A Numerical Study of the Nature of the Glaciation Process

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ABSTRACT

A model is presented which simulates the glaciation of a cloud. In this model both vapor transfer and accretion processes are computed, the latter being treated stochastically. Two model clouds have been examined, one a modified continental cloud and the other a maritime cloud. These clouds have been seeded with ice crystal concentrations of 0.5, 10 and 500 liter^{-1}. In all numerical experiments the resulting shape of the ice particle spectrum was a function of the initial water drop spectrum. At the lowest ice crystal concentration the rate of formation of ice in the clouds is relatively slow.

The importance of the vapor transfer processes in the calculation is discussed and a comparison is made between this model and one that uses a continuous formulation for the accretion processes.

From the point of view of the production of precipitation-size particles it appears that seeding a cloud with 10 ice crystals per liter would be the most productive of the three ice crystal concentrations tested. For the assumed cloud liquid water content of $\sim$1 gm m^{-3} it is unlikely that even the largest seedings used in the experiments would initiate marked dynamical changes in the cloud by the glaciation process.

1. Statement of the problem

To simulate completely the microphysical processes in a glaciating cloud would be a complex problem, even if the physics of all the processes were known. Apart from the dynamical effects there are at least three important microphysical processes simultaneously taking place in a glaciating supercooled cloud: 1) the change in the water drop and ice hydrometeor spectra arising from the transport of vapor in the cloud; 2) the change in the water drop spectrum resulting from the coalescence processes; and 3) the modification of the ice hydrometeor spectrum resulting from the collision of supercooled water drops and ice particles.

Many of the features of ice crystal growth from vapor can be successfully modelled (Koenig, 1971; Jayaweera, 1971). The microphysics of the coalescence of water drops is relatively well developed and the necessity of using a stochastic model when considering the coalescence of small drops has been clearly demonstrated (Telford, 1955; Twomey, 1964, 1966; Berry, 1967). The collision efficiencies of various sizes of drops have been computed theoretically and measured experimentally.

However, there has been relatively little work concerned with the interaction of water drops and ice crystals. The work that has been done is primarily directed to the riming process, whereby a large ice particle is accreting small water drops (Ludlam, 1958; Macklin, 1961; Bailey and Macklin, 1968). These authors have concerned themselves with a continuous model of the process, which is a reasonable approximation for the riming of large ice particles. The dependence of this approximation on the distribution of supercooled water drops in a cloud is discussed elsewhere (Ryan, 1972). Koenig (1966) is one of the few investigators to consider seriously the interaction of small drops and small ice particles. For simplicity he used a continuous model in his numerical experiment and assumed that the errors arising from this approximation would not be significant; however, he emphasized that this assumption needed to be tested. Danielsen *et al.* (1972) have published a detailed model of hailstone growth in a cumulonimbus cloud. The model is used to determine the ice and hail distribution in the cloud, and the radar reflectivities are computed and compared with observations. An important feature of this model is that it is the first to incorporate the stochastic collection of water drops by the ice particles in the cloud. A rapid computational technique developed by Bleck (1970) is used to solve the coalescence equations for both the ice/water and water/water interactions. However, the complexity of the model of Danielson *et al.* makes it difficult to separate the contributions of the separate microphysical and dynamical processes incorporated in the numerical calculations.

In this present paper the stochastic equations describing the interaction of waterdrops and ice crystals are discussed. A numerical model for the glaciation of clouds is developed that simulates the ice-hydrometeor/waterdrop interactions and the vapor exchange between the ice particles and the waterdrop spectrum. The model is used to study the evolution of the ice-hydrometeor spectrum in two supercooled clouds that have markedly different waterdrop spectra.

2. The coalescence equations

If a supercooled water cloud contains N_w drops per unit volume, the cloud waterdrop spectrum is defined by

$$N_w=\int_0^\infty n_w(\sigma)d\sigma, \quad (1)$$

where $n_w(\sigma)d\sigma$ is the number of drops per unit volume in the radius interval between σ and $\sigma+d\sigma$.

In the present formulation ice hydrometeors in the cloud are considered to fall into two categories, namely "ice crystals" and "ice pellets." An ice crystal is defined as a hydrometeor that has grown only from the vapor phase and has not taken part in any collisions between ice crystals and waterdrops. An ice pellet is defined as an ice hydrometeor that has undergone at least one ice-crystal/waterdrop collision. The total number of ice hydrometeors per unit volume, N_I, is defined by

$$N_I=\int_0^\infty n_c(\phi)d\phi+\int_0^\infty n_d(\rho)d\rho, \quad (2)$$

where $n_c(\phi)d\phi$ is the number of ice crystals per unit volume having radii between ϕ and $\phi+d\phi$, and $n_d(\rho)d\rho$ is the number of ice pellets per unit volume in the interval having radii between ρ and $\rho+d\rho$. If there is no secondary source of ice particles and if all the ice nuclei form ice crystals simultaneously, then

$$\frac{dN_I}{dt}=0. \quad (3)$$

Once the initial ice crystal spectrum has been introduced the cloud begins to glaciate. There is a decrease in the initial ice crystal spectrum given by

$$\frac{dn_c(\phi)}{dt}=-n_c(\phi)\int_0^\infty K_c(\phi,\sigma)n_w(\sigma)d\sigma, \quad (4)$$

where $K_c(\phi,\sigma)$ is the coagulation coefficient for waterdrops and ice crystals and is given by

$$K_c(\phi,\sigma)=\begin{cases}\pi\phi^2E(\phi,\sigma)[u(\phi)-u(\sigma)], & \phi\geq\sigma\\ \pi\sigma^2E(\phi,\sigma)[u(\sigma)-u(\phi)], & \sigma>\phi\end{cases} \quad (5)$$

where ϕ and σ are the radii of the ice crystal and water drop, $u(\phi)$ and $u(\sigma)$ the respective fall velocities, and $E(\phi,\sigma)$ the collection efficiency of the interaction. The coagulation coefficient for the interaction between an ice pellet and a waterdrop, $K_d(\rho,\sigma)$, is similarly defined.

