Numerical Models of the Circulation of the Atmosphere of Venus

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ABSTRACT

The deep circulation of the atmosphere of Venus is simulated by means of two-dimensional numerical models. Two extreme cases are considered: first, rotation is neglected and the subsolar point is assumed to be fixed; second (and more realistically), the solar heating is averaged over a Venus solar day and rotation is included. For each case a Boussinesq model, in which density variations are neglected except when coupled with gravity, and a quasi-Boussinesq model which includes a basic stratification of density and a semi-gray treatment of radiation, are developed. The results obtained with the Boussinesq models are similar to those obtained by Goody and Robinson and by Stone. However, when the stratification of density is included and most of the solar radiation is absorbed near the top, the large-scale circulation is confined to the upper layers of the atmosphere during the $4 \times 10^7$ sec of simulated time. We cannot be sure that on a much longer time scale ($10^9$ sec) the circulation will not penetrate the interior, but our results suggest that radiation will tend to make the lower atmosphere highly stable. When solar radiation is allowed to penetrate the atmosphere, so that at the equator 6% of the incoming solar radiation reaches the surface, then the combination of a more deeply driven circulation and a partial greenhouse effect is able to maintain an adiabatic stratification.

The effect of symmetrical solar heating is to produce direct Hadley cells in each hemisphere with small reverse cells near the poles. Poleward angular momentum transport in the upper atmosphere produces a shear in the zonal motion with a maximum retrograde velocity of the order of $10$ m sec$^{-1}$ at the top of the atmosphere.

The numerical integrations were performed using non-uniform grids to allow adequate resolution of the boundary layers.

1. Introduction

The planet Venus is subject to approximately the same dynamical driving as Earth, i.e., the incident minus the reflected solar radiation differ by less than a factor of 2. However, other parameters important to meteorologists are quite different. For example, the rotation rate of Earth is fast, but Venus has a rotation period of 243 days and in a retrograde direction; the inclination of the equator of Venus with respect to the ecliptic plane is less than 2°, compared with 23° for Earth; and the Venusian atmosphere has about a hundred times more mass than the Earth’s mass.

The effective temperature ($T_{\text{e}}$), which is approximately equal to the temperature at the top of the cloud deck covering most of the Venus atmosphere, is about 230K. However, at the beginning of the 1960’s, microwave emission temperatures indicated the existence of surface temperatures of at least 600K. This was confirmed by the Venera 7 spacecraft which measured a surface temperature of $747 \pm 20$K (Avduevaevsky et al., 1971). There were several theories offered to explain these high temperatures. Opik (1961) proposed an “aerospheric” model for the atmosphere of Venus between the planet’s surface and the top of the clouds. Strong winds driven by the differential heating at the top were responsible for both grinding and raising dust from the surface, making the atmosphere opaque to radiation, and for heating of the surface layers due to frictional dissipation of kinetic energy. In the more popular “greenhouse” model proposed by Sagan (1962) and others, most of the solar radiation is assumed to penetrate through the atmosphere to the planet’s surface, but the atmosphere is very opaque in the infrared region, so that emission into space takes place in the colder regions near the top of the cloud layers. The main objection to this model is not the large opacity required in the longwave region, but the relative transparency in the shortwave region necessary to heat up the lower layers of the atmosphere.

The first dynamical model offered to explain the high surface temperatures was that of Goody and Robinson (1966). They used a two-dimensional Boussinesq model on a flat surface with the subsolar and antisolars points represented by fixed vertical planes. The atmosphere was considered to be completely opaque so that radiation was absorbed and emitted at the top of the cloud deck. In the interior, radiative and turbulent transfer were parameterized as a diffusion process. Using scale and boundary layer analysis, they developed a model...
for the circulation of the atmosphere of Venus with slow rising motion in most of the atmosphere and a narrow region of sinking motion (mixing region) at the antisolunar point. There was a thin horizontal upper boundary layer with strong horizontal motion toward the antisolunar point and in the interior a slow return motion toward the subsolar point. Goody and Robinson's analysis suggested that the large-scale atmospheric motions convect heat downward, maintaining an adiabatic lapse rate throughout the lower atmosphere. In this way the high temperatures at the surface (about 60 km below the cloud top level) could be explained even if the solar radiation was absorbed near the top of the atmosphere.

Stone (1968) developed a similar two-dimensional Boussinesq model on a flat surface with an improved scaling of the mixing region at the antisolunar point. He did not deal with the problem of the maintenance of the adiabatic lapse rate in the interior. The main difference between Stone's and Goody and Robinson's results was the width of the mixing region, 150 km and 3 km, respectively. Furthermore, Stone pointed out that the magnitude of the vertical velocity would decay slowly away from the mixing layer so that downward motion would not be confined to the mixing region. Both Goody and Robinson, and Stone concluded that the Rossby number would be large due to the small rotation rate of the planet so that the effects of rotation would be minor.

Hess (1968) performed a numerical computation with a similar non-rotating, two-dimensional model in Cartesian geometry. He used pressure as a vertical coordinate so the Boussinesq approximation was not made. After the equivalent of 160 earth days of uneven heating at the top the model had not converged. The results were nevertheless similar to Goody and Robinson's except that the circulation was confined to the top third of the atmosphere, probably because of the increase of density with depth. The grid that Hess used was too coarse to resolve the boundary layers so that the width of the mixing region was much larger than in Goody and Robinson's and Stone's analyses. The negligible value of the winds near the surface made Öpik's "aerospheric" model improbable.

In their 1961 ultraviolet photographs of Venus, Boyer and Camichel found cloud patterns shaped like a horizontal Y which seemed to move in a zonal direction with a rotation speed corresponding to a period of about 4 days (Dollfiltr, 1968). In addition, there was a tendency for certain cloud patterns to recur every 4 or 5 days. More recent observations, reviewed by Smith (1967) and Schubert and Young (1970), support the evidence for the existence of a retrograde rotation of the Venus upper atmosphere with a period of 4–5 days. This implies zonal velocities of the order of 100 m sec⁻¹, i.e., about 50 times larger than the speed of rotation of the planet itself at the equator.

Schubert and Whitehead (1969) and Schubert and Young (1970) suggested that this rotation was due to the apparent (prograde) rotation of the Sun during a Venusian solar day, implying that the Reynolds stresses that arise from the vertical circulation driven by the periodically moving vertical forcing are able to sustain the mean zonal flow. Schubert et al. (1971) found that the average motion of a fluid driven by a moving thermal source was prograde or retrograde with respect to the heating source depending on the magnitude of the Prandtl number \( \nu/\kappa \). Only if the heat is well-diffused \( \nu/\kappa \ll 1 \) can retrograde motion occur. Gierasch (1970) showed that the radiative time constants at the cloud top level are of the correct magnitude to cause a strong zonal flow by the mechanism suggested by Schubert and Whitehead. Malkus (1970) and Thompson (1970) suggested that while the zonal flow could be started by the Schubert-Whitehead mechanism, the interaction of a shearing flow with the tilted cells via the Reynolds stresses could intensify the shear and produce an upper flow of the required magnitude.

The present paper is an attempt to study the deep circulation of the atmosphere of Venus from a dynamical point of view. The complexity of the processes to be considered and the obvious importance of nonlinear effects deduced from simple analytical models and from the strong high-altitude cloud motions, make necessary the use of numerical models. Although observational data on atmospheric structure are rather scarce, we know enough to develop some simple models. Astronomical data are by this time well established, and from the observations made by the space vehicles Mariner 5 and Veneras 4–7, we now have some data on the lower atmospheric structure of Venus (Avduevskiy et al., 1971).

Based on these data, a series of two-dimensional numerical models was developed. Two extreme cases were considered: first, we neglected (following Goody and Robinson) the effect of planetary rotation and assumed that the subsolar point is fixed; and second, we included the planetary rotation but assumed that the effects of the diurnal variation of the solar heating were negligible. In each case we developed two models: a Boussinesq model in which there is no basic stratification of density and no radiation since the transfer of heat is assumed to be effected only by eddy diffusivity and large-scale advection; and a quasi-Boussinesq model which includes both the basic density stratification and a semi-gray approximation of radiation. All of these models were developed for flow on a sphere.

