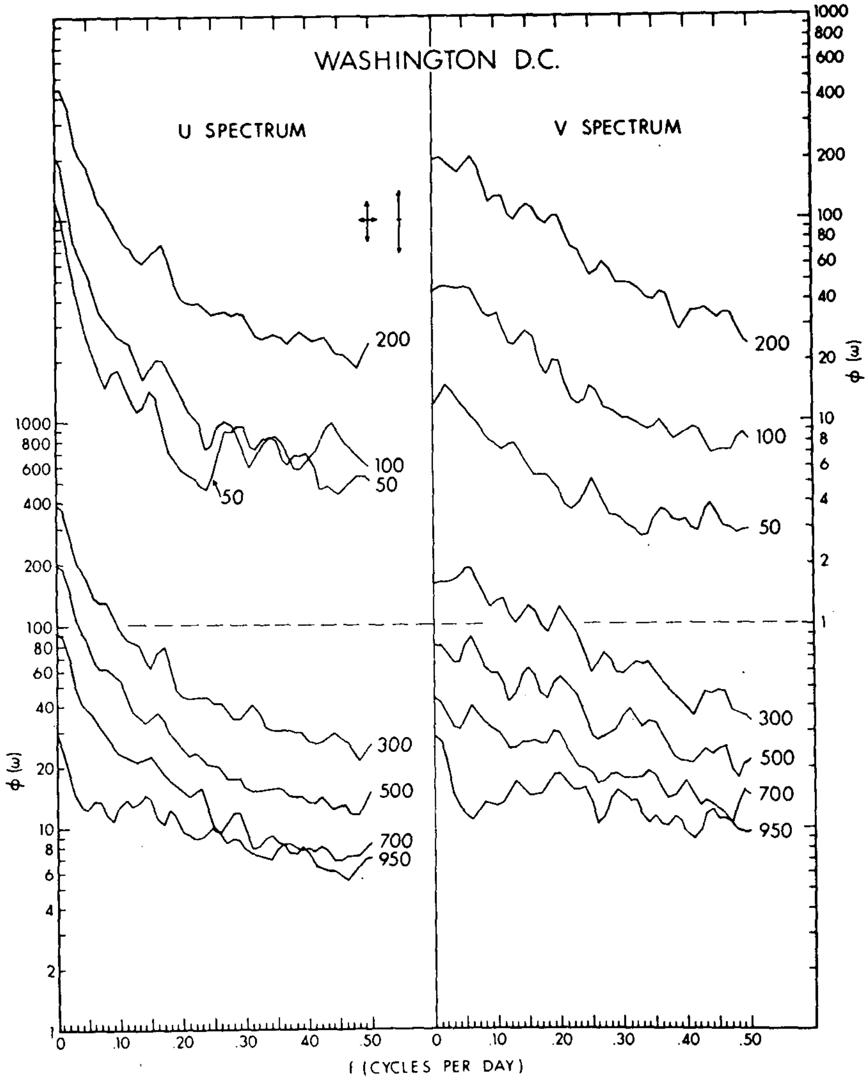

FIG. 6. Same as Fig. 2 except for Washington, D. C.

