Synoptic Features and Energetics of Wave-Amplitude Vacillation in a Rotating, Differentially-Heated Fluid

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ABSTRACT

Wave-amplitude vacillation in a thermally driven, rotating cylindrical annulus of fluid is studied using temperature and speed data from a 98-probe network of tiny bead thermistors suspended in the fluid. Mid-depth synoptic charts and estimates of heat and momentum fluxes, spectral characteristics and energetics are presented as a function of time. Relationships to time-dependent atmospheric phenomena are discussed. A section is included which deals with the effects of probes.

1. Introduction

As a result of research over the past 25 years or so, there exists now a reasonably good understanding of the “steady state” or long-time-averaged general circulation of the earth’s atmosphere and related experimental circulations in the laboratory. The time-dependent behavior of both the atmosphere and the laboratory circulations, however, is not nearly as well understood. During the last several years the writers and their colleagues have been exploring time-dependent phenomena in thermally driven rotating fluids with applications to the atmospheric general circulation (Pfeffer and Chiang, 1967; Pfeffer and Fowlis, 1968; Fowlis and Pfeffer, 1969; Pfeffer et al., 1969, 1970; Fein, 1973; Fein and Pfeffer, 1973). Recognizing that experiments with small numbers of internal measurements furnish limited information and render analysis tedious at best, we have devoted considerable time and effort to the development and improvement of networks of tiny sensors to provide field distributions of fluid temperature and velocity for studying such phenomena quantitatively (Fowlis and Pfeffer, 1969; Fowlis, 1970; Fowlis et al., 1971, 1972; Buzyna et al., 1973). To date we have been encouraged by our successes in producing and measuring meteorologically significant flows in experiments with increasing densities of probes.

This paper describes a continuation of our efforts to employ multi-probe networks in laboratory experiments to define more precisely the behavioral characteristics of time-dependent meteorological flows as a function of the imposed conditions (thermal Rossby number, \( R_{\sigma} \); Taylor number, \( T_a \); Prandtl number, \( \text{Pr} \)). In particular, we shall focus attention here on “wave-amplitude vacillation,” a phenomenon first discussed by Pfeffer and Chiang (1967) and studied experimentally in greater detail by Fowlis and Pfeffer (1969) using a 48-probe network of thermocouples. Efforts to explain this phenomenon theoretically have been made by Drazin (1970), Pedlosky (1970, 1971) and Merilees (1972), the most realistic of which appears to be the viscous theory of Pedlosky (1971).

As in the case of earlier experiments, the working fluid in the present experiment was a differentially heated liquid contained in a circular cylindrical annulus, the axis of which was oriented vertically. The inner and outer cylindrical walls were good thermal conductors held at different uniform temperatures, with the outer wall maintained at a higher temperature than the inner wall in order to simulate tropical heating and arctic cooling. The base of the annulus and the upper lid (which was not in contact with the free surface of the liquid) were good thermal insulators. Amplitude vacillation in such a fluid is characterized by the periodic growth and decay of a baroclinic wave mode. This is accompanied by a periodic fluctuation in the zonally (azimuthally) averaged radial temperature gradient which increases when the waves are weak and decreases more rapidly when they are strong. Amplitude vacillation differs from tilted-trough vacillation by the absence of noticeable changes in the horizontal tilt of the waves with time.

Pfeffer and Chiang (1967) suggested that the main energy conversions responsible for amplitude vacillation are between zonal and eddy available potential energy (\( A_z \rightarrow A_e \)) and between eddy available potential and eddy kinetic energy (\( A_e \rightarrow K_e \)). Fluctuations in the direction of the angular momentum flux, and exchanges of kinetic energy between the waves and the mean zonal current, which appear to be important in tilted-trough
vacillation, seem to be of secondary importance here. In contrast to tilted-trough vacillation, which (in dimensionless parameter space) is found near the border between the regular wave regime and the irregular flow regime, Fowlis and Pfeffer (1969) found that amplitude vacillation is confined to a limited region of dimensionless parameter space near the border between the wave regime and the upper symmetric flow regime. Their investigation, utilizing 48 thermocouples, was limited in scope, however, by the fact that they had available only a six-channel pen recorder which they used to study the time variations of the zonally averaged fluid temperatures at six different radii. In the present investigation an array of 98 tiny bead thermistors was employed to measure the distribution of both fluid temperature and speed, mainly at mid-depth, and the data were recorded with a 100-channel digital data acquisition system. The annulus gap width was chosen to be three times larger than that used by Fowlis and Pfeffer to accommodate the larger number of probes with a minimum influence on the flow. In order to conduct experiments conveniently in the same range of Taylor numbers at which amplitude vacillation was observed by Fowlis and Pfeffer in water in a small annulus, a more viscous fluid (silicone fluid, kinematic viscosity 5 centistokes) was used in the larger annulus.

The data from the 98-probe network, together with streak photographs of the flow direction in the upper layers of the fluid, were used to exhibit synoptic features, calculate heat and momentum fluxes, and determine spectral characteristics and energetics of amplitude vacillation as a function of time. The results are presented in the following sections and the relationship between amplitude vacillation and atmospheric phenomena is discussed. A later section of the paper is devoted to a discussion of probe effects.