After the calculations reported in this paper were completed, two other numerical computations of the circulation of Venus atmosphere were published (Sasamori, 1971; and Turikov and Chalikov, 1971 or Chalikov et al., 1971). We will compare our results with theirs in the last section.
2. Non-rotating Boussinesq model

a. The numerical model

A numerical model was developed with the following approximations:

(a) Boussinesq.
(b) Hydrostatic. This is based on the small aspect ratio $H/a$, where $H$ is the height of the cloud top layer (60 km) and $a$ the radius of Venus (6050 km). This approximation may be not very accurate in the mixing region.
(c) No rotation.
(d) The subsolar point remains fixed.
(e) The atmosphere is very opaque so that short- and longwave radiation is absorbed and emitted only at the top of the cloud layer. The heat flux is parameterized as a diffusion process.
(f) Constant horizontal and vertical coefficients of eddy viscosity $\nu_H$ and $\nu_V$, and of eddy diffusivity $\kappa_H$ and $\kappa_V$.
(g) Unit Prandtl number $\nu/\kappa$.

The Boussinesq equations are as follows:

The meridional component of the equation of motion

$$\frac{\partial \nu}{\partial t} = \frac{(\nu \text{ siao} \alpha)}{a \text{ siao} \alpha} - \frac{\nu \text{ g}}{\rho_0} \frac{\nu_H}{\sin \alpha} \left[ \frac{\nu}{\sin \alpha} - \frac{\nu}{\sin^2 \alpha} \right] + \nu \nu_{zz}$$

The hydrostatic equation

$$\frac{\partial p}{\partial t} = -\rho g$$

The continuity equation

$$\frac{(\nu \text{ siao} \alpha)}{a \text{ siao} \alpha} - w_z = 0$$

The thermal equation

$$\frac{\partial \rho}{\partial t} = \frac{(\rho \nu \text{ siao} \alpha)}{a \text{ siao} \alpha} - \left( \frac{\kappa_H}{a^2} \frac{\rho \text{ siao} \alpha \nu}{\sin \alpha} \right) + \kappa_V \nu_{zz}$$

In the above, $\alpha$ is the co-latitude measured from the antisolar point, $\nu$ the meridional velocity $(\partial \nu/\partial t)$ positive toward the subsolar point, $w$ the vertical velocity $(\partial \nu/\partial t)$, $p_0$ the surface pressure, $\rho$ the potential density departure from the mean value $\rho_0$ divided by $\rho_0$, and $\rho_0 = p_0 / (gH)$. From (1) and (2) the pressure term can be eliminated to obtain a forecast equation for the vorticity $\nu$. This vorticity equation contains nonlinear terms generated by the convergence of the meridians which produce numerical instabilities near the antisolar point. It is preferable to work with the vortex strength $\eta = \nu / \sin \alpha$, which, for an inviscid, homogeneous fluid, is individually conserved over the whole sphere. The vortex strength equation is

$$\frac{\partial \eta}{\partial t} = \frac{(\nu \text{ siao} \alpha \alpha)}{a \text{ siao} \alpha} - \frac{\nu_H}{\rho_0} \left[ \frac{(\nu \text{ siao} \alpha \alpha \sin \alpha)}{a \text{ siao} \alpha} \right] + \nu \nu_{zz}.$$  

We define a mass streamfunction $\psi$:

$$\eta = \sin \alpha \psi_{zz},$$

so that

$$v \sin \alpha = \psi_z, \; w \sin \alpha = -\psi / a.$$  

Eqs. (4)-(7) are the ones used in the model.

We assume a non-stress rigid top at the cloud top level and a non-slip rigid bottom at the surface. From the geometry of the model the horizontal velocity is zero at the subsolar and antisolar points. Therefore, the boundary conditions for the mass streamfunction are

$$\psi = \psi_{zz} = 0, \quad \text{at } z = H \left\{ \begin{array}{l} \text{if } 0 \leq \alpha \leq \pi/2 \\ \text{if } \pi/2 < \alpha \leq \pi \end{array} \right.$$  

The shortwave radiation absorbed at the top is

$$F_\alpha = \left\{ \begin{array}{l} 0, \quad \text{if } 0 \leq \alpha \leq \pi/2 \\ -(1 - A)S_0 \cos \alpha, \quad \text{if } \pi/2 < \alpha \leq \pi \end{array} \right.$$  

where $S_0$ is the Venusian solar constant, $A$ the planetary albedo, and $(1 - A)S_0/4 = \sigma T_\alpha^4$. Assuming that temperature or density departures from the mean value at the top are small and using approximation (e), we get the following boundary condition for $\rho$ at the top:

$$\rho_z = \frac{\sigma T_\alpha^4}{\kappa_V} \left[ (1 - 4\rho) + \left\{ 0, \quad \text{if } 0 \leq \alpha \leq \pi/2 \\ 4 \cos \alpha, \quad \text{if } \pi/2 < \alpha \leq \pi \right\} \right]$$  

At the bottom we assume that the heat flux is negligible:

$$\rho_z = 0, \quad \text{at } z = 0.$$  

From the previous studies by Stommel (1962), Goody and Robinson, and Stone, we expected the appearance of horizontal boundary layers at the top and possibly at the bottom of the atmosphere, and a vertical boundary layer at the antisolar point. In these narrow regions, a fine grid is needed to resolve them numerically, but in the interior much less resolution is necessary. The solution of the problem with a regularly spaced grid that is fine enough to resolve the boundary layers is not possible because it would consume too much computer time. We used non-uniform grids defined through the use of stretched coordinates to obtain better resolution in the regions where boundary layers
were expected. The finite-difference equivalent of
\[
\frac{\partial f}{\partial x} = \frac{d\xi}{dx} \frac{\partial f}{\partial \xi}, \quad \frac{\partial^2 f}{\partial x \partial \xi} = \frac{d\xi}{dx} \frac{d\xi}{dx} \frac{\partial f}{\partial \xi} - \frac{d\xi}{dx} \frac{d^2 f}{d\xi^2}
\]
was used, where \(x\) is the physical coordinate and \(\xi\) the
stretched coordinate. It has been shown (Kálmán de
Rivas, 1972) that in this form the "extra truncation error"
introduced by the use of nonuniform grids is of
second order in \(\Delta \xi\). Furthermore, the use of the stretched
coordinate \(x = \xi\) has several advantages: (i) the extra
truncation error is independent of \(x\), except for the
variations of \(f\) itself; (ii) the density of grid points near
the boundary \(x = 0\) is proportional to the square of the
total number of grid points; and (iii) at the worst
point, the other end of the interval, the resolution is
equal to one-half of that obtained with a regular grid
with the same number of grid points. The stretched
coordinate \(x = \sin[(\pi/2)\xi]\) has similar properties,
except that it gives excellent resolution at both ends of
the interval \(0 \leq x \leq 1\) and is rather linear in the interior.
Therefore, we used in the horizontal direction the
stretched coordinate \(y\) such that \(x = y^2\), and in
the vertical direction \(z\), such that \(z = H \sin[(\pi/2)s]\).
Whenever necessary because of the boundary conditions
we defined an external grid point located at the same
distance from the boundary as the first interior point
so that normal derivatives were computed on a locally
uniform grid with very small intervals. In this model
we used 20 grid intervals in both the horizontal and
vertical directions. Even with this sparse grid the
first interior point was only at 47 km from the antisolar
point, and 340 m from top and bottom (Fig. 2).
A staggered grid was used (Fig. 1) which decreases
by a factor of 2 the distance over which many of the
derivatives were computed and simplifies the numerical
treatment of all of the boundary conditions. The

simplest spatially-centered finite-difference scheme with
the nonlinear terms written in flux form was used,
which conserved the mean and the mean-squared den-
sity and the mean but not the mean-squared vorticity.
The lack of conservation of mean-squared vorticity
was accompanied by a nonlinear instability that ruined
the computation after \(7.5 \times 10^8\) sec when \(\nu_H = 10^9\ \text{cm}^2\)
sec\(^{-1}\). This instability was overcome when the hori-
izontal coefficient of eddy viscosity was increased to
the perhaps unrealistically large value of \(10^4\ \text{cm}^2\)
sec\(^{-1}\). The boundary conditions were also expressed in cen-
tered finite differences except for the non-slip boundary
condition at the bottom. The value of the vortex
strength at the bottom was obtained in terms of the
streamfunction through a procedure similar to the one
used by Pearson (1965) and Williams (1967), but also
taking into account the non-uniformity of the grid so
that even though differences were not centered, truncation
errors were kept of second order.

The leap-frog method (centered differences in time)
was used except for the viscosity and conductivity
terms. For these a forward time difference was used to
avoid the unconditional instability that occurs when
centered time differences are used with diffusion. The
uncoupling of the solutions at even and odd time steps
associated with the use of the leap-frog method was
avoided by averaging two successive solutions after
every 40 time integrations. After the new fields of
vorticity and density were forecast using the finite-
difference equivalents of (4) and (5), the new mass-
streamfunction field was obtained solving the system
of finite-difference equations corresponding to (6). The
matrix inversion involved in this solution had to be
done only once.