The change in the waterdrop spectrum per unit volume is given by

$$\frac{dn_w(\sigma)}{dt}=-n_w(\sigma)\int_0^\infty K_c(\phi,\sigma)n_c(\phi)d\phi -n_w(\sigma)\int_0^\infty K_d(\rho,\sigma)n_d(\rho)d\rho. \quad (6)$$

The integral equation expressing the change in the ice pellet spectrum can be derived using the mathematical techniques demonstrated by Twomey (1964). The rate of change of the number of ice pellets in the size range from ρ to $\rho+d\rho$ is given by

$$\begin{aligned}\frac{d}{dt}n_d(\rho)=&\int_0^\rho n_w(\sigma)K_c[(\rho^3-\sigma^3)^{\frac{1}{3}},\sigma]\\ &\times n_c[(\rho^3-\sigma^3)^{\frac{1}{3}}](1-\sigma^3/\rho^3)^{-\frac{2}{3}}d\sigma\\ &+\int_0^\rho n_w(\sigma)K_d[(\rho^3-\sigma^3)^{\frac{1}{3}},\sigma]\\ &\times n_d[(\rho^3-\sigma^3)^{\frac{1}{3}}](1-\sigma^3/\rho^3)^{-\frac{2}{3}}d\sigma\\ &-n_d(\rho)\int_0^\infty K_d(\rho,\sigma)n_w(\sigma)d\sigma. \quad (7)\end{aligned}$$

In this equation the first integral on the right-hand side expresses the number of the ice pellets of radius ρ that are formed as a result of the collision of an ice crystal with a waterdrop. In the present work, the ice crystals are of uniform size, and therefore the function $n_c[(\rho^3-\sigma^3)^{\frac{1}{3}}]$ has the properties of a delta function. The second integral expresses the number of ice pellets of radius ρ arising from the collision of an ice pellet and a waterdrop. The last integral expresses the decrease in the population of the drops of radius ρ resulting from their collision with a water drop.

The forms of the integrals are identical to those derived for the coalescence of waterdrops. However, for the waterdrop/waterdrop interaction there is a factor of $\frac{1}{2}$ in the integral defining the rate of increase of the droplets. This factor arises because each drop is counted twice. In addition, in the case of the coalescence of waterdrops the gain integrand is symmetrical. This is not so for the two gain integrands in (7).

Frequently it is convenient to use the hydrometeor distribution as a function of mass rather than function of radius. Eqs. (4), (6) and (7) then become

$$\left.\begin{aligned}\frac{dn_c(p)}{dt}=&-n_c(p)\int_0^\infty K_c(p,q)n_w(q)dq\\ \frac{dn_w(q)}{dt}=&-n_w(q)\int_0^\infty K_c(p,q)n_c(p)dp\\ &-n_w(q)\int_0^\infty K_d(s,q)n_d(s)ds\\ \frac{dn_d(s)}{dt}=&\int_0^s n_w(q)K_c(s-q,\,q)n_c(s-q)dq\\ &+\int_0^s n_w(q)K_d(s-q,\,q)n_d(s-q)dq\\ &-n_d(s)\int_0^\infty K_d(s,q)n_w(q)dq\end{aligned}\right\}, \quad (8)$$

where p is the mass of an ice crystal, q the mass of the waterdrop, and s the mass of the frozen drop.

3. The cloud glaciation model

a. *Assumptions*

In the development of any model that examines the microphysical processes taking place during the glaciation of a cloud it is necessary to include several simplifying assumptions.

In the present model the two most important assumptions are that the cloud is isothermal and that there are no updrafts in it. These two assumptions imply that there is no additional release of water vapor in the cloud. Although they are not generally valid it should be noted that in the middle regions of a cloud observations of liquid water suggest that it does not vary rapidly with height. The constant-temperature assumption also implies a constant difference between the vapor pressure over water and over ice. Near the temperature of −10C for which the computations have been made the vapor pressure over water and ice is a slowly varying function of temperature; consequently, the change in temperature which would occur in a real cloud for moderate changes in height would not produce a sharp change in the vapor pressure difference. Another advantage in the choice of −10C for the model is that the use of a spherical shape factor is a reasonable approximation for growth by vapor diffusion.

Changes in the waterdrop spectrum resulting from the coalescence of waterdrops have been ignored in this model. Large drops play an important role in the glaciation process and consequently the suppression of the continual creation of large drops by the cloud reduces the rate of glaciation.

The ice crystals are introduced into the cloud simultaneously and they are all of uniform size. This assumption permits no time dependence in the nucleation of ice crystals. In principle there is no difficulty in using a non-uniform spectrum of ice crystals; alternatively, ice crystals can be introduced as a function of time. The simpler system of initiating the glaciation process has been chosen because of the current uncertainty in the sources of ice crystals in clouds.

Finally, there is no explicit mechanism by which hydrometeors fall out of the cloud, although implicitly they do so upon exceeding the maximum radius size used in the computations.

b. *Numerical procedure*

The numerical procedure in the model is first to compute the changes in the ice-hydrometeor and waterdrop spectra arising from the coalescence of solid and liquid particles and then to compute the vapor exchange between the solid and liquid hydrometeors.

The changes in the hydrometeor spectra are evaluated by solving the integrals in (8) by a trapezoidal quadrature method. In order to solve these equations numerically it is necessary to determine the appropriate collection kernels using (5). In the literature there are virtually no experimental or theoretical determinations of the collection efficiencies between solid and liquid hydrometeors. For the present numerical experiments it has been assumed that all the ice/water collection efficiencies are identical to those for two waterdrops of the equivalent radii. The values used here have been based on a combination of the values obtained by Hocking (1959), Shafrir and Neiburger (1963) and Mason (1971). The second unknown parameter in (5) is the difference in fall velocities between solid and liquid hydrometeors. The fall velocity of the waterdrop can be simply deduced, but the computation of the fall velocity of an ice hydrometeor is much more difficult. Several parameters would need to be specified, such as the density and shape and the relationship between the Reynolds number and drag coefficient. Since it is not feasible to specify most of these parameters, it has been assumed that both ice crystals and ice pellets are spherical and that both have the bulk density of ice. The collection kernels were stored in a 29×29 matrix.