2. List of symbols and dimensionless parameters

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda )</td>
<td>azimuth angle measured counterclockwise</td>
</tr>
<tr>
<td>( R )</td>
<td>radius measured from axis of rotation</td>
</tr>
<tr>
<td>( z )</td>
<td>height above base of annulus</td>
</tr>
<tr>
<td>( t )</td>
<td>time</td>
</tr>
<tr>
<td>( a )</td>
<td>radius of inner cylinder (7.303 cm)</td>
</tr>
<tr>
<td>( b )</td>
<td>radius of outer cylinder (14.923 cm)</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>radial (poleward) eddy flux of heat per centimeter of fluid depth ( \left[ = -2\pi \rho c R^2 T' \right] )</td>
</tr>
<tr>
<td>( H )</td>
<td>depth of working fluid (15.0 cm)</td>
</tr>
<tr>
<td>( u )</td>
<td>azimuthal velocity measured positive in the counterclockwise direction</td>
</tr>
<tr>
<td>( v )</td>
<td>radial velocity measured positive outward</td>
</tr>
<tr>
<td>( w )</td>
<td>vertical velocity measured positive upward</td>
</tr>
<tr>
<td>( \rho )</td>
<td>fluid density at 25C (0.920 gm cm(^{-3}))</td>
</tr>
<tr>
<td>( T )</td>
<td>fluid temperature</td>
</tr>
<tr>
<td>( \Delta T )</td>
<td>imposed temperature difference between the inner and outer cylinders containing the working fluid (10.0C)</td>
</tr>
<tr>
<td>( \Omega )</td>
<td>rate of rotation of annulus (1.718 sec(^{-1}))</td>
</tr>
<tr>
<td>( g )</td>
<td>acceleration of gravity</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>coefficient of volume expansion of the fluid at 25C ( [1.05 \times 10^{-4} \text{ C}^{-1}] )</td>
</tr>
<tr>
<td>( c )</td>
<td>specific heat capacity of the fluid at 25C ( [0.347 \text{ cal gm}^{-1} \text{ C}^{-1}] )</td>
</tr>
<tr>
<td>( \nu )</td>
<td>coefficient of kinematic viscosity of the fluid at 25C ( (5.0 \times 10^{-5} \text{ cm}^2 \text{ sec}^{-1}) )</td>
</tr>
<tr>
<td>( \kappa )</td>
<td>coefficient of thermometric diffusivity of the fluid at 25C ( (8.8 \times 10^{-2} \text{ cm}^2 \text{ sec}^{-1}) )</td>
</tr>
<tr>
<td>( \tau )</td>
<td>radial (poleward) eddy flux of angular momentum ( \left[ = -2\pi \rho \nu \right] )</td>
</tr>
<tr>
<td>( Pr \equiv \nu / \kappa )</td>
<td>Prandtl number (57)</td>
</tr>
<tr>
<td>( Ro_T )</td>
<td>imposed thermal Rossby number (0.900)</td>
</tr>
<tr>
<td>( Ta )</td>
<td>Taylor number (1.6 x 10(^9))</td>
</tr>
<tr>
<td>( K,K_2,K_B )</td>
<td>total, zonal and eddy kinetic energies per unit mass at mid-depth in the fluid</td>
</tr>
<tr>
<td>( A,A_2,A_B )</td>
<td>total, zonal and eddy available potential energies per unit mass at mid-depth in the fluid</td>
</tr>
<tr>
<td>( A_2 \rightarrow K_2 )</td>
<td>rate of conversion from zonal available potential energy to zonal kinetic energy per unit mass at mid-depth in the fluid</td>
</tr>
<tr>
<td>( A_B \rightarrow K_B )</td>
<td>rate of conversion from eddy available potential energy to eddy kinetic energy per unit mass at mid-depth in the fluid</td>
</tr>
<tr>
<td>( A_2 \rightarrow A_B )</td>
<td>rate of conversion from zonal to eddy available potential energy per unit mass at mid-depth in the fluid</td>
</tr>
<tr>
<td>( K_B \rightarrow K_2 )</td>
<td>rate of conversion from eddy to zonal kinetic energy per unit mass at mid-depth in the fluid</td>
</tr>
<tr>
<td>( D_B,D_Z )</td>
<td>rate of dissipation of eddy kinetic and zonal kinetic energy</td>
</tr>
<tr>
<td>( G_B,G_Z )</td>
<td>rate of generation of eddy and zonal available potential energy</td>
</tr>
<tr>
<td>( \bar{q} )</td>
<td>azimuthal (&quot;zonal&quot;) average of quantity ( q ) at a specified radius (&quot;latitude&quot;)</td>
</tr>
<tr>
<td>( \bar{q} )</td>
<td>average of quantity ( q ) over the entire horizontal area of the fluid at a specified depth</td>
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</table>

3. Apparatus and procedures

The apparatus and procedures employed in the present and related investigations are described in detail by Fowlis (1970), Fowlis et al. (1972) and Pfeffer et al. (1962). Here we shall review only the information
The internal distribution of fluid speed and temperature was measured with an array of 98 thermistors supported by vertically-oriented leads. The array consisted of a distribution of 84 thermistors at mid-depth (7 equally spaced at each of 12 "latitude" circles, as shown in Fig. 1), and a distribution of 14 thermistors at mid-radius at two depths (7 equally spaced at 1 cm depth and 7 equally spaced at 14 cm depth) to give some measure of the vertical stratification of the fluid. The radii of the 12 latitude circles at which thermistors were located at mid-depth were 7.58, 7.87, 8.15, 9.00, 9.84, 10.69, 11.54, 12.38, 13.23, 14.08, 14.36 and 14.64 cm.

Each thermistor unit consisted of a thermistor bead 0.036 cm in diameter encased and sealed in glass such that its dimensions were between 0.04 and 0.05 cm in diameter and ~0.25 cm in length, with 0.0025 cm diameter tungsten wire leads extending from opposite ends. Each thermistor had a nominal electrical resistance of 2000 Ω and a temperature coefficient of $-3.3\%$ (°C)$^{-1}$ at 25°C. The thermistors were located in the fluid at desired heights and radii by suspending them by their long leads. The leads were copper plated and soldered to special top and bottom frames. The top frames bridged the annular gap above the fluid just below the glass cover. The bottom frames were flush with the base.