The numerical values of the physical parameters
are given in Table 1. They are those given by Goody
and Robinson except for the cloud top height \(H\), which
was taken as 60 km instead of 40 km, \(p_0\) which was
taken as the mean atmospheric density, and the hori-
zontal coefficients of eddy viscosity and conductivity
which, as noted before, were taken as \(10^4\) instead of
\(10^9\ \text{cm}^2\ \text{sec}^{-1}\).

The initial conditions were taken as a state of no
motion and constant potential density \((v = w = p = 0)\).
The constant differential heating by the sun was
allowed to build up a circulation during \(133 \times 10^8\) sec
(about 154 earth days). At this time a steady state had
been reached in most of the atmosphere. Tables 2 and 3
show the balance of terms in the vorticity and thermal
equations at nine different points in the atmosphere
placed as in Fig. 2. Even at the interior points where

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**Table 1. Numerical values of the physical parameters used in the Boussinesq model without rotation.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>(6.05 \times 10^4) cm</td>
</tr>
<tr>
<td>(H)</td>
<td>(6 \times 10^3) cm</td>
</tr>
<tr>
<td>(g)</td>
<td>(870) cm sec(^{-2})</td>
</tr>
<tr>
<td>(C_p)</td>
<td>(1.01 \times 10^7) cm(^2) sec(^{-2})</td>
</tr>
<tr>
<td>(p_0)</td>
<td>100 atm</td>
</tr>
<tr>
<td>(p_4)</td>
<td>(p_0/(gH) = 1.94 \times 10^2) gm cm(^{-2})</td>
</tr>
<tr>
<td>(T_e)</td>
<td>230K</td>
</tr>
<tr>
<td>(\kappa_H)</td>
<td>(\nu_H = 10^9) cm(^2) sec(^{-1})</td>
</tr>
<tr>
<td>(\kappa_e)</td>
<td>(p_0 = 10^4) cm(^2) sec(^{-1})</td>
</tr>
</tbody>
</table>
the time derivative was not much smaller than the leading terms, time variations were extremely small and no variations could be detected over several days.

b. Results

Fig. 3a shows the mass streamfunction. As in the analyses by Goody and Robinson and by Stone, there is a single, strongly asymmetric Hadley cell. Its center is near $70^\circ$ from the antisolar point and slightly below the central level. The existence and strength of the boundary layers is more apparent in Figs. 3b, 3c and 3d which depict the meridional and vertical velocity and the density fields. In the upper boundary layer the typical meridional velocity is $10$ m sec$^{-1}$ with a maximum of $18$ m sec$^{-1}$; in the interior the meridional velocities are of the order of $2$ m sec$^{-1}$ and vary slowly. Although the flow toward the antisolar point is very strong in the narrow boundary layer at the top, it is not confined to it. Essentially, the upper half of the atmosphere moves toward the antisolar point and return flow occurs in the lower half. The vertical boundary layer at the antisolar point, or mixing region, is characterized by a strong and concentrated downward flow with a maximum velocity of 60 cm. In the interior the vertical velocity is of the order of 1 cm sec$^{-1}$. One of the features of the Goody and Robinson model was that downward motion was confined to the mixing region, which could explain the complete cloud coverage of Venus if the clouds were formed by condensation. However, our numerical results, as well as Stone's analysis, differ from the Goody-Robinson model as our upward motions are confined mainly to the illuminated hemisphere. In Fig. 3d we see that the interior is almost completely adiabatic, or more precisely, neutrally stratified. The departures of density from the mean value are less than 0.1%, and this agrees well with the adiabatic interior obtained by Goody and Robinson. However, this result may be due to the fact that in our model, as in Stone's, internal radiative transfer is not included; therefore, there is nothing to counteract the tendency for turbulent diffusion to bring about an adiabatic lapse rate. The strong density gradients are confined to the top boundary layers with a thickness of about 1 km. The density difference between the subsolar and the antisolar points is about 1%, corresponding to a temperature difference of 23K. This is large compared to the few degrees observed temperature difference between equator and pole and the very small, if any, difference between the illuminated and

![Fig. 2. Non-uniform grid used in the computations. Here, as in the following diagrams, the abscissa is the co-latitude $\alpha$ (deg) and the ordinate height $z$ (km). The position of the points referred to in Tables 2 and 3 is indicated.](image)

**Table 2.** Balance of terms in the vorticity equation at 9 points (units of $10^{-18}$ sec$^{-2}$). Italicized values are the principal balancing terms at each point. See Fig. 2 for the position of the points A through I.

<table>
<thead>
<tr>
<th>Point</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horizontal advection</td>
<td>$-23706$</td>
<td>$-42.86$</td>
<td>$115.74$</td>
<td>$138.81$</td>
<td>$-0.58$</td>
<td>$0.24$</td>
<td>$-4632.1$</td>
<td>$54.78$</td>
<td>$-1.88$</td>
</tr>
<tr>
<td>Vertical advection</td>
<td>$25368$</td>
<td>$42.15$</td>
<td>$35.12$</td>
<td>$-356.79$</td>
<td>$1.60$</td>
<td>$-1.18$</td>
<td>$141.3$</td>
<td>$2.72$</td>
<td>$-2.30$</td>
</tr>
<tr>
<td>Horizontal viscosity</td>
<td>$15924$</td>
<td>$24.72$</td>
<td>$33.29$</td>
<td>$-82.62$</td>
<td>$2.04$</td>
<td>$0.89$</td>
<td>$765.9$</td>
<td>$74.00$</td>
<td>$-2.02$</td>
</tr>
<tr>
<td>Vertical viscosity</td>
<td>$684$</td>
<td>$282.92$</td>
<td>$310.75$</td>
<td>$-0.04$</td>
<td>$0.00$</td>
<td>$0.00$</td>
<td>$4346.9$</td>
<td>$20.32$</td>
<td>$-1.09$</td>
</tr>
<tr>
<td>Driving</td>
<td>$-13850$</td>
<td>$-306.86$</td>
<td>$-424.56$</td>
<td>$-123.12$</td>
<td>$-2.98$</td>
<td>$0.04$</td>
<td>$-4.3$</td>
<td>$1.47$</td>
<td>$-0.79$</td>
</tr>
<tr>
<td>Time derivative</td>
<td>$43990$</td>
<td>$0.06$</td>
<td>$0.00$</td>
<td>$-423.67$</td>
<td>$-0.02$</td>
<td>$-0.01$</td>
<td>$-913.4$</td>
<td>$1.45$</td>
<td>$-0.28$</td>
</tr>
</tbody>
</table>

**Table 3.** Balance of terms in the density equation at 9 points (units of $10^{-10}$ sec$^{-3}$). Italicized values are the principal balancing terms at each point. See Fig. 2 for the position of the points A through I.

<table>
<thead>
<tr>
<th>Point</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horizontal advection</td>
<td>$15324.3$</td>
<td>$-198.1$</td>
<td>$163.6$</td>
<td>$-139.1$</td>
<td>$2.40$</td>
<td>$-0.63$</td>
<td>$-131.6$</td>
<td>$0.213$</td>
<td>$0.434$</td>
</tr>
<tr>
<td>Vertical advection</td>
<td>$-6127.6$</td>
<td>$48.0$</td>
<td>$196.5$</td>
<td>$213.4$</td>
<td>$-2.91$</td>
<td>$1.10$</td>
<td>$133.4$</td>
<td>$-0.072$</td>
<td>$-0.431$</td>
</tr>
<tr>
<td>Horizontal diffusion</td>
<td>$9394.3$</td>
<td>$-20.6$</td>
<td>$41.3$</td>
<td>$-74.0$</td>
<td>$0.90$</td>
<td>$-0.02$</td>
<td>$-2.2$</td>
<td>$-0.295$</td>
<td>$0.262$</td>
</tr>
<tr>
<td>Vertical diffusion</td>
<td>$197.8$</td>
<td>$171.1$</td>
<td>$-401.1$</td>
<td>$0.0$</td>
<td>$-0.00$</td>
<td>$-0.02$</td>
<td>$0.9$</td>
<td>$0.573$</td>
<td>$0.097$</td>
</tr>
<tr>
<td>Time derivative</td>
<td>$0.3$</td>
<td>$0.4$</td>
<td>$0.4$</td>
<td>$0.4$</td>
<td>$0.38$</td>
<td>$0.43$</td>
<td>$0.4$</td>
<td>$0.419$</td>
<td>$0.352$</td>
</tr>
</tbody>
</table>
dark hemispheres. However, these temperatures are measured at the cloud top level, and, if the clouds are formed by condensation, their tops may correspond roughly to an isothermal surface. There is a small region near the antisolar point with a gravitationally unstable stratification.