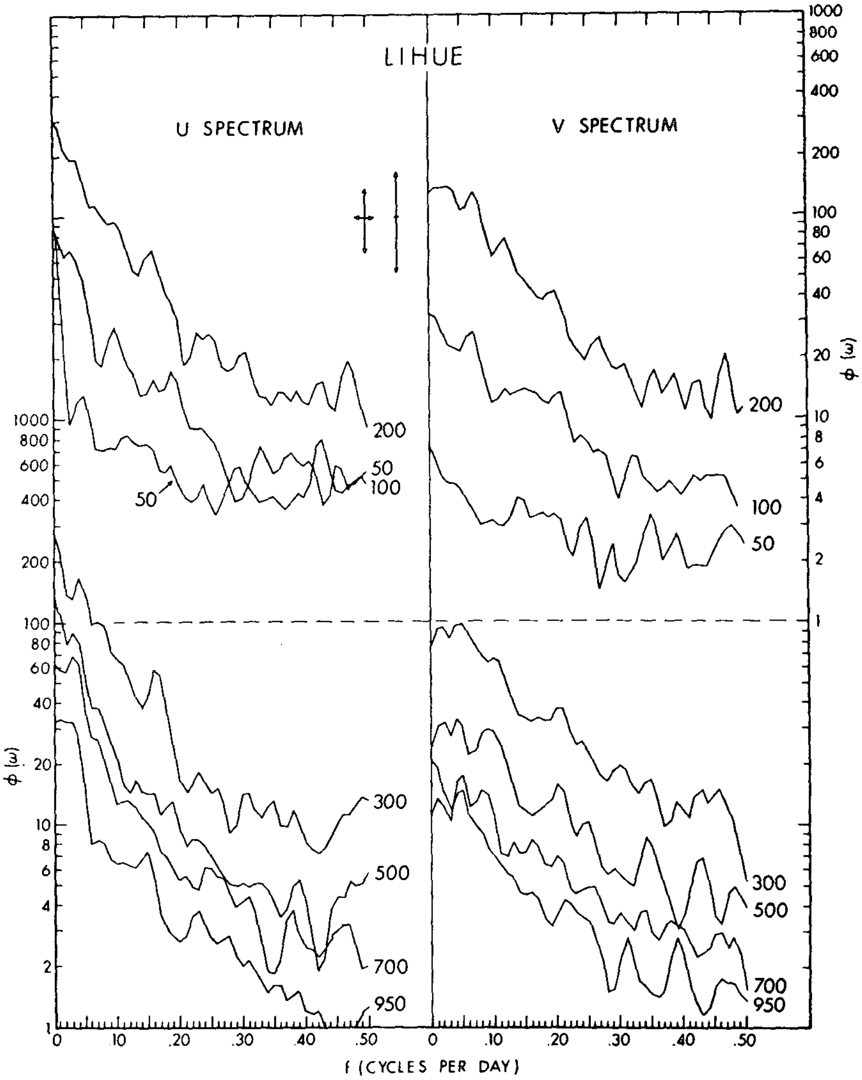

Fig. 7. Same as Fig. 2 except for Lihue, Hawaii.

severely *a posteriori* (Mitchell *et al.*, 1966; Julian, 1971). In this study, we shall employ 95% confidence limits for *a priori* evaluation, and 99.8% confidence limits for *a posteriori* evaluation (equivalent to about 96% *a priori* limits for our spectra). Both of these limits are indicated in Figs. 2–8.

If a smooth curve purporting to represent the general decrease of spectral density with frequency is drawn through each spectrum (one such curve is shown in Fig. 3), the significance of the peaks and valleys against such a background curve will depend on whether *a priori* or *a posteriori* criteria are used to evaluate them.

In general, *a posteriori*, rather than *a priori*, criteria will be used to judge these spectra. Because, after we have taken measures to minimize the seasonal and diurnal variations in the data, we really do not have very strong physical reasons to expect a peak or valley in these (frequency) spectra. However, I do not completely rule out the possibility of the emergence of a minor but physically real spectral maximum at some frequency related to cyclone activities at stations where such activities are known to be strong and frequent, and, whenever appropriate, shall employ *a priori* criteria for its evaluation.

Judging *a posteriori* with the 99.8% confidence limits, we may say that none of the peaks and valleys appears to be statistically significant, and that they could very well be the results of sampling fluctuations from a basically smooth red-noise spectrum.

There are, however, a few cases among our spectra in which a spectral peak repeatedly shows up at the same frequency at different levels, and at nearly the same frequency at several stations. These are: 1) at Anchorage with v spectrum peaks at 0.25 cpd and u spectrum peaks near the same frequency; 2) at International Falls with v spectrum peaks at or near 0.20 cpd; 3) at Caribou with v spectrum peaks at or near 0.06 cpd, and minor v spectrum peaks at or near

0.20 cpd; 4) at Denver with v spectrum peaks at or near 0.21 cpd; and 5) at Washington, D. C., with u spectrum peaks at or near 0.20 cpd, and v spectrum peaks at or near 0.06 cpd.