The ice pellet and waterdrop spectra were stored for 51 discrete points. Initially the radius intervals were $2\,\mu$m between 0 and $20\,\mu$m and $5\,\mu$m between 20 and $225\,\mu$m. The maximum radius of $225\,\mu$m was chosen for reasons relating to computing time and storage. Care was taken in choosing a computing time interval so that significant errors arising from multiple collisions could be avoided: it was found that an interval of 1 sec was satisfactory. On completion of this section of the computation a check was made on the total number of ice hydrometeors. Failure to find conservation of the number of ice hydrometeors could arise from either the numerical procedures used or from ice pellets growing to more than $225\,\mu$m radius and moving outside the range of the calculation. This latter effect would produce a net loss to the system. In the present experiments it was not possible to differentiate between these two causes, and therefore conservation of the number of ice hydrometeors was forced at each stage. This was achieved by uniformly adjusting the ice pellet spectrum. The deviation from conservation before it was forced generally varied from 1% to 5% and mostly indicated a loss of hydrometeors. The problem of the loss of particles from the system will be discussed later.

In the second section of the computation the vapor exchange between the solid and the liquid hydrometeors is calculated. Here it is assumed that the total increase in the mass of ice due to vapor deposition is balanced by the loss of water by the evaporation of waterdrops. The ice pellet and waterdrop distributions are transformed into their cumulative forms. (The ice crystal spectrum is already formulated in this manner.) The radii at the various discrete points in the cumulative distribution are modified according to

$$\delta r_i = D(\rho_e - \rho_i)[1+0.3(\mathrm{Re})^{\frac{1}{2}}]/(r_i d_i), \quad (9)$$

$$\delta r_w = D(\rho_e - \rho_w)[1+0.3(\mathrm{Re})^{\frac{1}{2}}]/(r_w d_w), \quad (10)$$

where δr_i and δr_w are the increase in radii of the ice particles and waterdrops, respectively, D is the diffusion coefficient of water vapor in air, ρ_e the vapor density of the environment, ρ_i and ρ_w the saturation vapor densities over ice and over water, Re the Reynolds number, r_i and r_w the radii of the ice and water hydrometeors, and d_i and d_w the density of ice and water. If temperature and pressure are fixed, ρ_i and ρ_w are known. Eqs. (9) and (10) can be evaluated if ρ_e can be found. By assuming vapor conservation, and a constant ρ_e in the period of time for the hydrometeor to grow from r to $r+\delta r$, and by neglecting curvature effects in the vapor density over the various hydrometeors, the vapor density of the environment can be simply calculated. With the new radii the cumulative distribution is changed back to its original form. The new distribution was tested to check that mass had been conserved. In general, the numerical procedure introduced a discrepancy of about 1% between the mass lost from the waterdrops and the mass gained by the ice hydrometeors. However, in the first 50 sec of glaciation of a cloud with a broad-sized drop spectrum the error was greater than this. Mass conservation was forced on the system to prevent the error from becoming cumulative. This was achieved by uniformly adjusting the water drop spectrum.

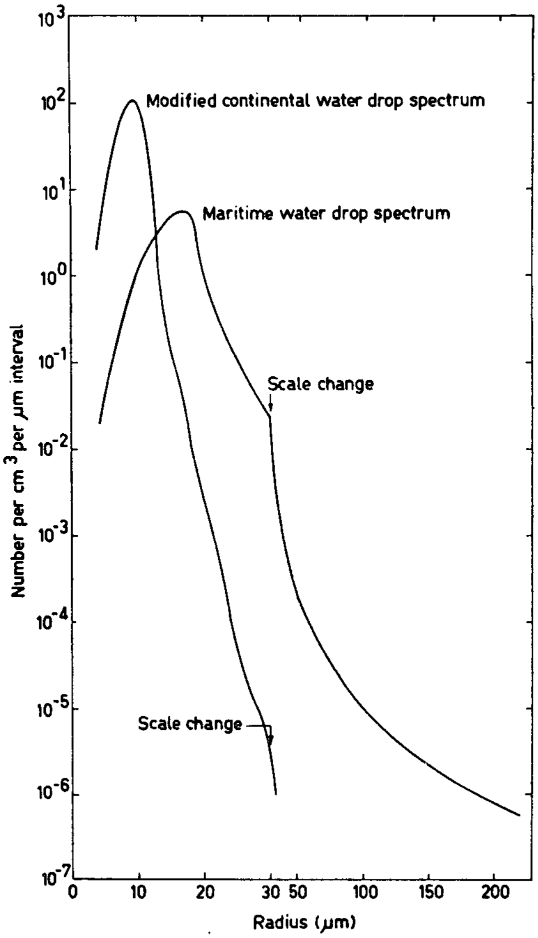

FIG. 1. The size distribution of the waterdrop spectrum for the maritime and modified continental clouds.

4. The numerical experiment

Two series of numerical experiments have been carried out in which ice crystals (10 μm radius) were introduced into a supercooled cloud at a temperature of −10C and 750 mb pressure. In the first series of experiments a modified continental cloud containing 460 drops cm^{-3}, a liquid water content of 1.5 gm m^{-3}, and a dispersion of 0.18 was seeded with 0.5 and 500 ice crystals $liter^{-1}$. In the second series of experiments a maritime cloud containing 43 drops cm^{-3}, a liquid water content of 0.8 gm m^{-3}, and a dispersion of 0.22 was seeded with 0.5, 10 and 500 $liter^{-1}$. Both cloud drop spectra were taken from field data collected by S. C. Mossop and are shown in Fig. 1.