The irregularities in the upper right region of Figs. 3a and 3b are due to truncation errors in the finite-difference approximation of the transport terms. They are most noticeable there because in a region where spacial variations are exponential, as in a boundary layer, transports are overestimated, and the relative error made in the estimation of time derivatives is therefore larger if the flow is toward the boundary layer than if it is away from it.

Tables 2 and 3 have the principal balancing terms italicized in the vorticity and density equations. It is clear that nonlinear terms are important everywhere in the atmosphere. There is a good agreement in the overall balance of terms between our results and the scale analysis of Goody and Robinson and of Stone,

Table 4. Comparison of the width and velocity magnitudes at the upper boundary layer and the mixing region obtained by Stone, by Goody and Robinson, and by using the numerical non-rotating Boussinesq model.

<table>
<thead>
<tr>
<th>Magnitude*</th>
<th>Stone's analysis</th>
<th>Numerical model</th>
<th>Goody and Robinson</th>
</tr>
</thead>
<tbody>
<tr>
<td>$z_{av}$ [km]</td>
<td>$\frac{\kappa_{av}(\rho_0)}{\kappa_{av}(\rho_1)}$</td>
<td>$=0.43$</td>
<td>$\sim5$</td>
</tr>
<tr>
<td>$y_{av}$ [km]</td>
<td>$\frac{\kappa_{av}(\rho_0)}{\kappa_{av}(\rho_1)}$</td>
<td>$=1.350$</td>
<td>$\sim1500$</td>
</tr>
<tr>
<td>$w_{av}$ [cm sec$^{-1}$]</td>
<td>$\frac{\kappa_{av}(\rho_0)}{\kappa_{av}(\rho_1)}$</td>
<td>$=0.23$</td>
<td>$\sim20$</td>
</tr>
<tr>
<td>$\omega_{av}$ [m sec$^{-1}$]</td>
<td>$\frac{(\kappa_{av}(\rho_0))^{1/4}}{\kappa_{av}(\rho_1)}$</td>
<td>$=7.4$</td>
<td>$\sim10$</td>
</tr>
<tr>
<td>$z_{av}$ [km]</td>
<td>$\frac{\kappa_{av}(\rho_0)}{\kappa_{av}(\rho_1)}$</td>
<td>$=1.5 \times 10^{-2}$</td>
<td>$\sim2 \times 10^{-3}$</td>
</tr>
<tr>
<td>$w_{av}$ [cm sec$^{-1}$]</td>
<td>$\frac{\kappa_{av}(\rho_0)}{\kappa_{av}(\rho_1)}$</td>
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<td>$\sim1$</td>
</tr>
<tr>
<td>$\omega_{av}$ [m sec$^{-1}$]</td>
<td>$\frac{(\kappa_{av}(\rho_0))^{1/4}}{\kappa_{av}(\rho_1)}$</td>
<td>$=0.43$</td>
<td>$\sim5$</td>
</tr>
<tr>
<td>$\rho_{av}$</td>
<td>$\frac{(\kappa_{av}(\rho_0))^{1/4}}{\kappa_{av}(\rho_1)}$</td>
<td>$=3.7 \times 10^{-3}$</td>
<td>$\sim6 \times 10^{-2}$</td>
</tr>
</tbody>
</table>

* Mixing region, $mr$; upper boundary layer, $bl$. 

Fig. 3. Mass streamfunction $\psi$ (10$^9$ cm$^3$ sec$^{-1}$). a, velocity components $v$(m sec$^{-1}$), b, and $w$(cm sec$^{-1}$), c, and relative density departure from the initial value, d, in the non-rotating Boussinesq model. The antisolar (AS) point is on the left and the subsolar point (SS) on the right.
except in some interior points where horizontal viscosity and diffusivity are not small because of the excessively large values of $\nu_H$ and $\kappa_H$. In Table 4 we compare the orders of magnitude of the velocity components, the density departures, and the width of the boundary layers in our numerical model with those in Stone's analysis. We also include the results obtained by Goody and Robinson. The comparison is more difficult because of their use of radiative-diffusive boundary conditions, whereas ours are diffusive only, and because we use a larger value of $\rho_0$, corresponding to the mean density of the atmosphere. In the mixing region the agreement with Stone's results is good, except for the vertical velocity which is much larger in our results. This is probably due to the convergence of the meridians in the spherical geometry. In the upper boundary layer there is a discrepancy in the scale of the velocity components which are ten times larger in our results. However, in this region Stone's analysis is valid only if $H^3/(\pi a)^6 \lesssim \nu_V/\nu_H$ which is not true with $\nu_H = 10^{11}$ cm$^2$ sec$^{-1}$.

3. Boussinesq model including rotation and axisymmetric heating

a. The numerical model

It was first suggested by Thaddeus (1968) that the atmosphere of Venus has such a large heat capacity that it cannot respond to diurnal changes even for the long Venusian day. Gierasch et al. (1970) found that the ratio between the estimated radiative time constant and the length of the day is about 100 for Venus. This is roughly the same value as for Earth, where diurnal (tidal) effects are known to have a negligible effect on the general circulation of the atmosphere. The smallness of the diurnal effects has been confirmed by observations both at the cloud top level and at the ground. The brightness-temperature maps by Murray et al. (1963) show no clear-cut night-to-day variation at the cloud top level, although they do show an appreciable contrast between equator and poles. The surface temperatures measured by Veneras 7 and 8 in night and day regions were almost the same (747$\pm$20K and 727$\pm$20K, respectively). Also the Mariner 5 radio occultation day-side and night-side temperature profiles between 30 and 60 km are remarkably similar (Fjeldbo et al., 1971). Ultraviolet pictures show a marked zonal structure, and an absence of the radial structure that could be expected if rotation were negligible (Boyer and Newell, 1967; Dollfus, 1968).

There is thus ample motivation to explore a two-dimensional model that represents another limiting case of the possible circulation of the atmosphere of Venus, one in which rotation is included and the daily variations are neglected so that the solar heating is assumed to be symmetrical about the axis of rotation. Such a model was therefore developed using the Boussinesq approximation and rotating coordinates. In what follows, $\alpha$ is the co-latitude, $u$ the zonal component of the velocity [positive in a retrograde direction (direction of rotation of Venus)], $v$ the meridional component (positive southward), and $w$ the vertical velocity (positive upward).
Because of the geometry of the problem, only two changes have to be made in Eqs. (4)–(7):

(i) We have to include the zonal component of the equation of motion:

\[
\frac{\partial u}{\partial t} = \frac{(uw \sin \alpha)_{\alpha}}{a \sin \alpha} - (uw)_{\alpha} + f v - \frac{\partial u}{\partial a} + \nu H \frac{\partial^2 (u_{\alpha} \sin \alpha)_{\alpha}}{a^2} = \frac{u}{\sin \alpha} + \nu H \frac{\partial^2 u}{\partial a^2},
\]

(10)

where \( f = 2 \Omega \cos \alpha \) is the Coriolis parameter and \( \Omega = -2.993 \times 10^{-7} \) \( \text{sec}^{-1} \) is the angular velocity of Venus, corresponding to a period of 243 earth days.

(ii) The terms \(-f \sin \alpha + \frac{\cota}{(a \sin \alpha)_{\alpha}}(a^2)_{\alpha}\) have to be added to the right-hand side of (5).

The boundary conditions are similar to those described in the Section 2. However, there is now a symmetry condition at the equator, and the solar heating at the top is \( T_e = (1 - A) T_0 \sin \rho \). We used stretched coordinates as before with 13 grid intervals in both directions. The method of integration was similar to that used in the non-rotating model, with a staggered grid as in Fig. 1. The physical data were the same as in the non-rotating case except that \( K_H = \nu_H = 10^9 \) \( \text{cm}^2 \) \( \text{sec}^{-1} \), the values suggested by Goody and Robinson. There was no computational instability in this case, suggesting that rotation had a stabilizing effect.