However, a peak's appearance at more than one level is not alone enough to certify that it has significance. The large-scale motions are known to be correlated in the vertical. Therefore, when the data used for different levels of a station pertain to the same period (as is usually the case in meteorological studies, and, in particular, in this study), it is possible that the same sampling error, in the form of a peak or valley, shows up repeatedly at different levels of the same station. If the peak (or valley) can withstand the test with data of similar length, but pertaining to different periods (or else in a situation to be discussed below), then its physical reality will be much less open to doubt.

What seems to be quite significant is that all the above cases have a consistent peak in the frequency interval from 0.20–0.25 cpd. Since these are the frequencies appropriate to the cyclones, and since these are the stations where cyclones are active, it appears as though these peaks are related to cyclones. This would mean that despite the fact that cyclone's varying speeds and directions tend to dull or depeak its spectral intensity in an Eulerian frequency spectrum (Chiu and Crutcher, 1966; Julian, 1971), at these stations the cyclone activities may be strong and frequent enough, and other meteorological conditions favorable enough, to show a peak in such a spectrum. In lieu of other more plausible explanations, we believe that this is what happened (and presumably will continue to happen) at these stations.

If we consider these peaks to be related to cyclone activities, and use the 95% *a priori* limits to evaluate

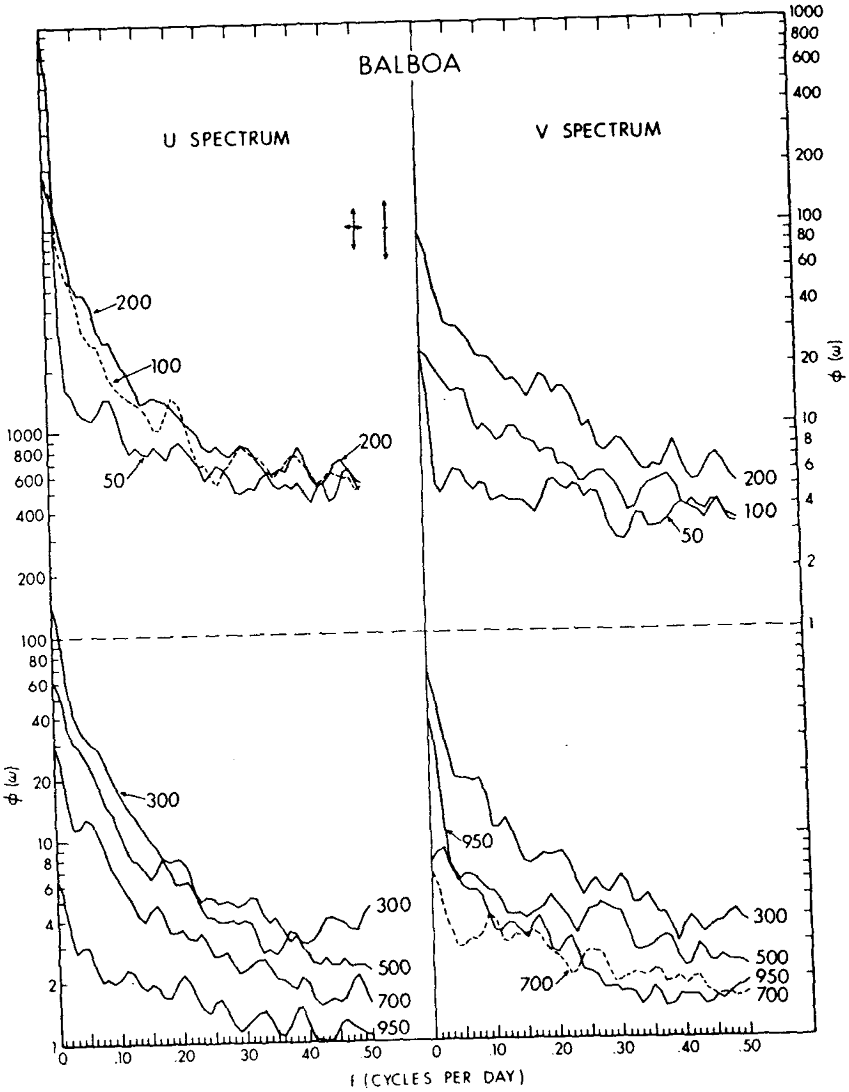

FIG. 8. Same as Fig. 2 except for Balboa, Canal Zone.