An experiment was conducted using the maritime cloud from which vapor growth was excluded and a comparison experiment was run to compare the glaciation rate as predicted by the present stochastic model and that predicted by a continuous glaciation model. In the context of this paper the glaciation rate is considered to be the rate of increase in the mass of ice in the cloud. In the continuous glaciation model the cloud is seeded with the appropriate concentration of ice crystals. These crystals are permitted to grow both by vapor deposition and by accretion. The rate of increase of mass collected by one of these ice pellets as a result of the accretion process is given by

$$\frac{dM(R)}{dt} = \int_0^\infty K(R,r)n(r)X(r)dr, \quad (12)$$

where $K(R,r)$ defines the volume per second swept by an ice crystal of size R collecting water drops of size r and numerically equal to the coagulation coefficient, $n(r)$ is the waterdrop distribution function, and $X(r)$ the mass of a waterdrop of size r. It should be noted that in this model all of the ice pellets in the cloud grow at the same rate and thus the size distribution of the ice particles in the cloud is always uniform.

The numerical experiments have been conducted on a CDC 3600 computer and the programs are coded in Fortran IV. An experiment using the stochastic glaciation model and following the life of the cloud for 600 sec requires approximately 90 min of computing time.

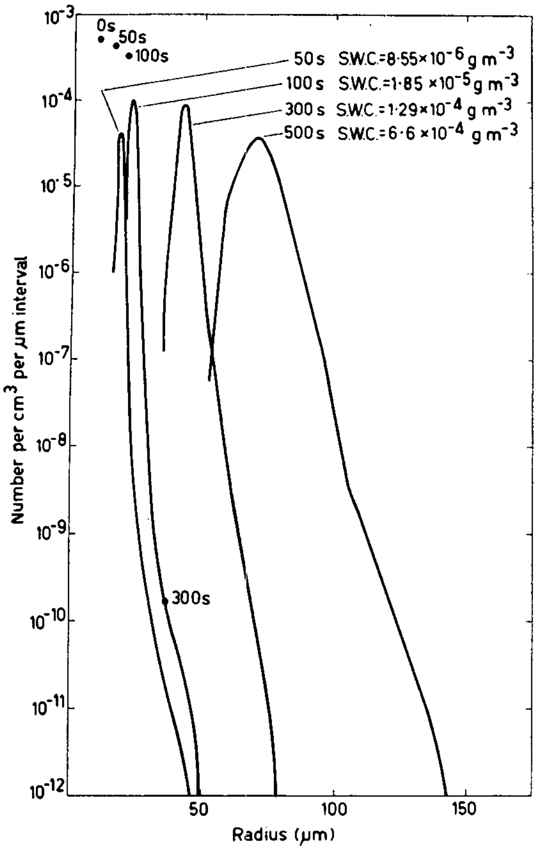

FIG. 2. The size distribution of the ice pellet spectra and the solid water contents (SWC) for the modified continental cloud after 50, 100, 300 and 500 sec: the filled circles show the number of ice crystals after 0, 50, 100 and 300 sec.

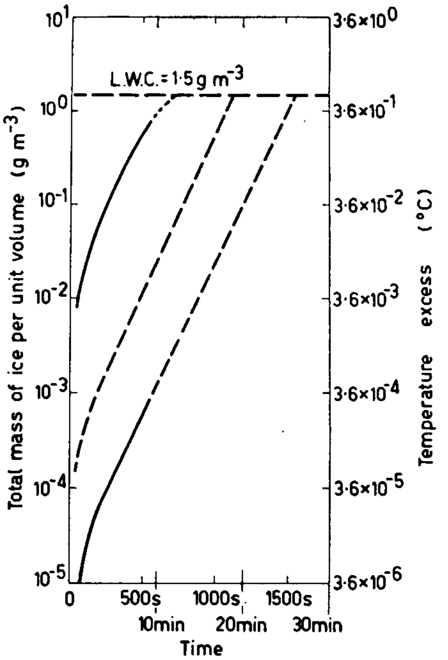

FIG. 3. The total mass of ice and the temperature excess in the continental cloud as a function of time for ice crystal seedings of 0.5, 10 and 500 liter^{-1}. Note that the liquid water content of the cloud is 1.5 gm m^{-3}.

5. Results

a. The modified continental cloud

Fig. 2 shows the variation in the ice pellet spectrum during the first 500 sec in the life of the modified continental cloud when seeded with 0.5 ice crystal liter^{-1}. Also shown in the figure is the change in the number of ice crystals and the increase in the mass of ice in the cloud. After 300 sec more than 99% of these have taken part in at least one collision with a waterdrop. The ice pellet spectrum is continually broadening and this is a direct consequence of the stochastic equation. With an initial concentration of 0.5 liter^{-1} there is in the first 500 sec of growth very little mass of ice formed in the cloud and a negligible change in the size distribution of the waterdrops. In the time of the experiment a negligible number of ice particles with radii $>150\,\mu$m are produced.

When the same cloud is seeded with 500 ice crystals liter^{-1} then 0.5 gm m^{-3} of ice is produced in the first 500 sec after seeding and there is a corresponding reduction in the liquid water content from 1.5 to 1 gm m^{-3}. The total number of cloud drops present drops from 460 to 370 cm^{-3} but the shape of the waterdrop spectrum is not greatly changed. A negligible number of ice particles with radii $>150\,\mu$m are produced.

The mass of ice in the cloud for these two ice crystal concentrations is shown as a function of time in Fig. 3

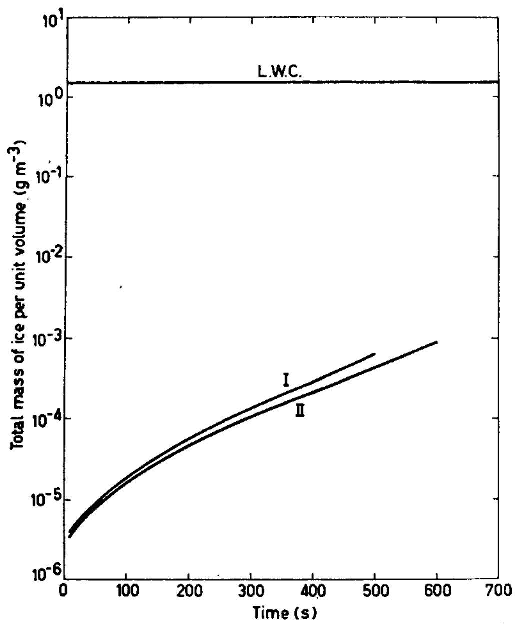

FIG. 4. The mass of ice (and liquid water content) in the modified continental cloud as a function of time. Curve I refers to the calculation using the stochastic equations, curve II to the calculation using the continuous equations.