The initial conditions were a state of solid rotation \((u=v=w=0)\) and of neutral stability \((\rho = 0)\). The differential heating between equator and pole was allowed to act for about \( 2 \times 10^2 \) sec (2 Venus solar days), at which time the model had converged.

b. Results

Fig. 4a shows a cross section of the meridional mass streamfunction. As might be expected, it consists mainly of a direct Hadley cell. The main difference with the non-rotating case is that the region of strong downward motion (mixing region) is not at the pole where the maximum cooling occurs but between 5° and 8° from the pole. There is a narrow reverse cell 5° wide in the polar region. This is because the conservation of angular velocity (except for turbulent viscosity) would create infinite zonal velocities and shears if the parcels at the top of the atmosphere coming from the equator were to reduce their radius of rotation to zero at the pole. (The circulation resembles the vortex formed when a bath tub is being emptied). The center of the positive cell is lower than in the non-rotating case.

The order of magnitudes of the velocities is the same as in the non-rotating case, but the maximum velocities in the boundary layers are half as strong. The cross section of the zonal velocity field (Fig. 4b) shows that the relative rotation is in the same direction as the planetary rotation in most of the atmosphere, and attains the rather large value of 14 sec\(^{-1}\) near the pole. The shear of the zonal momentum is also positive except in the narrow reverse cell, and has its maximum near the pole. The angular momentum field \( AM = (-2 \Omega \sin \alpha + u \sin \alpha)\) shows (Fig. 4c) that the meridional circulation has produced a poleward transport of angular momentum in the upper atmosphere. If the flow were nonviscous and symmetric, the relative zonal velocity at the equator would be strictly zero, but the existence of horizontal eddy viscosity makes it non-zero. The strong density contrasts are again confined to a top boundary layer of less than 5 km, and the interior is neutral stable. The relative density difference between the equator and the pole is only 0.025 at the top, corresponding to about 6K. This is of the order of magnitude of the observed temperature contrast.

Tables giving the numerical balance of terms in the vorticity, thermal and zonal momentum equations are given in Káhnay de Rivas (1971). They show that the balance is similar to the non-rotating Boussinesq case, except that the relative rotation terms are important away from the equator, and horizontal viscosity and diffusion are small away from the vertical boundary layer. This behavior is due to the smaller value of \( \nu_H \) and \( \kappa_H \) used here.

4. Non-rotating quasi-Boussinesq model

The atmosphere of Venus is much deeper than the Earth’s atmosphere: the cloud top level is located at about 60 km from the solid surface, the ratio of the density at the surface level to the density at the cloud top is ~100, the temperature ratio is ~3, and the pressure ratio ~400. The Boussinesq approximation neglects density variations except when coupled with gravity so that the basic density stratification is not taken into account. The use of this approximation for a compressible fluid can be justified only if the vertical extent of the model is less than any scale height, and this is not so in the Venus atmosphere. For this deep atmosphere, a better approximation is to use horizontal mean values of temperature, pressure and density which vary with height, rather than constant mean values. The observations made by Venera 7 (Avdudevsky et al., 1971) showed that the stratification of the atmosphere of Venus is nearly adiabatic. This allows us to use the “anelastic” or “quasi-Boussinesq” approximation (Ogura and Phillips, 1962; Charney and Ogura, 1960), where the distribution of pressure and density is assumed to be always close to that in an adiabatically stratified atmosphere. Variations of potential temperature from its mean value are neglected except when coupled with gravity. Both the quasi-Boussinesq and the Boussinesq approximations eliminate sound waves that are present in the original hydrodynamic equations.

We developed a quasi-Boussinesq model which included radiation through a semi-gray approximation.
a. Quasi-Boussinesq equations

Following the procedure outlined by Ogura and Phillips, we define a nondimensional vertical coordinate \( \tau = z / D \), where \( D = C_p \beta_0 / g \) is the adiabatic height of the atmosphere, and \( \theta_a \) the mean potential temperature which is equal to the surface temperature. Potential temperature is defined by \( \theta = T / \Pi \), where \( \Pi = (\rho / \rho_0)^{\kappa} \) is a reduced pressure variable. The subscript zero indicates a surface value and \( \kappa = R / C_p \). We expand all variables as \( \Pi = \Pi_0 + \Pi' \), where \( \Pi_0 \) is the value of \( \Pi \) in an adiabatic stratification and \( \Pi' \), the departure from the adiabatic value, is assumed to be small everywhere. The mean adiabatic values are

\[
\begin{align*}
\theta_a &= T_0, \quad \Pi_a = 1 - \tau, \quad T_a = T_0 (1 - \tau), \\
\rho_a &= \rho_0 (1 - \tau)^{\kappa}, \quad \rho_a = \left[ \rho_0 / (RT_0) \right] (1 - \tau)^{(1/\kappa) - 1}.
\end{align*}
\]

The quasi-Boussinesq equations corresponding to (4)-(7) are as follows: the thermal equation\(^2\)

\[
\begin{align*}
\frac{\partial \theta'}{\partial t} &= -\frac{(\theta' \sin \alpha)_{\alpha}}{a \sin \alpha} - \frac{(\theta' \rho_a)_{\alpha}}{\rho_a} a^2 \sin \alpha + \frac{q_v}{c_p \rho_a \Pi_a},
\end{align*}
\]

where \( q_v \) is the radiative energy absorbed per unit volume per unit time; and the vorticity equation\(^3\)

\[
\begin{align*}
\frac{\partial \eta}{\partial t} &= -\frac{(\eta \sin \alpha)_{\alpha}}{a \sin \alpha} + \frac{g}{a^{\nu \eta}} \frac{\partial \theta'}{\partial x} \\
&\quad + \left[ \frac{\nu \eta}{a^2} \left( \frac{(\eta \sin \alpha)_{\alpha}}{\sin \alpha} - \eta \right) \right] + \frac{\nu \eta}{\rho_a} \frac{\partial^2 \eta}{\partial z^2}.
\end{align*}
\]

The mass streamfunction is defined by

\[
\eta_{\alpha} \sin^2 \alpha = (\psi_{\alpha} / \rho_a)_{\alpha},
\]

so that

\[
v \sin \alpha = \psi_z, \quad w \sin \alpha = -\psi_\alpha / a.
\]

b. Radiative transfer

We assumed that the atmosphere is semi-gray, i.e., constant absorption coefficients but with different values for solar and thermal radiation. In what follows, the subscripts \( T \) and \( S \) refer to the thermal and solar radiation respectively, and a superscripted asterisk indicates the value of a variable at the surface level.

1) Longwave radiation

The net upward flux by thermal radiation is given (Goody and Gierasch, 1970) by

\[
F_T(\tau) = \int_{\tau}^{\tau*} \sigma T^4(t) \exp[-\Gamma(t - \tau)] \Gamma \, dt
+ \sigma T^4 * \exp[-\Gamma(\tau^* - \tau)]
- \int_{0}^{\tau*} \sigma T^4(t) \exp[\Gamma(t - \tau)] \Gamma \, dt.
\]

Here \( \sigma \) is the Stefan-Boltzmann constant, \( \Gamma = 1.66 \) is a diffusivity factor that compensates for the neglect of the angular dependence of the radiation field, and \( \tau^* = f_{T*} k_T dz' \) is the thermal optical depth. We assumed that \( k_T \), the volume absorption coefficient, is proportional to the mean density below the cloud top level, and zero above, so that the optical thickness between two heights is proportional to the pressure differences.

The thermal radiation was separated into two parts, \( F_{T*} \) due to the mean adiabatic stratification, which was computed only once, and \( F_T' \) due to the small departures of the temperature from its mean adiabatic value, which had to be computed along with the other variables as the time integration went on. For the perturbation we assumed that \( T' = \Pi_0 \theta' \). This is not totally consistent with the quasi-Boussinesq approximation, but was numerically justified by the results. We also assumed that due to the large thermal optical depth of Venus, only the contiguous layers gave a significant contribution to \( F_T' \) at any level. This is not very accurate near the top. With these approximations, a linear expression for \( F_T' \) in terms of \( \theta' \) and \( d\theta'/dz \) was obtained. (Káltnay de Rivas, 1971).