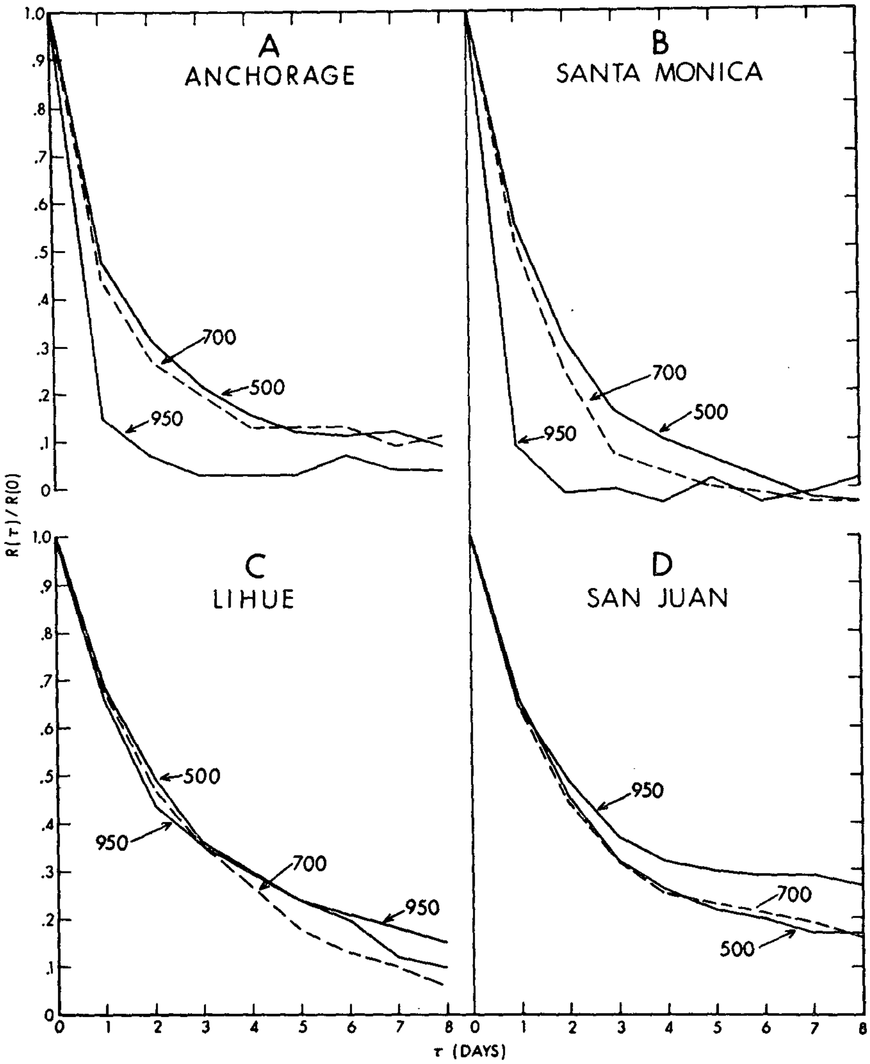

Fig. 9. Autocorrelation coefficients, $R(\tau)/R(0)$, of u component for lags τ (days) from 0 to 8, at the pressure levels and stations indicated.

them, some of them appear statistically quite significant.

In this connection, it might be mentioned that the results of a previous study by this author (Chiu, 1960) were cited by Kolesnikova and Monin (1965) as evidence for the presence of a peak in the u spectrum at the frequency corresponding to the period of 4 days on the 700-mb level over Belmar, N. J. (40° 11'N, 74° 02'W). Actually, that study alone, with much less data and much smaller edf, is not enough to support this claim. From the statistical point of view, the peak at Belmar could very well be due to sampling fluctuations. It would be prudent to consider it so, as was done in that study. In light of evidence for a similar spectral peak at the above-mentioned stations, and the fact that Belmar is located in the same general area as most of those stations, the spectral peak at Belmar may be real.