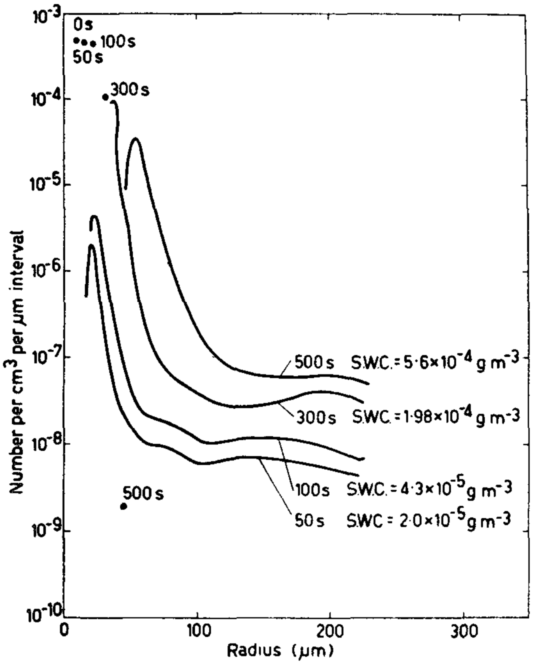

FIG. 5. The size distribution of the ice pellet spectra and the solid water content for the maritime cloud after 50, 100, 300 and 500 sec: the filled circles show the number of ice crystals after 0, 50, 100, 300 and 500 sec.

and an estimate of the glaciation time has been made by extrapolating these curves. Included in the figure is an interpolated curve for a concentration of 10 ice crystals liter^{-1}. The glaciation times when the cloud is seeded with 0.5, 10 and 500 liter^{-1} are of the order of

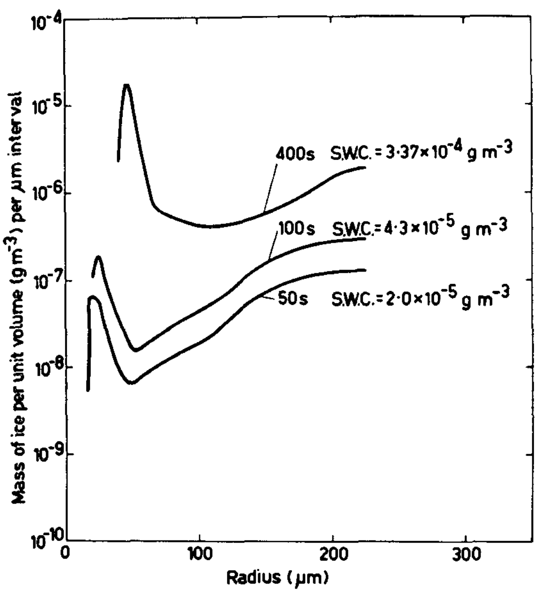

FIG. 6. The mass distribution of the ice pellet spectra and the solid water content for the maritime cloud after 50, 100 and 400 sec.

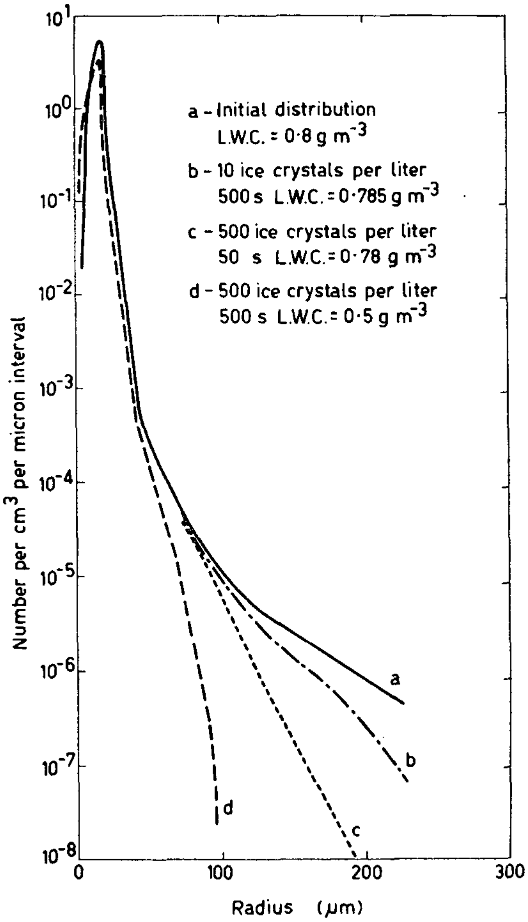

FIG. 7. The liquid waterdrop spectra and the liquid water content of the maritime cloud with 10 ice crystals liter^{-1} 500 sec after seeding, and with 500 ice crystals liter^{-1} at 50 and 500 sec after seeding.

25, 19 and 12 min, respectively. Fig. 3 also shows the temperature excess that the latent heat of fusion releases into the cloud. Fig. 4 shows the difference in the mass of ice predicted by the model when the continuous growth equation is used to describe the accretion process.

b. The maritime cloud

Fig. 5 shows the variation in the ice pellet spectrum during the first 500 sec in the life of the maritime cloud when seeded with 0.5 liter.$^{-1}$ Also shown in the diagram is the decrease in the initial ice crystal spectrum with time and the increase in the mass of ice in the cloud. After the first 300 sec more than 60% of the initial ice crystals have undergone at least one collision with a water drop. There are noticeably more ice crystals remaining than in the modified continental cloud. The ice pellet spectrum rapidly assumes a rather broad aspect. The contribution to the total mass of ice from the large ice pellets is significant (Fig. 6). However, the total mass