Silicone fluid was used as the working fluid in order to avoid electrolysis, which occurs in fluids with higher electrical conductivities (such as water) when uninsulated thermistor leads are used. The choice of bead thermistors was motivated by their small size and large temperature sensitivity (100 times greater than that of thermocouples) and the fact (Rasmussen, 1962) that a heated bead can be made sensitive to flow speed in the range of interest in thermally driven rotating annulus experiments.

The thermistors were wired into dc Wheatstone bridge networks. The out-of-balance voltages, the network bridge voltage, and the time of initiation of the measurements were scanned at a rate of 125 channels per second by a 100-channel digital data acquisition system and were recorded on magnetic tape. The time required for a complete set of readings was 0.8 sec, which is small in comparison with the time required for significant changes to occur in the flow field.

Flow direction in the working fluid was recorded by streak photography using a 35-mm remote-controlled motor-driven camera mounted on the turntable 82 cm above the annulus, concentric with the axis of rotation of the system. The streaks were formed by photographing a suspension of aluminum flakes in the interior of the fluid using a photographic exposure of 2 sec.

The digital data acquisition system, power supplies and camera were all interfaced with a digital timing unit which was programmed for the type and time of measurement to be made and/or photograph to be taken.
b. Experimental procedure

Since it is not possible to obtain simultaneous determinations of both the temperature and the speed of the fluid with a single thermistor, a technique had to be employed whereby the measurements of temperature and speed were separated in time. The procedure was as follows: The bridge voltage of the thermistor network was alternated from a low value (0.5 V) to a high value (5 V). (When the bridge voltage is low the thermistors are effectively sensitive to temperature alone; when it is high they are sensitive to both temperature and speed.) The bridge voltage was maintained at 0.5 V for 8.6 sec, and then stepped up to, and maintained at, 5 V for 6.0 sec. Subsequently it was stepped down again to 0.5 V, and so on, repeating the cycle. The period of each cycle, approximately 14.6 sec, was equal to four rotation periods of the turntable. The 8.6- and 6.0-sec durations of the low- and high-bridge voltages, respectively, were required to allow the thermistors to reach thermal equilibrium with their environment (within the accuracy of measurement) after each change in their heating state. During the last second of each step the out-of-balance bridge voltages of all thermistors were automatically scanned and recorded by the data system. At the high voltage step the command to scan also triggered a 2-sec photographic exposure of the flow field.

The time sequence of recorded out-of-balance bridge voltages (each record containing 98 thermistor readings obtained in 0.8 sec) was converted into a time sequence of temperatures and speeds by means of calibration experiments. Calibrations were performed with the thermistors in place in the annulus at several known constant ambient temperatures. The inner and outer baths were maintained at the same temperature and the fluid in the annulus was allowed to reach thermal equilibrium. The temperature calibration constants for individual thermistors were determined with the low-bridge voltage. The root-mean-square uncertainty of the temperature measurement, determined from the calibration results, was \( \pm 0.028 \) C in the temperature range 22.5–27.5 C. The speed calibration constants were determined at the high-bridge voltage in a constant temperature environment. The relationship between the rate of heat dissipation from a thermistor and the relative fluid speed was assumed to be linear over the speed range 0–1.0 cm sec\(^{-1}\). The dissipation rate at zero speed was determined for each thermistor individually, whereas the linear variation constant was determined from a "representative" thermistor and assumed to be the same for all thermistors. We estimate the resulting uncertainty of the speed measurement to be on the order of \( \pm 0.15 \) cm sec\(^{-1}\).

Conversion of voltage readings into speed obtained during the high-voltage scans requires knowledge of the ambient temperature as well as of the calibration constants. The ambient temperature during the high-voltage scan was determined by linear interpolation from the measured temperatures (low-voltage scans) preceding and following the high-voltage scan. The time variation of the temperature field was sufficiently slow for these interpolations to be acceptable. Thus, the final experimental data, suitable for analysis, was in the form of a time sequence of temperature fields obtained from primary measurements (low-voltage scans) every four rotation periods (\(~14.6\) sec) and a time sequence of speed fields and interpolated temperature fields 6 sec after each of the primary temperature fields.

Experimental data were taken for a half-hour beginning 2 hr after the start of the experiment. The experiment was begun by first establishing the desired temperature difference (\( \Delta T = 10.0\) C with the inner cold bath at 20.0 C) and then starting the rotation of the table (\( \Omega = 1.718 \) sec\(^{-1}\)). Based on visual observation, the fluid flow in the annulus was well established one-half hour after the start of rotation. The additional 1/2 hr allowed before data gathering insured that the flow was in a non-transient state relative to the externally imposed constant driving forces. The dimensionless parameters corresponding to the conditions of the experiment were: \( Pr = 57 \), \( Ta = 1.6 \times 10^7 \) and \( Ro_T = 0.900 \).