There are only two stations, Caribou and Washington, D. C., that show prominent and consistent peaks at or near 0.06 cpd (corresponding to a period of about 17 days). These peaks should also be tested against data of different periods. For want of other evidence, we note their appearance, but refrain from assigning physical significance to them at the moment.

c. Height and latitudinal variation of the spectral structure

The general rate of decrease of the spectral density with frequency, i.e., the shape of the would-be smooth spectrum, except for some of the spectra at 950 and 50 mb, is very much the same at all levels of each station. This means that the spectral structure of the atmospheric kinetic energy is very similar throughout most of the troposphere and part of the lower stratosphere, although the spectral density at the same fre-

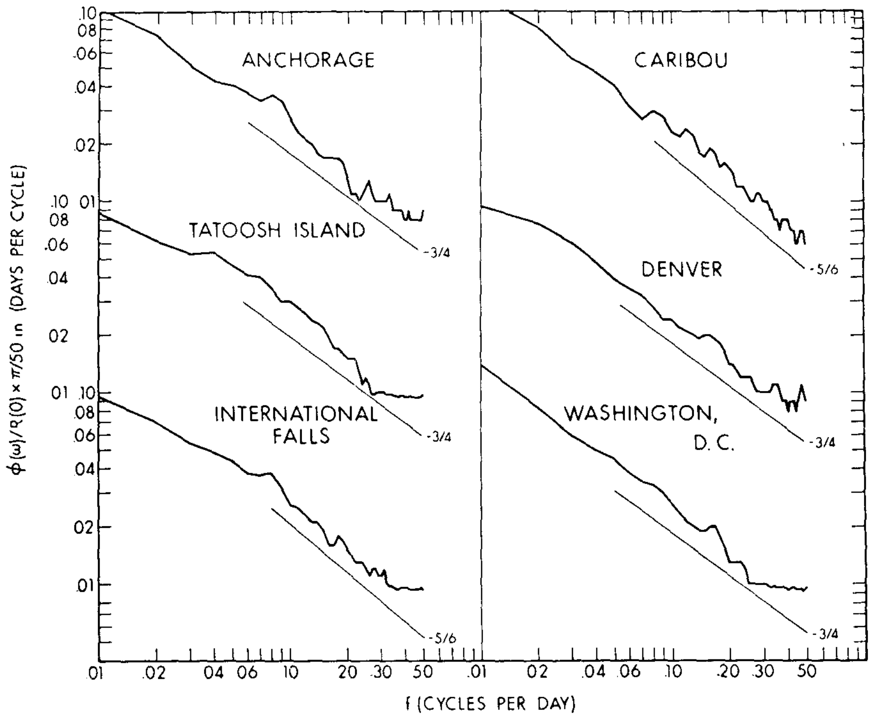

FIG. 10. The average normalized *u* spectra for the stations indicated. Each straight line segment approximates the slope of the high-frequency portion of its nearby spectrum.