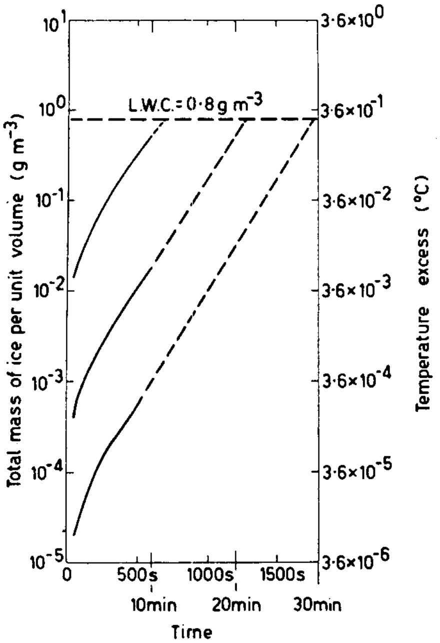

FIG. 8. The total mass of ice and the temperature excess in the maritime cloud as a function of time for an ice crystal seeding of 0.5, 10 and 500 liter^{-1}. Note that the liquid water content of the cloud is 0.8 gm m^{-3}.

of water in the cloud has not been appreciably reduced and there has been a negligible change in the distribution of sizes in the waterdrop spectrum.

The form of the ice pellet spectrum and rate of decrease in the number of ice crystals when the maritime cloud is seeded with either 10 or 500 liter^{-1} are similar to those shown for the smaller concentration of 0.5 liter^{-1}. After 500 sec the cloud with 10 liter^{-1} contains 1.1×10^{-2} gm m^{-3} while the cloud with 500 liter^{-1} contains 0.3 gm m^{-3} of ice and is nearly 40% glaciated. Fig. 7 shows the changes in the waterdrop spectrum for these two seedings. When the cloud is seeded with 10 liter^{-1} then at a time 500 sec after seeding there is a definite change in the tail of the waterdrop spectrum but a negligible change in the number of drops with radii $<75\,\mu$m. When the cloud is seeded with 500 liter^{-1} the tail of the water drop spectrum has been significantly changed in only 50 sec and there is a noticeable change in the whole spectrum after 500 sec.

Fig. 8 shows the mass of ice in the maritime cloud as a function of time for the three different ice crystal concentrations. Included in this figure is the temperature excess produced by the release of latent heat of fusion into the cloud.

In Fig. 9 a comparison has been made between the mass of ice in the cloud as predicted by the stochastic and continuous models. In this experiment the mass of ice predicted by the continuous formulation exceeds that predicted by the stochastic formulation. There is an inconsistency between these results and those derived using the modified continental spectrum. In this case the imposition of a relatively small maximum radius (initially 225 μm) in the stochastic model probably leads to significant removal of mass from the system by the creation of ice pellets with radii $>225\,\mu$m. The computation has been repeated using 10 μm radius steps for ice pellets with radii $>50\,\mu$m. The increase in the size of the steps permits the maximum radius to rise to 420 μm for the 51 steps allowed for in the computations. The mass of ice present in the cloud during the first 150 sec of its life is shown in Fig. 9, where it is seen that the mass of ice predicted by the stochastic equations now exceeds that predicted by the continuous equations.

A consequence of increasing the size of the steps is the development of an oscillation in the tail of the distribution. In addition to this, at a radius of $\sim100\,\mu$m, the distribution curve begins to have radii less than the corresponding values computed using the smaller radius steps. Both of these features are believed to be a consequence of using too coarse a radius step in the computation.

A computation using the maritime cloud has been run in which all vapor transfer processes have been suppressed. A compression between this and the complete computation is shown in Fig. 10 at a time 100 sec after seeding. It is seen that the contribution from the vapor transfer processes is significant, at least until the ice particle radii $>175\,\mu$m. For larger radii the difference is not so marked.

6. Discussion

The stochastic equations used in the model imply that the coalescence processes are self-accelerating, as Elton *et al.* (1958) and Twomey (1966) have shown for the coalescence of waterdrops in a warm cloud. The statistics of the process increase both the mean radius and the dispersion in the size distribution of the waterdrops. Twomey (1964) has compared the changes in the size distribution of the waterdrops in two numerical experiments, one using a continuous formulation and the other a stochastic formulation. The stochastic equations permit the rapid growth of a few specially favored drops whereas the continuous formulation makes no allowance for such events. The author has previously pointed out that the continuous equations used in modeling the glaciation of a cloud are subject to the same limitations. In the glaciation process there are two sources of favored ice pellets. The first is the collision of a small ice crystal with a large waterdrop and the second is the collision of a medium-sized ice pellet with a medium-sized drop. The first of these processes is important in the early stages of glaciation if there is a significant number of large waterdrops present. A comparison between the distribution of ice pellets formed in the two model clouds used in the present experiments shows that the waterdrop spectrum has a marked influence on the subsequent ice-hydrometeor spectrum.

Another deficiency in the continuous formulation is that it seriously underestimates the mass of ice in the cloud and hence predicts an excessive glaciation time for the cloud. The experiments using the maritime spectrum show that the rate of glaciation in the cloud can be underestimated even when the stochastic equations are used (Fig. 9). However, this is a limitation resulting from the numerical techniques used in solving the stochastic equations and it occurs when a significant proportion of the mass of ice in the cloud is contained in particles having a radius close to the maximum radius used in the calculation. Effectively, the cloud is precipitating all ice pellets that form with a radius larger than the maximum radius. In the present experiments the use of a maximum initial radius of 225 μm is unrealistic.