The basic data at mid-depth in the fluid consisted of a network of 84 temperature and speed measurements (Fig. 1). For the purpose of drawing synoptic temperature and speed charts, or obtaining temperature and speed data at locations other than those measured, it was necessary to construct a finer grid network of data by interpolating between the points of the basic network. Such operations were performed by a weighted least-squares polynomial fitting scheme (Pfeffer et al., 1962). Briefly, the scheme consisted of describing the temperature and speed fields by a second-degree polynomial surface in the vicinity of each point at which a value was desired. The six constants of the polynomial were evaluated from the nearest 12 measured values (thermistor locations) by weighted least-squares fitting in which the assigned weight was inversely proportional to the distance between the desired and measured data points. The basic 84-point data field was first expanded by interpolation to a 168-point field corresponding to the intersection of the 12 latitude circles with the 14 radial thermistor frames (see Fig. 1). The data field was then interpolated further to three equally-spaced points between each pair of radial frames on each of the 12 latitude circles. These interpolations provided a grid network of measured and interpolated values at 672 points (56 equally spaced points on each of the 12 latitude circles) at mid-depth of the fluid. The above network was used as a basis for drawing synoptic temperature and speed charts of the flow field. The coordinates of isotherm and isolatich lines were determined by two-dimensional linear interpolation from the 672-point network, and the results were displayed and photographed on a cathode ray tube plotter.
During the experiment five mid-depth thermistors malfunctioned. For completeness, the temperature and speed at these points were determined by the weighted least-squares procedure using the 12 nearest locations at which the thermistors were functioning properly.

Streak photographs of the flow field served both a qualitative and a quantitative function in the present experiment; a qualitative function by providing an overall view of the character of the flow field, and a quantitative function by providing specific information concerning flow direction (i.e., the streamline field) which was combined with the speed data to give flow velocity. Since the aluminum flakes were suspended in the interior of the fluid, the flow field represented by the streak photographs is some integral of the motion in the upper layers of the fluid. While such photographs have the disadvantage that they do not explicitly define the flow at a single depth, they do provide a good repre-

**Fig. 2. Upper left:** Streak photograph 125 of aluminum flake tracers depicting the flow in the upper part of the fluid 799.6 sec after the sequence of photographs was begun. The flow is in the counterclockwise direction. The 14 radial lines are frames which support 98 thermistors in the fluid. Date of experiment, 16 September 1969. **Upper right:** Mid-depth streamlines (solid lines) and isotherms (dashed lines). The isotherms, interpolated to the time of the photograph, are drawn for every 0.25C. The shaded region covers the temperature range 26.75 ≤ T ≤ 27.25C. **Lower left:** Mid-depth streamlines (thin solid lines) and isolachs (heavy solid lines). The isolachs are drawn for every 0.2 cm sec⁻¹ with shading as follows: clear, 0-0.2 cm sec⁻¹; light stippling, 0.2-0.4 cm sec⁻¹; solid black, 0.4-0.6 cm sec⁻¹; vertical lines, 0.6-0.8 cm sec⁻¹. At this stage of the amplitude-vacillation cycle the eddy available potential energy and the processes associated with the horizontal and vertical eddy fluxes of heat are near or at their minimum magnitudes (see lower right).
Photograph #128

\[ \max: A_Z, K_Z, \]
\[ (A_Z - K_Z), \]
\[ \frac{\partial T}{\partial R}. \]

Fig. 3. Streak photograph 128 and corresponding streamlines, isotherms and isolachs 12 rotations (43.9 sec) after photograph 125 (Fig. 2). Note how narrow the shaded band of isotherms is. At this stage of the amplitude-vacillation cycle the symmetrical (i.e., zonally averaged) component of the general circulation is fully developed and is operating near or at its most vigorous rate (see lower right).

sentation of the motions in the upper body of the fluid, avoiding misrepresentations and distortions due to surface tension and other boundary layer effects which must be present to some degree in top surface photos. Top-surface streak photographs of the flow field were not attempted since aluminum flakes are wetted by the silicone fluid, and do not, therefore, remain on the top surface of the fluid.

Streamlines and flow directions at mid-depth were determined from the photographs by recognizing that the flow patterns in rotating annulus experiments with a rigid base and a free top surface change very little with height above mid-depth, the main change being a slight clockwise shift in phase as we approach the top of the fluid. This conclusion is supported by recent experiments of Douglas et al. (1972) in which they have viewed the horizontal component of flow at three different depths in such fluid flows by shining horizontal beams of light through glass side walls at different depths. Hence, the flow patterns depicted in the photographs are probably reasonably close to the patterns at mid-depth. Accordingly, we took the liberty of tracing these patterns and superimposing the resulting streamlines on the mid-depth temperature and speed fields (see Figs. 2–6, Section 4), keeping in mind that the actual mid-depth streamline fields might be shifted...
slightly downstream (counterclockwise) from the ones in these figures. For the purpose of calculating the $u$ and $v$ components of velocity, flow directions were obtained at points on the photographs corresponding to the locations of the temperature and speed data. Vertical velocities along five equally spaced latitude circles in the middle half of the fluid, $\frac{1}{4} \leq (R-a)/(b-a) \leq \frac{3}{4}$, were subsequently calculated from the thermodynamic energy equation.

4. Synoptic aspects and general considerations

Figs. 2–6 show streak photographs depicting the flow field in the upper part of the fluid at five successive stages of an amplitude-vacillation cycle, each stage being 12 revolutions (43.9 sec) later than the one before. In the photographs, the flow field nearest the inner wall (approximately 18% of the annulus gap width at mid-depth) is blocked from view due to the perspective effect of the inner cylinder. The figures also show the
corresponding fluid temperature and speed fields at mid-depth determined by weighted least-squares analysis of the 84 thermistor measurements at this depth. Superimposed on the temperature and speed fields are streamlines determined from the photographs, as described in the preceding section.

The fluid flow is counterclockwise at mid-depth with maximum speed in excess of 0.6 cm⁻¹ sec⁻¹ (see Fig. 4) in the jet stream. By means of Fourier analysis we have ascertained that the waves travel counterclockwise around the annulus at an average speed (at mid-radius) of 0.184 cm sec⁻¹. The time for the 4-wave pattern to make one complete circuit is 100 revolutions (366 sec).