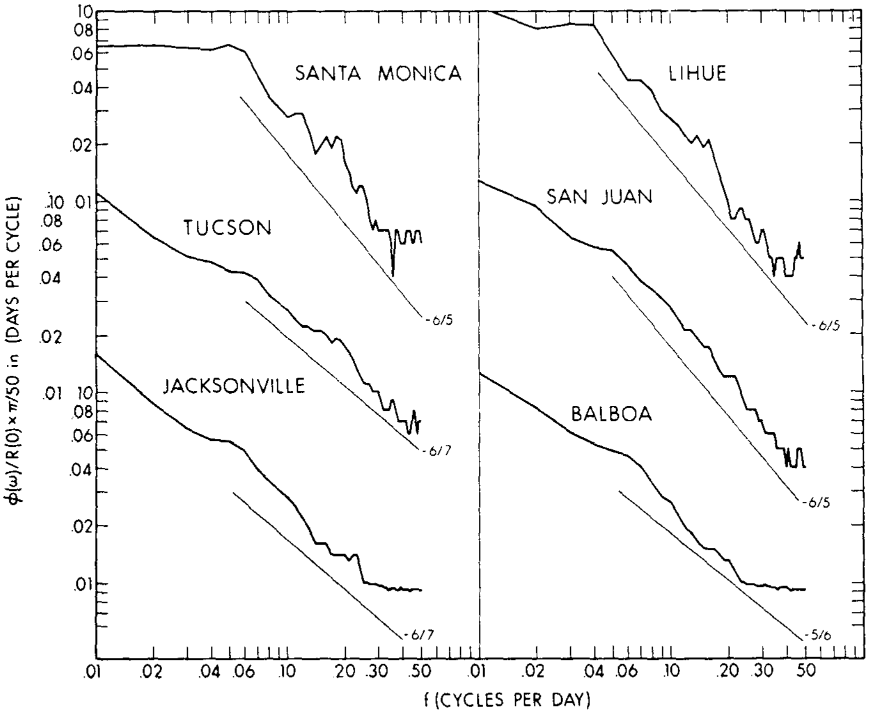

FIG. 11. Same as Fig. 10 except for other stations.

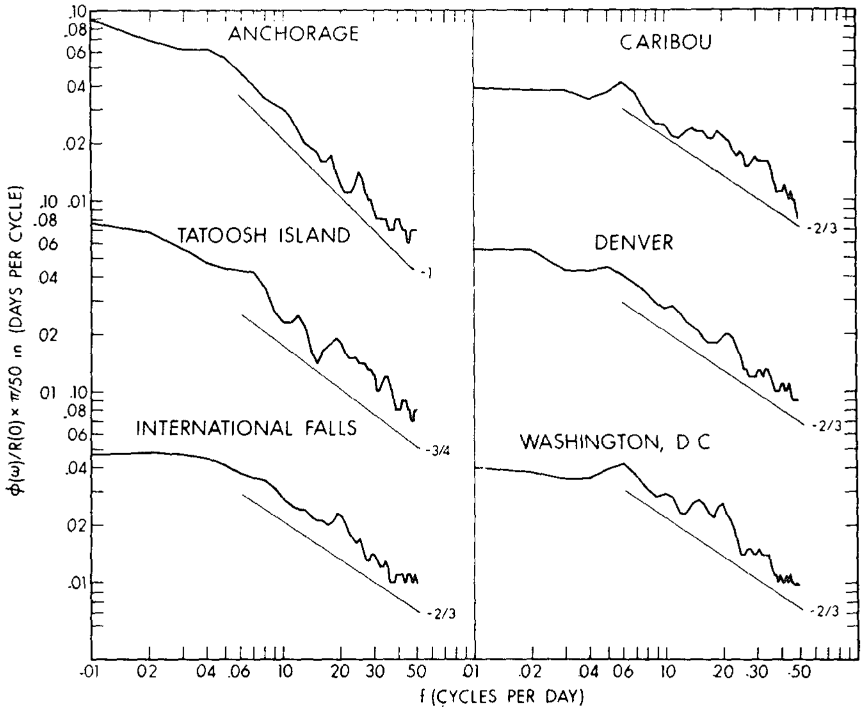

Fig. 12. Same as Fig. 10 except for the normalized v spectra.