The experiments in which the maritime cloud has been glaciated with and without the vapor transfer process operating show that mass growth by vapor deposition contributes significantly to the mass of ice in the cloud. The experiments suggest that the contribution to the distribution from the vapor is important at least until the ice particles reach 175 μm in radius (Fig. 10). The model ignores the warming of the ice pellet surface by the heat of fusion from the accretion process and by the heat of sublimation from the vapor deposition process. The continuous accretion model was used to show that this assumption was unlikely to produce gross errors in the range of ice pellet sizes and cloud drop distributions used in the numerical experiments. In the maritime cloud, when an ice pellet is

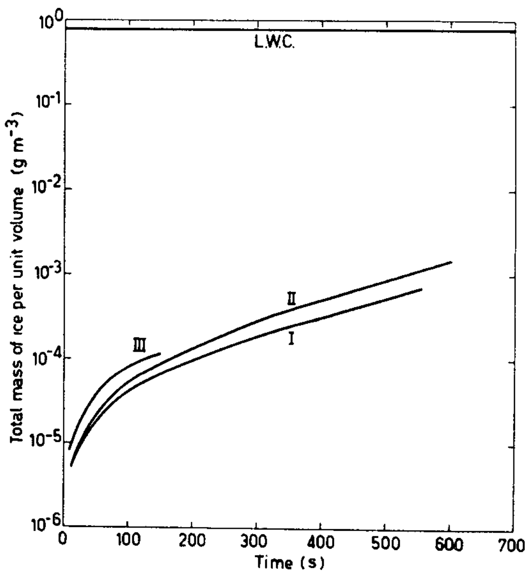

FIG. 9. The mass of ice in the maritime cloud as a function of time. Curve I refers to the calculation using the stochastic equations and using a maximum radius ice pellet of 225 μm. Curve II refers to the calculation using the continuous equations. Curve III refers to the calculation using the stochastic equations and using a maximum radius ice pellet of 420 μm.

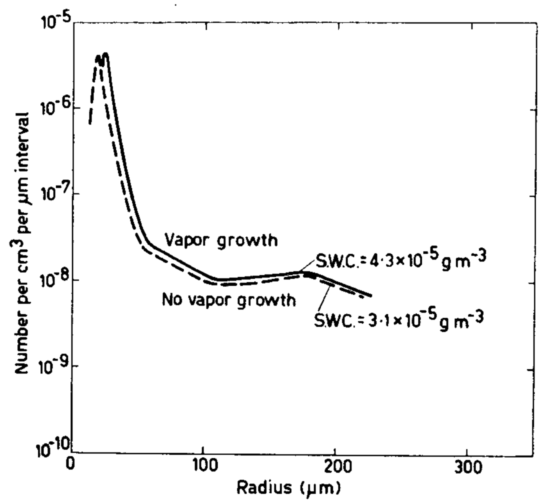

FIG. 10. The ice pellet size distribution after 100 sec when the maritime cloud is seeded with 0.5 ice crystal liter^{-1}. The full line shows the distribution when vapor diffusion is included in the calculation; the dashed line shows the distribution when vapor diffusion is excluded from the calculation. The respective solid water contents in the cloud are also included.

50 μm radius the surface temperature is −9.84C and the ratio of the mass of ice deposited by vapor growth to the mass of ice deposited by accretion is 0.63; at 100 and 200 μm radius the ratio is 0.15 and 0.067 respectively. Similar results apply to the continental cloud.

The experiments using both the modified continental spectrum and the maritime spectrum show that if the cloud contains a number of ice crystals similar to the number of natural ice nuclei active at −10C, the glaciation rate is very slow. Linear extrapolation of Figs. 3 and 8 suggest that both of these clouds take at least 25 min to glaciate completely. The conclusions are consistent with the results obtained by Koenig (1966).

The effect of increasing the number of ice crystals in the continental cloud from 0.5 to 500 liter^{-1} is to reduce the time for complete glaciation by a factor of 2. The initial rate of glaciation decreases as the cloud becomes more nearly glaciated. In the model the equations describing both vapor diffusion and accretion are linear with respect to the number of ice particles in the cloud. If the waterdrop spectrum remained invariant the total mass of ice in the cloud would be a linear function of the number of ice particles in the cloud. In the first 300 sec after seeding the continental cloud with both 500 and 0.5 liter^{-1} this is a reasonable approximation, and Fig. 3 shows that the mass of ice in the cloud at the higher concentration of ice crystals is very nearly 1000 times greater than that at the lower ice crystal concentration. At later times the change in the waterdrop spectrum becomes evident and the ratio of 1000 between the two curves in Fig. 3 is gradually reduced. From this relationship between the mass of ice and the number of ice particles in the cloud, a glaciation time of 19 min has been deduced for the continental cloud when seeded with 10 liter^{-1} (see Fig. 3).

The results of the simulation experiments using the maritime cloud provide an interesting contrast with those from the continental cloud. With the various ice crystal seedings of 0.5, 10 and 500 liter^{-1} the respective estimated glaciation times for the maritime cloud are 30, 21 and 11 min. However, the computational techniques used in the numerical simulation experiments produce an implicit loss in all ice particles with radii $>225\,\mu$m; consequently, the glaciation times can only be considered as an upper limit. It seems likely the glaciation times in the maritime cloud would be less than those in the continental cloud, since the maritime cloud has only a little more than half the liquid water content of the continental cloud.

In all three computations using the maritime waterdrop spectrum the presence of large drops had a dominant effect on the evolution of the ice pellet spectrum. The rapid removal of the large waterdrops is clearly demonstrated in Fig. 7 for the ice crystal seedings of 10 and 500 liter^{-1}. With the seeding of 10 liter^{-1} only the tail of the waterdrop spectrum has been noticeably modified 500 sec after seeding. In the same time and at the higher concentration of ice crystals in the cloud the waterdrop spectrum has experienced a general change in the number of waterdrops of all sizes. It should be noted that the large drops will tend to be regenerated by coalescence among cloud drops and consequently the number of large ice pellets in the cloud generated by frozen drops is likely to be greater than indicated in these numerical experiments.

Koenig (1966) and Cotton (1972) have each performed simulation experiments which can be compared with the present work. Both authors included in their glaciation model production of ice by vapor diffusion processes and by the collision of ice particles and waterdrops. In each case the authors used a "continuous" formulation to describe the formation of ice by the collision of ice particles and waterdrops. In addition, Cotton placed the microphysics of his model in a dynamical framework. The results obtained by Koenig and Cotton support the conclusion that the largest drops are rapidly removed from the waterdrop spectrum by conversion to ice pellets. Cotton points out that this implies that rapid glaciation initially promotes the formation of freezing rain. Koenig deduced from his experiments that the glaciation time was a function of the width of the waterdrop size distribution and the number of ice particles present. The present simulation experiments support the last conclusion, but the difference in the liquid water content of the continental and maritime clouds prevents a meaningful interpretation being placed on the effect of the distribution of waterdrops on the glaciation time.