2) Shortwave radiation

The volume absorption coefficient in the shortwave region of the spectrum was also assumed to be proportional to the mean adiabatic density and zero above \( z = H \). The downgoing flux of solar radiation was given by

\[
F_S(\tau) = \begin{cases} 
0, & \text{if } 0 \leq \alpha \leq \pi / 2 \\
-(1 - \delta_S \cos \alpha)^{s / \cos \alpha}, & \text{if } \pi / 2 < \alpha \leq \pi
\end{cases}
\]

3) Heating rate

\[
q_v = \partial (F_S - F_T') / \partial z.
\]

c. Other characteristics

The mechanical boundary conditions are similar to those used in the Boussinesq model: a no-stress rigid top at the cloud top level \( H \) at which \( \zeta_H = H / D < 1 \), and a non-slip rigid bottom. We assumed that there is no turbulent transfer of heat at the top (since radiation

\[\text{---}\]

\(^2\) It would have been more correct to write the vertical diffusion and viscosity terms as

\[
\frac{k_v \partial}{\partial \zeta} \left( \rho_a \frac{\partial \theta'}{\partial \zeta} \right) \quad \text{and} \quad \nu_v \frac{\partial}{\partial \zeta} \left( \frac{1}{\rho_a} \frac{\partial}{\partial \zeta} (\rho_a \eta) \right)
\]

respectively.
is considered as a separate term), so that $\theta_s = 0$ at $z = H$. At the bottom we assumed an instantaneous balance between the downward solar radiation and the upgoing thermal radiation and convective heat transfer (parameterized as $-\kappa_T \rho C_p \theta'_s$). The finite-difference scheme was similar to the one used in the Boussinesq models.

Table 5. Numerical values of the physical parameters used in the non-rotating quasi-Boussinesq model.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H$</td>
<td>$5.3 \times 10^4$ cm</td>
</tr>
<tr>
<td>$g$</td>
<td>$850$ cm sec$^{-2}$</td>
</tr>
<tr>
<td>$C_p$</td>
<td>$8.5 \times 10^8$ cm sec$^{-1}$ (°K)$^{-1}$</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>$0.224$</td>
</tr>
<tr>
<td>$\rho_0$</td>
<td>$100$ atm</td>
</tr>
<tr>
<td>$\tau^*$</td>
<td>$14$</td>
</tr>
<tr>
<td>$\tau^*$</td>
<td>$222$</td>
</tr>
<tr>
<td>$\nu_0$</td>
<td>$10^4$ cm$^2$ sec$^{-1}$</td>
</tr>
<tr>
<td>$\kappa_0$</td>
<td>$10^4$ cm$^2$ sec$^{-1}$</td>
</tr>
</tbody>
</table>

Stretched coordinates with 13 grid intervals were used both in the horizontal and vertical directions.

The numerical values of the physical parameters used in this model are given in Table 5. The specific heat was taken as constant with a value corresponding to a temperature of $300$ K, which introduces an error of the order of $10\%$. The temperature at the top was taken as $200$ K; and by trial and error a value for the optical thickness of the atmosphere in the longwave range was found such that under adiabatic stratification, the effective radiative temperature was $250$ K, thus balancing the shortwave incoming radiation. The value obtained ($\tau^* = 222$) is high, but not unreasonably so, and not large enough to allow the greenhouse effect to maintain the observed high surface temperatures [see Gierasch and Goody (1970) and our own radiative equilibrium computations in Section 5]. The values of the coefficients of viscosity and diffusivity are the same as those used in the non-rotating Boussinesq model.

\textit{d. Results}

Fig. 5a shows the heating rate $q_v/\sum_{i=0}^{\infty} \Pi_i \theta_i$ in an adiabatic stratification. The cooling rate due to longwave radiation is such that thermal relaxation times are of the order of several Venus solar days at the top, and more than a hundred Venus solar days in the interior.

The model was run for $1.5 \times 10^7$ sec of simulated time, corresponding to approximately 1.5 Venus solar days. At that time, the model had converged to equilibrium in the upper atmosphere but not in the lower atmosphere. The most striking result, as shown in Fig. 5b, is that the direct Hadley cell driven by differential heating has not penetrated into the lower atmosphere but remained confined to the upper third of the atmosphere. The smaller and weaker cells below are probably frictionally driven and correspond to horizontal velocities of less than 1 m sec$^{-1}$ at middle levels and 1 cm sec$^{-1}$ at low levels. The maximum velocity in the upper boundary layer is 30 m sec$^{-1}$, which is roughly twice that in the Boussinesq model. However, owing to the density stratification, the return flow occupies a much narrower layer than the direct flow, with a maximum of 6 m sec$^{-1}$. The vertical velocity field is similar to the one obtained with the Boussinesq model, except in its confinement to the upper atmosphere. The maximum downward velocity is 44 cm sec$^{-1}$ compared to 60 cm sec$^{-1}$ in the Boussinesq case. Again downward motion occurs not only in the mixing region,
but over a region only slightly less than half of the atmosphere. The relative potential temperature departure from the mean value, $\theta / \theta_0$, is shown in Fig. 5c. In the upper 20 km the atmosphere is almost neutrally stable, or slightly unstable, due to the strong circulation. The maximum horizontal temperature difference between the subsolar and the antisolar point is 2%, or 4K. This is much smaller than the 23K obtained in the Boussinesq model. Between 26 and 32 km, where the weak reverse cell occurs, there is a region with strong static stability ($\theta_s \approx 1.5K \ km^{-1}$). In the lower atmosphere the stratification is only slightly stable ($\theta_s \approx 0.3K \ km^{-1}$) but this is due to the fact that radiative processes have not acted long enough to produce a more isothermal stratification.

The numerical balance of terms in the vorticity and thermal equations is similar to the one in the Boussinesq model only in the upper atmosphere. In the lower atmosphere nonlinear terms cease to be important except perhaps at the antisolar point.

5. Quasi-Boussinesq model with rotation

In this section we describe a numerical model in which, as in Section 3, we assumed that the solar heating is symmetric about the axis of rotation ("toroidal sun") and rotation is included. However, in this case the effect of a basic near-adiabatic stratification of density, pressure and temperature was introduced. As we noted earlier, the large thermal capacity of the atmosphere of Venus, as well as the marked zonal structure of the temperature and cloud distribution, make this model more realistic than that of a fixed planet and sun.

Rotating coordinates were used as in Section 3, $\alpha$ being the co-latitude. The dynamic equations were the same as (11)–(14) except that rotational terms were added to Eq. (12) and the quasi-Boussinesq equation of zonal momentum equivalent to (10) was also included. Solar radiation was numerically averaged over one day, but otherwise the treatment of radiation was the same as in Section 4.
For this model we used a stretched vertical coordinate and a regular horizontal grid, since from the results obtained from the Boussinesq model with rotation, we did not expect a narrow boundary layer at the pole. We used 28 grid points in the horizontal, and 13 in the vertical. A conservative finite-difference scheme was designed for the nonlinear terms of the vorticity equation. For this purpose it was necessary to obtain the finite-difference continuity equation valid at the points where $\eta$ was located (Fig. 1). This was done making a weighted average of the finite-difference continuity equation valid at the center of the four neighboring cells, the weights being proportional to the mass contained in these cells. The finite-difference continuity equation obtained in this way, without cancelling any terms, was used according to Bryan's (1966) method to design the conservative scheme. A conservative scheme for nonlinear terms in the zonal momentum equation was designed in a similar way.

This model was run with four different specifications for the numerical values of the coefficients of eddy viscosity and diffusivity and of the solar optical depth. The other physical parameters were the same as in Table 4.

### a. Run I

Values of the eddy coefficients and optical depth were as follows: $\nu_H = \kappa = 10^{10}$ cm$^2$ sec$^{-1}$; $\nu_V = \kappa = 10^4$ cm$^2$ sec$^{-1}$ (the values suggested by Goody and Robinson); $r_S = 55$, so as to have the maximum heating contrast near the top (Fig. 6a).

The model was run during the equivalent of $4 \times 10^7$ sec. At this time the velocities and temperatures had converged in the upper atmosphere, but not in the lower atmosphere where time variations were small but significant compared to other terms in the equations. The direct Hadley cell has again remained largely confined to the top of the atmosphere (Fig. 6). The indirect cell near the pole is present but is weaker than in the Boussinesq model, perhaps due to less numerical resolution near the pole. The maximum zonal and meridional velocities in the upper boundary layer are 19 and 12 m sec$^{-1}$ compared with 14 and 10 m sec$^{-1}$, respectively, in the Boussinesq model. The return meridional flow has a maximum of only 60 cm sec$^{-1}$ at a depth of 6 km from the top. The downward jet is located 8° from the pole with a maximum speed of 28 cm sec$^{-1}$. In the interior the meridional velocities are of the order of a few cm sec$^{-1}$ and the zonal velocities are even smaller. The temperature contrast between equator and pole at the top is 1.1%, or about 2K. In the interior there is no horizontal contrast of potential temperature.

When a comparison of the numerical values of the different terms in the prognostic equations is made, it may be seen that nonlinear terms corresponding to large-scale advection of momentum and temperature are negligible in the interior and near the bottom. In particular, the balance in the thermal equation tends to be radiative-diffusive, not advective, except near the top. Furthermore, the system has not reached a steady state in the deep atmosphere since the relaxation time is radiative and much larger than 10$^5$ sec (at least 10$^3$ sec).