The vacillation period averages 53.4 revolutions (195.6 sec). The range of periods of 4-wave amplitude vacillation observed thus far by the present writers in experiments with different fluids (6 ≤ Pr ≤ 86), different annulus dimensions [2.5 ≤ (b−a) ≤ 12.5 cm; 1.73 ≤ b/a ≤ 3.94; 10 ≤ d ≤ 15 cm], and different numbers of probes (0 ≤ N ≤ 98) is from 24 to 92 revolutions. These periods are normally much greater than those for tilted-trough vacillation, which is more prevalent at the lower Prandtl numbers and higher Taylor numbers within the range in which we have worked. To date our survey has covered a range of imposed Taylor number.
from $5 \times 10^8$ to $1 \times 10^8$ and imposed thermal Rossby numbers from 0.15 to 2.0.

The sequence (Figs. 2–6) reveals clearly the phase relationships among the streamline, isotach and isotherm fields. The isotach fields determined from the thermistor measurements show maximum speeds tending to be elongated along the general direction of flow, as might be expected from observations of similar flows in the atmosphere and in related laboratory experiments (Riehl and Fultz, 1957). The speed maxima favor the region to the east of the cyclonic troughs. The isotherms are observed to be out of phase with the streamlines, with the cold fluid located to the rear of the cyclonic troughs and the warm fluid located to the rear of the anticyclonic ridges. This is known both synoptically and dynamically to be a configuration in which quasi-geostrophic waves grow by means of conversions from potential to kinetic energy. In the process of such growth, the waves distort the temperature field, transport significant amounts of heat both "poleward" (i.e., toward the inner cylinder) and upward, and substantially reduce the horizontal temperature gradients in the fluid. By so doing they deplete their supply of available potential energy.

Because of the approximate way in which the streamline pattern was derived, it is not considered worth-

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**Photograph #137**

\[
\min: \frac{K_E - K_Z}{u'v'}
\]

**Fig. 6.** Streak photograph 137 and corresponding streamlines, isotherms and isotachs 12 rotations (43.9 sec) after photograph 134 (Fig. 5). At this stage of the amplitude vacillation, 5.4 revolutions (20 sec) before the end of the cycle, the eddy kinetic energy and the processes associated with the eddy fluxes of momentum are near or at their minimum magnitudes (see lower right).
while to attempt to account for small, non-systematic time and space variations of the phase relationship between the streamlines and the isotherms. At present, we are in the process of calibrating a direction-sensitive transducer (Fowlis et al., 1972) for employment in the annulus. If successful, such transducers will enable us to make quantitative determinations of the flow direction at selected levels in the fluid and to investigate more thoroughly the phase relationships between the streamlines and each of the other field parameters. Although we cannot specify closely the time variation of the phase difference between the streamline and isotherm patterns, it is evident from the figures that this difference does not approach zero, and consequently that the vacillation cycle under consideration does not progress to an "occlusion" stage. The present vacillation is clearly not characterized by "overshooting" of the isotherm field and consequent adiabatic suppression of the waves. Inasmuch as the estimated viscous damping time turns out to be the same order of magnitude as the observed decay period of the waves, it is difficult to escape the conclusion that dissipation plays an important role in damping the waves during the second half of the cycle while the waves are still in a configuration of growth by adiabatic processes (the growth rate having been much reduced by the reduction in the horizontal temperature gradients). During the early stages of the cycle, however, when the wave amplitude

![Figure 7](image)

**Figure 7.** Streamlines (solid lines), isotherms (dashed lines) and vertical velocity contours (dotted lines positive, upward regions shaded) at mid-depth, corresponding to photograph 131. The streamlines and isotherms are identical to those in Fig. 4. The vertical velocity contours are drawn for every 0.01 cm sec$^{-1}$ with zero lines separating the shaded from the unshaded regions. Upward vertical velocities (+) are seen to coincide with the warm fluid to the "east" of the wave troughs, and downward vertical velocities (−) are seen to coincide with the cold fluid to the "west" of the troughs as observed in the earth's atmosphere.

(Fig. 8. Zonally averaged temperatures (T) as a function of time at eight equally-spaced radii at mid-depth in the main body of the fluid. The time covered corresponds to approximately 332 revolutions. Note that the time period $\Delta t = t_3 - t_1$ is shorter than the time period $\Delta t = t_5 - t_3$. (and hence the effect of viscosity) is small, the adiabatic rate of conversion from potential to kinetic energy must substantially exceed the rate of energy dissipation.

A characteristic feature of the baroclinic instability mechanism and the growth of wave cyclones is the rising of warm fluid to the east of the wave troughs and the sinking of cold fluid to the west of these troughs. East-west overturnings are, in fact, the primary mechanism for conversion from eddy available potential energy to eddy kinetic energy in the earth's atmosphere. It is of interest, therefore, to examine the phase relationships which exist in the experiment between the vertical velocity and the streamline field, and between the vertical velocity and the temperature field. These are shown in Fig. 7 for the data corresponding to photograph 131, at which time the waves are in their most active stage of growth. Inspection of this figure reveals that the phase relationships in the annulus are similar to those in the atmosphere, indicating that the experiment is indeed simulating processes which are fundamental to atmospheric behavior.

5. The zonally-averaged state and the eddy fluxes

In general circulation studies it is often found useful to resolve the dependent variables into zonally-averaged (or axially symmetric) components and eddy components (i.e., asymmetric departures from the zonal averages), and to study how the symmetric and asymmetric parts interact to influence each other. In the present example of amplitude vacillation it appears that the nearly symmetric flow seen in Fig. 2 becomes unstable, permitting macroscale perturbations of small amplitude to develop into large-amplitude wave disturbances (Fig. 4). The macroscale waves draw upon the store of zonal available potential energy in the basic
symmetric state for their growth. As in the case of their counterparts in the earth's atmosphere, these waves, or "eddies," once formed, can be expected to alter significantly the radial and vertical distributions of the zonally-averaged temperature, pressure and velocity.