Cotton (1972) showed that clouds that contain a relatively small amount of supercooled water are dynamically insensitive to moderate concentrations of ice crystals. In clouds of this type extensive riming and vapor deposition growth of ice particles in concentrations of several thousand per liter are required to make a substantial contribution to the dynamical structure of the cloud. Each cloud in the present experiments has only a relatively small mass of liquid water. Figs. 3 and 8 show the temperature excesses produced in these clouds as a function of time and ice crystal concentration. When the continental cloud and the maritime clouds are seeded with 500 ice crystals liter^{-1} the mean values of the rates of change of temperature for the first 600 sec after seeding are 5.5×10^{-4} and 3×10^{-4} C sec^{-1}, respectively. If this heating resulted in lifting which followed a moist adiabat of 7×10^{-3} C m^{-1}, the parcels would attain velocities of 7.8 and 4.3 cm sec^{-1}. In the absence of all other dynamic effects the two air parcels would rise a maximum of 47 and 25 m, respectively, in the first 10 min of growth. This somewhat crude analysis suggests that the temperature rise contributed by the latent heat of fusion alone is unlikely to modify significantly the dynamics of these clouds, and as such the numerical experiments are consistent with the assertion made by Cotton. Ice crystal concentrations of tens of thousands per liter would appear to be necessary to produce significant dynamic effects.

The simulation experiments provide some interesting insights into the optimum ice particle concentrations necessary to produce precipitation in the absence of marked dynamical modification of the clouds. At small concentrations of ice crystals it is possible that the cloud is unable to glaciate because all of the ice particles reach precipitation size before the cloud is glaciated. For example, consider the case of the continental cloud that has all of the ice particles precipitating upon reaching $500\,\mu$m radius. In 500 sec the modal radius of the ice pellet spectrum has grown to $70\,\mu$m. Using a continuous accretion model with vapor growth, we find that the time required for the modal radius of the ice pellet spectrum to reach $500\,\mu$m is about 20 min after seeding with 0.5 ice crystal liter^{-1}; the total mass of ice in the cloud is then 9×10^{-2} gm m^{-3}. Even when all the ice particles have grown to $500\,\mu$m radius only 0.23 gm m^{-3} of the liquid water is removed from the cloud. Seeding the continental cloud with 500 liter^{-1} completely glaciates the cloud in about 11 min. In this relatively short space of time the majority of ice particles are still smaller than precipitation size and unless ice crystal aggregation plays a significant role very little precipitation is likely to fall from the cloud. With the intermediate concentration of ice crystals the simulated glaciation time is comparable with the time taken for the modal radius of the ice pellet spectrum to reach $500\,\mu$m. Consequently, a relatively high percentage of the liquid water is likely to precipitate from the cloud.

When a similar analysis is applied to the maritime cloud for the three different seeding experiments the same basic conclusions are reached. It therefore appears from this rather simple analysis that when the concentration of ice crystals is 0.5 $liter^{-1}$, which is similar to the number of ice nuclei found at −10C, and when these clouds have liquid water contents $\geqslant 1$ gm m^{-3}, it is unlikely that more than 30% of the liquid water will precipitate as ice crystals. At a relatively high ice crystal concentration the clouds will most probably glaciate rapidly but very few ice crystals will reach precipitation size. At ice crystal concentrations of ~10 $liter^{-1}$ it appears that much of the liquid water can be precipitated from the cloud. The question that these simulation experiments is unable to answer is whether this concentration can be attained naturally or whether seeding can supplement nature in optimizing the number of ice crystals in the cloud.

The isothermal and zero updraft assumptions inherent in the model make these results most relevant to supercooled layer clouds at the −10C level. The experiments demonstrate that if the distribution of waterdrops and the number of ice crystals in these types of clouds are known then it is possible to estimate qualitatively whether or not any beneficial changes in the microstructure are likely to result from seeding. These numerical experiments are not applicable to cloud seeding experiments where dynamic growth is the process by which the precipitation is being induced.

There is a substantial body of field evidence to show that in a rapidly glaciating cumulus cloud there are many more ice crystals than ice nuclei. In relating the simple numerical glaciation experiments to the atmosphere this observation needs to be recognized. The numerical experiments, in fact, show that unless there are substantially more ice crystals than ice nuclei, clouds at −10C and of the type considered in the present experiments are unlikely to glaciate in under 25 min. At present, no convincing explanation for the physical nature of the ice multiplication process has come forth from either the laboratory or the field. However, field observations have produced some circumstantial evidence. In particular, Gagin (1971) and Mossop *et al.* (1972) have both presented evidence to show that in the absence of large supercooled water drops the glaciation process proceeds relatively slowly and that the numbers of the ice crystals in the cloud are not greatly in excess of the background ice nucleus count. The numerical experiments show that even when ice multiplication mechanisms are not considered the waterdrop spectrum has a marked influence on the subsequent ice-hydrometeor spectrum and, perhaps more significantly, it shows that large drops in relatively small concentrations contribute rapidly to the formation of ice in the cloud.

7. Conclusions

A model has been presented which simulates the glaciation of a cloud. Two model clouds have been examined, one a modified continental cloud and the other a maritime cloud. Both were considered to be at a temperature of −10C. When these clouds were numerically seeded with an ice crystal concentration of 0.5 $liter^{-1}$, which is typical of that observed in real clouds at −10C in the absence of multiplication processes, glaciation was found to proceed slowly.

The initial drop size distribution has a marked effect on the shape of the ice pellet spectrum; in particular, the presence of a relatively small concentration of large drops can contribute significantly to the mass of ice in the cloud.

The vapor transfer processes are important and cannot be ignored in the computation.

It has been confirmed that the continuous model underestimates the amount of ice in the cloud.

To optimize the production of precipitation size particles it appears that seeding a cloud with 10 ice crystals per liter would be the most effective of the three ice crystal concentrations tested. For the assumed cloud liquid water content of ~1 gm m^{-3} it is unlikely that even the largest seedings used in the present numerical experiments would initiate marked dynamical changes in the cloud by the glaciation process.

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