In the interior, radiation tends to produce a strongly stable lapse rate so that the use of a vertical coefficient of eddy diffusivity as large as $10^4$ cm$^2$ sec$^{-1}$ is not entirely justified. One dimensional calculations were made to obtain the vertical radiative-diffusive equilibrium state of the atmosphere, with radiation treated in the same linearized fashion as in the two-dimensional model. It was observed, that within this approximation, $\kappa_V = 10^{4}$ cm$^2$ sec$^{-1}$ forced the lower atmosphere to remain adiabatically stratified, with the potential temperature near its original value of 730K. The result with $\kappa_V = 10^{4}$ cm$^2$ sec$^{-1}$ was about the same as with no diffusion ($\kappa_V = 0$) and the surface temperature was 25% lower than the initial value so that the linearization procedure was no longer valid. When the time integration is carried out for much longer than 10$^6$ sec, the original radiative transfer equations should be used, as we have done at the end of this section.
b. Run II

As in Run I except that \( \nu = \kappa V = 10^4 \) cm sec\(^{-1}\). The meridional mass streamfunction obtained after \( 3.6 \times 10^7 \) sec is shown in Fig. 7a. The circulation in the upper atmosphere is almost identical to that obtained in Run I, the main difference being in the interior where the circulation is stronger with horizontal velocities of the order of \( 10 \) cm sec\(^{-1}\) instead of \( 1 \) cm sec\(^{-1}\). The balance of terms in the vorticity equation shows that nonlinear terms are still negligible both in the interior and near the bottom. In the thermal equation they are of the same order as the vertical diffusion terms, but smaller than the radiative terms.

c. Run III

As in Run I except that \( \nu_h = \kappa V = 10^{11} \) cm\(^2\) sec\(^{-1}\). This computation was made for the purpose of comparison, since even \( \nu = 10^{10} \) cm\(^2\) sec\(^{-1}\) is probably excessive, especially in the interior. The meridional streamfunction field obtained after \( 1.8 \times 10^9 \) sec is shown in Fig. 7b. The result is similar to Run I except that the circulation is slowed down by the large horizontal viscosity. The maximum zonal and meridional velocities are only \( 2.0 \) and \( 4.6 \) m sec\(^{-1}\), respectively. The counter cell near the pole is not present because before a ring of air reaches the pole, horizontal eddy viscosity has dissipated most of its angular momentum. If we compare the streamlines with those obtained in the non-rotating case (Fig. 5b), we see that the circulation is more confined to the upper atmosphere, probably because of the smaller penetration of solar radiation. In addition, we see that the upper part of the direct cell is the most compressed because of the stabilizing effect of relative rotation.

d. Run IV

As in Run I except that \( \tau_s = 2.3 \), corresponding to \( 6\% \) of the daily averaged solar radiation reaching the surface at the equator. In this run we allowed a deeper penetration of solar radiation, and consequently, a deeper circulation. In the upper atmosphere the resulting potential temperature stratification was unstable. The use of the hydrostatic approximation in a numerical model makes it necessary to include some mechanism to simulate the turbulent upward transport of heat that occurs in an unstable stratification (Phillips, 1970). Unfortunately, our model did not contain such a mechanism so that a "noodling effect" appeared in the upper half of the atmosphere, corresponding to "grid-size convection cells". Nevertheless, from the results after running for \( 2 \times 10^7 \) sec, a conclusion could be drawn, namely, that both the zonal and meridional velocities in the interior and lower atmosphere were much stronger than in previous runs (several m sec\(^{-1}\)). In the thermal equation nonlinear terms were everywhere much larger than radiation terms, so that such a deeply driven circulation is able to maintain a near-adiabatic lapse rate throughout the atmosphere.

e. Energy budget

From the dynamic equations, the following energy equations may be deduced (Kálñay de Rivas, 1971):

\[
\begin{align*}
\frac{\partial K_z}{\partial t} &= CT + UT - D_z = \{K_M, K_z\} - D_z \\
\frac{\partial K_M}{\partial t} &= \{P, K_M\} - CT - UT - D_M \\
\frac{\partial P}{\partial t} &= G - \{P, K_M\}
\end{align*}
\]

where \( K_z \), \( K_M \) and \( P \) are the zonal kinetic, the meridional kinetic, and the potential energies, respectively; \( D_z \) and \( D_M \) are the dissipation terms of zonal and meridional kinetic energy; and \( G \) represents the generation of both available and non-available potential energy due to non-adiabatic effects (radiation and diffusion). The term \( \{P, K_M\} \) represents conversion from potential to meridional kinetic energy, and the analogous conversion term \( \{K_M, K_z\} \) has one part due to planetary rotation (Coriolis term) and one due to the relative rotation (\( \nu \) term).
Table 6. Surface temperature (°K) for radiative equilibrium in a semi-gray atmosphere. Solar radiation is assumed to be normal and equal to $F_s = S_0 (1 - A)/4$.

<table>
<thead>
<tr>
<th>$\tau_s^*$</th>
<th>1.0</th>
<th>2.3</th>
<th>4.6</th>
<th>14</th>
<th>55</th>
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<tr>
<td>20</td>
<td>371</td>
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<td>50</td>
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<td>222</td>
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<td>661</td>
<td>565</td>
<td>415</td>
<td>271</td>
</tr>
<tr>
<td>500</td>
<td>909</td>
<td>805</td>
<td>685</td>
<td>495</td>
<td>300</td>
</tr>
</tbody>
</table>

Fig. 8 shows the energy budget of the model atmosphere after Run I. This is typical of the first three runs. If the model circulation had converged we should have observed the following equalities:

$$G = \{P, K_M\}$$

$$\{P, K_M\} = \{K_m, K_Z\} + D_M$$

$$\{K_M, K_Z\} = D_Z$$

Of these, the last two are satisfied within 0.1%. The first one is very far from being satisfied because of the tendency of the interior to reach radiative equilibrium with a much larger relaxation time than the time for which the integrations were made. In Run IV the last two equations were not satisfied, but the values of $G$ and $\{P, K_M\}$ were of the same order, indicating that radiation was not able to generate much unavailable potential energy in the interior. The kinetic energy was 100 times larger than the previous runs, because the more deeply driven circulation was able to stir the whole atmosphere.

f. Radiative equilibrium computations

The results of the quasi-Boussinesq models suggest that if solar radiation is absorbed near the cloud top level, the balance in the interior will be mainly radiative and the atmosphere will tend to become isothermal. In this case, the quasi-Boussinesq approximation and the linearized treatment of radiation become inaccurate.

To find what the radiative balance would be if large-scale convection and turbulent diffusion were negligible, a fast method to evaluate radiative fluxes in a semi-gray atmosphere was developed (Appendix). Table 6 shows the surface temperatures at radiative equilibrium computed for some combinations of $\tau_s^*$ and $\tau_f^*$, using 20 pressure intervals. A combination of $\tau_s^* = 50$ and $\tau_f^* = 222$, similar to the values used in Runs I–III, gives a radiative equilibrium surface temperature of 271K. If large-scale circulation were included, radiation would still tend to make the lower atmosphere very stable and the surface temperature would have a much lower value than the observed 747±20K. The combination $\tau_s^* = 2.3$, $\tau_f^* = 222$ used in Run IV gives a surface temperature of 661K without any correction for the convection that would result from superadiabatic conditions. Even if more than 90% of the solar radiation is absorbed before it reaches the surface, there is a considerable greenhouse effect, although not enough to explain the high surface temperature. However, the results of Run IV show that in this case the combined effect of a deep circulation which penetrates to the ground and a partial greenhouse effect can explain the adiabatic lapse rate and therefore the high surface temperatures of Venus.

6. Summary and conclusions

The behavior of two-dimensional models of the atmosphere of Venus has been studied. The results of the non-rotating Boussinesq model show an overall agreement with those of Stone and of Goody and Robinson. An asymmetric cell developed with a strong current at the top directed from the subsolar to the antisolar points and a strong downward jet at the antisolar point. However, weak downward motion occurred over most of the dark hemisphere, not only in the mixing region. The convergence time was given by the interior advective time $t_{int} \approx 10^9$ sec (1 Venus solar day). This shows that the approximation in which the subsolar point is assumed to be fixed is not really justified, and even less so in the quasi-Boussinesq model in which the interior velocities are much smaller. In the Boussinesq model with rotation and symmetric solar heating the result obtained is a Hadley cell in each hemisphere and a small indirect cell near the poles. These indirect cells occur because the rings of air near the top tend to increase the speed of rotation as they move poleward until the centrifugal force urges them back before reaching the pole. Angular momentum is transported poleward in the upper atmosphere with the result that in most of the atmosphere there is a shear of the zonal momentum in the same direction as the planetary rotation. In both Boussinesq models the circulation penetrates the atmosphere and the return flow has its maximum intensity near the bottom. In the interior the stratification remains neutrally stable, but this is not really significant, for radiation is not taken into account except for the influx and outflux of heat at the top, and it is the vertical eddy diffusivity, which is probably too large, that tends to maintain a neutrally stable lapse rate.