In the present section we shall examine the response of the zonally-averaged temperature and vertical velocity fields to the periodic growth and decay of the waves.

The zonally-averaged temperature ($\bar{T}$) was calculated by taking the mean of the seven thermistor measurements at each latitude. The time variations of $\bar{T}$ at eight equally-spaced radii ($R = 8.15, 9.00, 9.84, 10.64, 11.54, 12.38, 13.23$ and $14.08$ cm) in the main body of the fluid are shown in Fig. 8 over a time-period covering about $64^2$ vacillation cycles ($\sim 332$ revolutions). Because of temperature reversals near both boundaries, the additional temperature curves ($\bar{T}$) corresponding to $R = 7.58, 7.87, 14.36$ and $14.64$ cm, if plotted in this figure, would have interfered with the ones shown, making the figure somewhat more difficult to interpret. For this reason the data for these radii are omitted here but are shown in Fig. 9. In Fig. 8 the distance between any two curves is directly proportional to the radial temperature gradient ($\partial \bar{T} / \partial R$) between the corresponding two radial circles. The reader will observe that in each cycle the decrease in the radial temperature gradient in the main body of the fluid is more rapid than the subsequent increase. This is illustrated in the figure by comparing the rate of change of $\partial \bar{T} / \partial R$ in the time range $t_2 \leq t \leq t_3$ with that in the range $t_1 \leq t \leq t_2$, and is characteristic of amplitude oscillations reported in earlier experiments (Pfeffer et al., 1965; Fowlis and Pfeffer, 1969). The average times for the destruction and reintensification of the radial temperature gradient are $85 \pm 5$ sec ($23.3 \pm 1.4$ revolutions) and $110 \pm 5$ sec ($30.0 \pm 1.4$ revolutions), respectively. The $\pm 5$-sec uncertainty in these periods could be due to actual time differences among different cycles, or, more likely, to truncation errors due to the less optimum sampling frequency. Times $t_1$ and $t_2$ precede by 6 sec photographs 128 (Fig. 3) and 141 (not shown), respectively. They correspond to maxima in both the radial temperature gradient ($\partial \bar{T} / \partial R$) and the zonal available potential energy ($A_x$). Time $t_3$ precedes by 6 sec photograph 134, at which time both $\partial \bar{T} / \partial R$ and $A_x$ are at their minimum values.

Radial profiles of the zonally-averaged temperatures at times $t_1$ and $t_2$ are shown in Fig. 9. These are determined from measurements at all 12 radii sampled. The comparatively small slope in mid-latitudes ($9.8 \leq R \leq 12.4$ cm) at time $t_2$ is a feature noted in a previous amplitude-vacillation experiment by Fowlis and Pfeffer (1969). The present measurements which extend radially beyond the previous ones, show, in addition, temperature reversals near the inner and outer walls at both times. Such reversals have been reported in symmetric and asymmetric flows by Faller (1958) and Kaiser (1972), and have been obtained theoretically by Hunter (1967), Williams (1967, 1972), McIntyre (1968), and others.

Fig. 10 shows the radial profile of the change in the zonally-averaged temperature from time $t_2$ to $t_3$ (solid curve) and the contribution to the temperature change from heat conduction (dashed curve). The secondary peaks in the heat conduction curve $\sim 1$ cm from each wall are associated with the temperature reversals shown in the preceding figure. Clearly, heat conduction...
Fig. 11. “Poleward” (radial) eddy flux of heat ($\overline{\sigma}$) per centimeter of fluid depth at mid-radius ($R = 11.11$ cm) and mid-depth ($z = 7.5$ cm) as a function of time. The shaded band embodies estimates of $\overline{\sigma}$ for each of three successive amplitude-vacillation cycles represented by circles, triangles and diamonds. Squares and half-moons represent later portions of the cycles depicted by circles and triangles. Time scale (lower abscissa) and photograph number sequence (upper abscissa) correspond only to the circled data points (first cycle). Dashed curves through the dotted data points represent composite estimates of the time variations of the poleward eddy flux of heat $\overline{\sigma}$ at $R = 8.15$ and 14.08 cm for the same three vacillation cycles.

Fig. 11 reveals that the observed time variations of the poleward eddy flux of heat at mid-radius ($R = 11.11$ cm) and mid-depth ($z = 7.5$ cm) are largely responses to the time variable “eddy” flux of heat accomplished by the growing and decaying waves. The shaded band in this figure embodies estimates of the “poleward” (radial) eddy flux of heat per centimeter of fluid depth ($\overline{\sigma} = -2\pi\nu C' \overline{\nabla}' / \overline{\nabla}$) at mid-radius ($R = 11.11$ cm) and mid-depth ($z = 7.5$ cm) as a function of time. The calculated values of $\overline{\sigma}$ for each of three successive amplitude vacillation cycles are given in the figure by circles, triangles and diamonds, respectively. The squares and half-moons represent later portions of the cycles depicted by circles and triangles, respectively. The temperature and velocity components required for these calculations were extracted at 72 points around the mid-latitude circle from a sequence of synoptic charts and photographs such as those shown in Figs. 2–6.

In order to facilitate comparisons with these and other figures, we have used for reference purposes in Fig. 11 the time scale (lower abscissa) and the photograph number sequence (upper abscissa) corresponding to the circled data points (first cycle). The shaded envelope is the writers’ subjective interpretation of the “best” representation of all the points. The dashed curves through the dotted data points represent composite estimates of the time variations of the poleward eddy flux of heat $\overline{\sigma}$ at $R = 8.15$ and 14.08 cm for the same three vacillation cycles. The distance between the two dashed curves is a measure of the radial gradient of the poleward eddy flux of heat $\overline{\sigma}$ at mid-depth in the main body of the fluid.