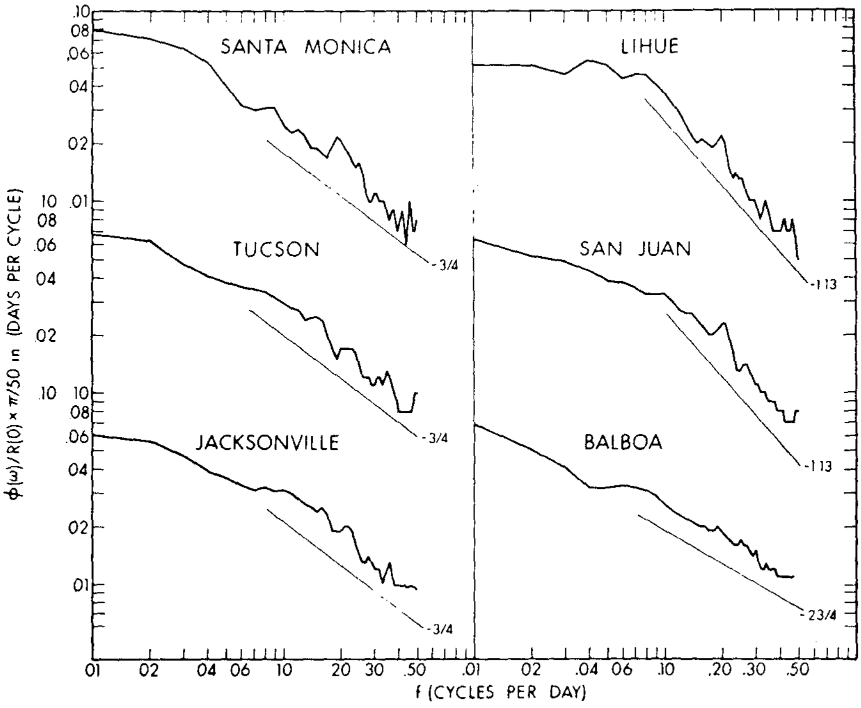

Fig. 13. Same as Fig. 11 except for the normalized v spectra.

quency (see Figs. 2–8), as well as the total amount of energy, vary greatly with altitude.

For the purpose of determining the slope of the smooth spectrum in a log-log plot of spectral density vs frequency, and to detect the variation of this slope with location, we first normalized the *u* (or *v*) spectrum at each of the 700-, 500-, 300- and 200-mb levels. From these normalized spectra an average normalized *u* (or *v*) spectrum representing the average situation for these four levels was obtained for each station. This average normalized spectrum should have an edf larger than that of the spectrum for each individual level, and thus should be better than the individual spectrum as an approximation of the true spectrum. Figs. 10 and 11 show the average normalized *u* spectrum, while Figs. 12 and 13 show the average normalized *v* spectrum at different stations. For the purpose of comparison, a straight line segment, with a slope closely approximating the slope of the high-frequency portion of the spectrum, i.e., for frequencies $\geqslant 0.06$ cpd, has also been drawn along side each spectrum on these figures. The low-frequency portion of the spectrum is not included in this slope determination because for quite a few spectra this portion begins to deviate from the slope of the high-frequency portion at about 0.06 cpd. Also the low-frequency portion of the spectra is more vulnerable to the distortion caused by the attempted removal of the mean and the first harmonic from the data, although previous discussions indicate that such a distortion, if any, would be very slight. By so excluding the low-frequency portion, we make doubly sure that the slope determination is free from this possible source of error.

The straight line segments representing the slopes were drawn by eye. Different persons could conceivably draw them in slightly different ways. But as long as the drawing is done for the same spectral density points, room for maneuvering is small. Certainly the lines are, in general, representative.

It is seen from Figs. 10 and 11 that the slope of the average normalized *u* spectrum is rather similar for most of the stations studied. It ranges from $-3/4$ to $-6/5$, with the stations at low latitudes having, in general, the larger slopes. The stations having the largest slope, $-6/5$, are Santa Monica, Lihue and San Juan. Except for Balboa, Lihue and San Juan are the southernmost stations among those studied. Figs. 12 and 13 show that the slope of the average normalized *v* spectrum varies somewhat more, and in a less systematic way, from station to station than the slope of the average normalized *u* spectrum does. The larger slopes are found in both high- and low-latitude stations, with Anchorage having -1 slope, and Lihue and San Juan both having about -1.13. Most other stations have a slope of either $-3/4$ or $-2/3$, and Balboa has the smallest slope of $-2.3/4$.

From this examination, especially of the average normalized *v* spectrum, we find no strong evidence for a systematic variation of the slope of the spectrum with latitude.

To examine the possible influences of the Coriolis parameter on atmospheric energy spectrum still further, we also calculated the ratio of the average normalized *u* spectral density to the average normalized *v* spectral density for every frequency point of the spectrum. The reason for this calculation is that if Coriolis force is related to the transfer of energy between velocity components, perhaps it might influence the pattern of this ratio and influence it to different degrees at different latitudes. Again no systematic variation of the pattern of these ratios with latitude is indicated.

Thus, this study shows that within the range of latitudes covered by our stations there is no definite, systematic connection between the shape of the spectrum and the latitude. It is not clear at the moment whether it means that the variation of the Coriolis parameter among our stations is still not large enough to generate a detectable difference in their spectra, or whether it means something else.

d. *Effect of the quasi-biennial oscillation*

Fig. 8 shows that the spectral density of *u* spectrum at the 50-mb level over Balboa is very high at and near zero frequency. This is due to the contribution to the kinetic energy from the quasi-biennial oscillation of the zonal wind, which is known to exist at this level over this station (Reed, 1965; Lee, 1972).