In the quasi-Boussinesq models with solar radiation absorbed near the top of the atmosphere, the results obtained were similar to those for the Boussinesq models. The main difference was that during the $4 \times 10^7$ sec of simulated time, the stratification of density confined the circulation to the upper part of the atmosphere. The circulation in the interior was weak and time variations were small but there was no convergence because the relaxation time was radiative ($\sim 10^9$ sec). The balance in the thermal equation was radiative-diffusive when a value of $\kappa_T = 10^6$ cm$^2$ sec$^{-1}$ was used, and radiative-advective when $\kappa_T = 10^6$ cm$^2$ sec$^{-1}$. We cannot be sure that on a much longer time scale, the large-scale circulation will not be able to penetrate the
lower atmosphere, but our results suggest that if solar radiation does not penetrate into the lower atmosphere, longwave radiation will tend to make it highly stable.

When some penetration of solar radiation was allowed in the quasi-Boussinesq model with rotation, the circulation was able to stir the interior of the atmosphere, and it was found that the combination of a deep large-scale circulation and some greenhouse effect can maintain an adiabatic lapse rate. 2 The effect of increasing the horizontal coefficient of eddy viscosity was to slow down the circulation, especially the zonal velocities. With \( \kappa_H = 10^{10} \text{ cm}^2 \text{ sec}^{-1} \) the maximum relative rotation was found to be 18 m sec\(^{-1} \), whereas it was only 2 m sec\(^{-1} \) when \( \kappa_H = 10^{11} \text{ cm}^2 \text{ sec}^{-1} \). Smaller (and probably more realistic) values of \( \kappa_H \) would allow larger zonal velocities, perhaps of the order of 100 m sec\(^{-1} \), although not near the equator.

It was found that planetary rotation has a considerable effect upon the circulation even though the rotation period is very long. In the interior the relative velocities are very small so that the Rossby number is \( \ll 1 \), or \( \lesssim 1 \) in the case of a deep driven circulation. Near the top, it is the relative rotation that is important.

A possible explanation of the 4-day circulation at high levels may be a combination of symmetrical heating introduced in Sections 3 and 5, and Thompson's idea that an initial zonal flow can amplify beyond the linear limit by Reynolds stress interaction with the tilted thermally-induced disturbances. Let us assume that the main effect of the sun is to produce a direct Hadley cell in each hemisphere, and a corresponding shear of the zonal momentum with maximum retrograde velocities at the cloud top level as in our quasi-Boussinesq model with rotation. The heating contrast between the day and night hemispheres will be important only near or above the cloud top level, where the radiative relaxation time is smaller than 1 Venus solar day. There, as Thompson has suggested, the basic shear of the zonal velocities will tilt the convective cells in such a manner as to increase the shear and produce even stronger zonal velocities at the top. Although the zonal shear produced by the Hadley cells is a minimum at the equator, the heating contrast is maximum. Both effects may thus combine to give an appearance of near-solid rotation. Furthermore, the UV cloud pictures show that the motion is actually three-dimensional, with the bifurcated horizontal \( \Psi \) patterns suggesting that there is eddy transport of retrograde zonal momentum from middle to low latitudes.

Sasamori (1971) has also made calculations of the circulation of the atmosphere of Venus using a two-dimensional version of the NCAR general circulation model (Kasahara and Washington, 1967) and a careful treatment of radiation. When the sun was assumed to be fixed, a Hadley circulation was obtained but no boundary layers were observed due to the lack of sufficient resolution. The circulation penetrated the atmosphere, probably because there was some penetration of solar radiation. The most striking result was the observation of strong oscillations with a period of 4-5 earth days, which according to Sasamori, are gravity waves. We think that the oscillations are due to insufficient horizontal resolution, a problem we also encountered in several computations. In Sasamori's computations, when the horizontal length was increased from 180° to 360°, and presumably lower resolution was used, the oscillations became several times stronger. When the sun was allowed to move along the equator assuming uniformity in the meridional direction, no time lag was observed in the response of the atmosphere. This is probably due to the presence of the short-period oscillations.

A Soviet group (Turikov and Chalikov, 1971) made the first three-dimensional computation of the Venus circulation with a two-layer global model. The results obtained correspond to a bipolar circulation between the morning and evening terminators, which are the coldest and warmest points in their model. Unfortunately, planetary rotation was not taken into account, and the insufficient resolution, especially in the vertical direction, makes it impossible to test the validity of previous models with their results.

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APPENDIX

A Simple Method to Compute Radiation in a Semi-Gray Atmosphere

In a gray atmosphere, where there is a constant absorption coefficient (Gierasch and Goody, 1970;
Goody, 1964), the upgoing and downgoing thermal radiation fluxes are, respectively

\[
\begin{align*}
\uparrow F_T(\tau_T) &= \int_{\tau_T}^{\tau_T^*} \sigma T^4(\tau') \exp[-\Gamma(\tau'-\tau_T)] d\tau' \\
&\quad + \sigma T^4 \exp[-\Gamma(\tau_T^*-\tau_T)] (A1)
\end{align*}
\]

\[
\downarrow F_T(\tau_T) = \int_0^{\tau_T} \sigma T^4(\tau') \exp[\Gamma(\tau'-\tau_T)] d\tau' 
\]

We may assume that the volume absorption coefficient is proportional to density, so that the thermal optical depth is proportional to pressure, i.e.,

\[
\tau_T/\tau_T^* = p/p^*. \quad (A2)
\]

Eqs. (A1) and (A2) can be integrated if temperature is known in terms of pressure, but if the atmosphere has a large optical depth, truncation errors due to the rapid variation of the exponentials will make the results inaccurate. This problem can be solved in the following way. Divide the atmosphere into \( N \) pressure layers and assume that the temperature varies linearly with pressure within each layer, i.e., at each layer set

\[
T = T_n + (T_{n+1} - T_n)(p - p_n)/\Delta p. \quad (A3)
\]

Using (A2) and (A3) we can integrate (A1) directly and obtain

\[
\begin{align*}
\uparrow F_T(N\Delta p) &= \sigma T N^4 \\
\uparrow F_T(n\Delta p) &= \uparrow F_T((n+1)\Delta p) e^{-\alpha} \\
&\quad + \sigma \left(T_n, T_{n+1} - T_n/\alpha\right), \\
&\text{for } n = N-1, N-2, \ldots, 0 \\
\downarrow F_T(0) &= 0 \\
\downarrow F_T(n\Delta p) &= \downarrow F_T((n-1)\Delta p) e^{-\alpha} \\
&\quad + \sigma \left(T_n, T_{n-1} - T_n/\alpha\right), \\
&\text{for } n = 1, 2, \ldots, N
\end{align*}
\]

where

\[
\alpha = \Gamma \Delta \tau_T = \Gamma (\tau_T^*/p^*) \Delta p,
\]

\[
f(T, \beta) = \sum_{i=0}^{i=N} \left( \frac{4}{i} \right) \mu C_i T^4 - i \beta, \\
C_0 = 1 - e^{-\alpha}, \\
C_i = C_{i-1} + \frac{\alpha}{i} e^{-\alpha},
\]

The radiative equilibrium computations in Section 5 were made using the procedure described here, with a semi-gray assumption, i.e., with the solar volume absorption coefficient also proportional to density, so that

\[
\downarrow F_S(\tau_S) = \downarrow F_S(0) e^{-\tau_S}, \quad (A5)
\]

where \( \tau_S = \rho \tau_S^*/p^* \) is the effective solar optical depth. The boundary condition at the bottom was taken as \( \uparrow F_T = \downarrow F_T^* + \downarrow F_S \) and the hydrostatic equation was used to express the heating rate as

\[
\frac{\partial T}{\partial t} = -\frac{g}{c_p} \frac{\partial (\downarrow F_S + \downarrow F_T - \uparrow F_T)}{\partial p}. \quad (A6)
\]

This method can be easily adapted to the use of non-constant pressure intervals.

REFERENCES