Using the upper abscissa (viz photograph number) in Fig. 11 as our time reference we now describe the sequence of events depicted in this and related figures. Starting with photograph 125 we see that when the wave amplitude is smallest and the poleward eddy flux of heat $\overline{\sigma}$ is correspondingly close to zero, $\partial \overline{\sigma}/\partial R$ is increasing most rapidly. Soon afterward the baroclinic waves begin to grow and $\overline{\sigma}$ begins to increase. At $t = t_1$ (photograph 128) $\overline{\sigma}$ is sufficiently large to reverse the intensification of the radial temperature gradient. The radial gradient $\partial \overline{T}/\partial R$ then decreases at an accelerating rate until $\overline{\sigma}$ reaches its maximum value (photograph 131). As a result of the intense poleward eddy flux of heat during the period $t_1 \leq t \leq t_2$ (photographs 128 to 134) the radial temperature gradient in the main body of the fluid is reduced to its minimum value by the end of this period. Since diabatic heating in the main body of the fluid was shown in Fig. 10 to be ineffective, the
Fig. 12. "Poleward" (radial) eddy flux of angular momentum ($\tau$) per centimeter of fluid depth at mid-radius ($R = 11.11$ cm) and mid-depth ($z = 7.5$ cm) as a function of time. The shaded band embodies estimates of $\tau$ during the same three cycles represented in Fig. 11.

reestablishment of this gradient during the period $t_2 \leq t \leq t_3$ (photographs 134 to 142), when the eddy flux of heat is again small, can be accomplished only by the axisymmetric circulation which concentrates the radial temperature gradient in the main body of the fluid at the expense of the temperature gradients near the side walls.

It is also of interest to examine the time variation of the poleward eddy flux of angular momentum ($\tau = -2\pi pR\bar{u}\bar{v}$) accomplished by the growing and decaying waves. The shaded band in Fig. 12 displays features which are qualitatively similar to those in Fig. 11, although the band is wider and there are more data points lying outside it. Comparison of the two figures reveals a slight time phase lag between the band representing the momentum flux and that representing the heat flux. Higher quality velocity data will be needed, however, before one can assign significance to this feature. It will be recalled, in this connection, that the $u$ and $v$ components of the velocity were obtained by combining mid-depth speed measurements with estimates of flow direction obtained from upper-layer streak photographs.

The lower quality of such velocity data may also account for the greater scatter of the points in Fig. 12 in comparison with those in Fig. 11, since $\tau$ depends upon the product of the two components of the horizontal velocity, whereas $\Phi$ depends upon the product of one component of velocity and the more accurately determined temperature departure. Inspection of the streamline and isostach field in Fig. 4 reveals that the eddy flux of momentum depicted in Fig. 12 is associated with an asymmetry of the speed field across the waves rather than with a tilted-trough, ridge asymmetry in the streamline pattern.

It is known from the work of Eliassen (1952) and Kuo (1956a, b) that secondary symmetric circulations in a stably stratified, rotating fluid can be driven by heating and friction and/or by eddy fluxes of heat and momentum. In quasi-geostrophic, quasi-adiabatic wave regimes such as the general circulation of the earth's atmosphere and of the fluid in the present experiment, the axially symmetric meridional circulation in the main body of the fluid (outside the boundary layers) must fluctuate in magnitude and direction in response
to the time variations of the eddy fluxes of heat and momentum. The intense poleward eddy flux of heat shown in Fig. 11 is sufficient to generate a comparatively strong thermally-indirect “Ferrel” circulation in the main body of the fluid, with upwelling of cold, dense fluid at high latitudes and downwelling of warm, light fluid at low latitudes. In the earth’s atmosphere a similar thermally-indirect, middle-latitude Ferrel cell, driven by eddy fluxes of heat and momentum, serves to support desert conditions in the Northern and Southern Hemispheric subtropics and predominantly cloudy conditions in the sub-arctic zones. For meteorological reasons, as well as for other fundamental reasons, it is, therefore, of interest to examine the time variation of the secondary vertical circulation in the laboratory experiment as the amplitude-vacillation cycle proceeds.

Fig. 13 shows five radial profiles of the zonally-averaged vertical velocity \( \omega \) at mid-depth corresponding to photographs 125, 128, 131, 134 and 137. Consistent with flow patterns obtained in computer integrations of the Navier Stokes equations by Williams (1967), there is an upward drift of fluid across mid-depth in this region at the time of photograph 125 when the flow is nearly symmetric. Subsequently, as the baroclinic cyclone waves develop and the eddy fluxes of heat and momentum intensify, the maximum upward velocity is shifted toward the cold source (photograph 128). When the waves are fully developed and the eddy fluxes of heat and momentum reach their peak values (photograph 131) the maximum upward vertical velocity reaches the poleward limits of the data, and downwelling appears at the equatorward limits of the data. This pattern is consistent with the circulation in the earth’s atmosphere and in the numerical solutions of the hyrodynamical equations for wave flows obtained by Williams (1972) and by Dietrich (1973). The magnitude of the reverse Ferrel circulation decreases as the eddy fluxes decrease in intensity (photographs 134 and 137) and the flow again returns toward its symmetric configuration.