Acknowledgments. This research has been in part supported by the National Science Foundation, under Grant GA-21158. Thanks are due to Dr. Harold L. Crutcher of the National Weather Record Center for his help and supervision of the construction of spectra, to Prof. Colin S. Ramage for his reading of this paper, to Mr. Louis K. Oda for his help in drafting some of the figures, and to Mrs. S. Arita for the typing of this paper.

REFERENCES

Blackman, R. B., and J. W. Tukey, 1958: *The Measurement of Power Spectra*, New York, Dover, 190 pp.

Chiu, Wan-Cheng, 1960: The wind and temperature spectra of upper troposphere and lower stratosphere over North America. *J. Meteor.*, **17**, 64–77.

——, 1967: On the interpretation of energy spectrum. *Amer. J. Phys.*, **35**, 642–648.

——, 1970: On the spectral equations and the statistical energy spectrum of atmospheric motions in the frequency domain. *Tellus*, **22**, 608–619.

——, and Harold L. Crutcher, 1966: The spectrums of angular momentum transfer in the atmosphere. *J. Geophys. Res.*, **71**, 1017–1032.

Fultz, D., *et al.*, 1959: Studies of thermal convection in a rotating cylinder with some applications for large-scale atmospheric motions. *Meteor. Monogr.*, **4**, No. 21, 104 pp.

Gilman, D. L., F. J. Fuglister and J. M. Mitchell, Jr., 1963: On the power spectrum of "red noise." *J. Atmos. Sci.*, **20**, 182–184.

Hide, R., 1969: Some laboratory experiments on free thermal convection in a rotating fluid subject to a horizontal temperature gradient and their relation to the theory of the global atmospheric circulation. *The Global Circulation of the Atmosphere*, G. A. Corby, Ed., London, Roy. Meteor. Soc., 196–221.

Jones, Richard H., 1971: Spectrum estimation and time series analysis—A review. *Preprints Intern. Symp. Probability and Statistics in the Atmospheric Sciences*, Honolulu, Amer. Meteor. Soc., 35–40.

——, and William M. Brelsford, 1967: Time series with periodic structure. *Biometrika*, **54**, 403–408.

Julian, Paul R., 1971: Some aspects of variance spectra of synoptic scale tropospheric wind components in midlatitudes and in the tropics. *Mon. Wea. Rev.*, **99**, 954–965.

Kao, S. K., and Larry L. Wendell, 1970: The kinetic energy of the large-scale atmospheric motion in wavenumber-frequency space. *J. Atmos. Sci.*, **27**, 359–375.

Kolesnikova, V. N., and A. S. Monin, 1965: Spectra of meteorological field fluctuations. *Izv. Atmos. Oceanic Phys.*, **1**, 377–386.

Lee, Seung-Man, 1972: Observational study of quasi-biennial oscillation of zonal wind. M.S. thesis, University of Hawaii, 47 pp.

Mitchell, J. M., Jr., *et al.*, 1966: Climatic change. W. M. O. Tech. Note No. 79, Geneva, 79 pp.

Monin, A. S., 1963: Stationary and periodic time series in the general circulation of the atmosphere. *Time Series*, M. Rosenblatt, Ed., New York Wiley, 145–151.

Oort, Abraham H., and Albion Taylor, 1969: On the kinetic energy spectrum near the ground. *Mon. Wea. Rev.*, **97**, 623–636.

Reed, R. J., 1965: The present status of the 26-month oscillation. *Bull. Amer. Meteor. Soc.*, **46**, 374–387.

Wallace, J. M., and Chih-Pei Chang, 1969: Spectrum analysis of large-scale wave disturbances in tropical lower troposphere. *J. Atmos. Sci.*, **26**, 1010–1025.

Ward, Fred, and Ralph Shapiro, 1961: Meteorological periodicities. *J. Meteor.*, **18**, 635–656.

Yanai, M., T. Maruyama, T. Nitta and Y. Hayashi, 1968: Power spectra of large-scale disturbances over the tropical Pacific. *J. Meteor. Soc. Japan*, **46**, 308–323.