6. Fourier analysis

Any set of scalar readings at 2\( N \) equally-spaced points around a “latitude” circle may be represented by the purely formal expression

\[
f_i = \bar{f} + \frac{1}{N} \sum_{i=-N}^{N} (A_n \sin \lambda_i + B_n \cos \lambda_i) + B_N \cos \lambda_i, \quad (1)
\]

where

\[
\lambda_i = \frac{2\pi i}{2N}, \quad i = 0, \pm 1, \pm 2, \ldots, \pm N,
\]

and

\[
A_n = \frac{1}{N} \sum_{i=-N}^{N} f_i \sin \lambda_i, \quad (2a)
\]

\[
B_n = \frac{1}{N} \sum_{i=-N}^{N} f_i \cos \lambda_i, \quad (2b)
\]

\[
\bar{f} = \frac{1}{2N} \sum_{i=-N}^{N} f_i, \quad (3a)
\]

\[
\frac{1}{2N} \sum_{i=-N}^{N} (-1)^i f_i. \quad (3b)
\]

If the readings represent point measurements in a continuum, and if the variation of the scalar quantity \( f \) around the latitude circle consists of only wavenumbers 1 through \( N-1 \), plus the zonal average \( \bar{f} \), then the subscript \( i \) may be dropped, the summations in (2a, b), and (3a, b) may be replaced by integrals, and (1) may be used to represent the variation of the scalar quantity in the continuum \( 0 \leq \lambda \leq 2\pi \) as well as at the grid points \( i = 0, \pm 1, \pm 2, \ldots, \pm N \).

Although the network of measurements employed in the present experiment was not originally intended to be used for Fourier analysis, it was found to be suitable for such purposes, since only a few low wavenumbers dominate the flows in the region of dimensionless parameter space in which this experiment was performed. The probes on each radial frame are sufficiently closely spaced, and the radial distributions of temperature \( T \) are sufficiently smooth in the main body of the fluid \((8 < R < 14 \text{ cm})\), such that reasonable estimates of \( T \) are obtainable by linear interpolation along frames A (Fig. 1) to radii at which there are thermistor probes on frames B, and vice versa. With the present arrangement of probes it is thus possible to obtain 7 measured and 7 radially interpolated values of temperature at each interior radius, the latter being independent of the former. With 14 independent values of temperature on each “latitude” circle, Fourier analysis enables us to determine undistorted amplitudes and phases of wavenumbers 1 through 6, provided there is not a significant amount of temperature variance in wavenumbers \( > 7 \). The amplitude of wavenumber 7 is formally obtainable but is, in general, very much distorted.

Comparisons made at a sampling of interior points revealed that linearly interpolated temperatures were indistinguishable from weighted least-squares, polynomial-interpolated temperatures. For convenience, therefore, the least-squares interpolated temperatures already obtained for the purpose of synoptic analysis were used in the Fourier analysis. Near the boundaries, neither linear nor least-squares polynomial interpolation is as good as in the interior because the reversals in the zonally-averaged radial temperature profiles (see Fig. 9) are on the same scale as the distance between the grid points. It is for this reason that only seven measured temperature readings at each latitude were used to calculate \( T \) (Section 5). Because of the orthogonality property of the modes, it is easily shown, however, that calculations of the sine and cosine amplitudes of wave-
Fig. 14. Time variations through more than eight vacillation cycles of the mid-depth temperature amplitudes of wavenumbers 1–7 at the 12 radii at which thermistors were located. The numbers in the last column on the right indicate the radii of the different curves. The numbers to the left of each curve indicate the mean amplitude (°C) of each wavenumber at each radius. The scale of the temperature fluctuations is given at the extreme right of the figure. Note the dominance of wavenumber 4 which is undergoing strong amplitude vacillation.
numbers 1 through 6 using all 14 points are not affected by interpolation errors in \( \dot{T} \).

We have already examined in Fig. 9 the radial distribution of the zonally-averaged temperature field (wavenumber 0) and in Fig. 8 the time variation of \( \dot{T} \) at 8 of the 12 latitude circles sampled. Fig. 14 shows the time variation through more than eight cycles of the temperature amplitudes of wavenumbers 1 through 7 at all 12 latitudes at which thermostors were located. Wavenumber 7 is included for completeness although it is subject to considerable distortion. Fourier analysis of the speed field was also performed but is not shown here because a 4-wave pattern has eight speed maxima, and aliasing misrepresents wavenumber 8 as wavenumber 6 in such an analysis. Fig. 14 shows clearly the predominance of wavenumber 4 which is undergoing strong amplitude vacillation. This wave is particularly strong at middle radii \( 9.00 < R < 13.23 \) (the temperature amplitude ranging from 0.04 to 0.49°C at \( R = 10.69 \text{ cm} \)) and becomes considerably weaker near the inner and outer walls. The writers are not certain about the source of the superimposed noise which is a small fraction of the maximum signal. Aliasing of high-frequency noise by time sampling every 14.6 sec also makes it impossible to determine the true frequency of the noise. It should be noted that the amplitude of wavenumber 7 is extremely small at every radius, indicating that the presence of seven probes around each latitude circle is not introducing spurious wavenumbers. The probes do, however, represent an additional source of dissipation which makes the behavior of the fluid appear more like that of a fluid with a higher coefficient of viscosity (see Section 8). It is interesting to note that wavenumbers 3 and 5 at some radii have clearly identifiable amplitude vacillations with the same period as wavenumber 4. This is particularly true of wavenumber 3 near the inner and outer walls and of wavenumber 5 at middle radii.

A different perspective concerning the relationships among the Fourier components is provided by examination of Fig. 15 which shows the temperature amplitude vs wavenumber at \( R = 11.54 \text{ cm} \) at successive times during the cycle. Here we see four complete cycles representing roughly a 13-min time period. The similar
shapes of the spectra in the first two cycles and those in the second two cycles are worth noting, suggesting that even the weaker modes display a systematic and measurable behavior. Synoptic fields of temperature and velocity corresponding to every third spectrum in the first cycle were shown in Figs. 2–6, the time of each spectrum being 6 sec earlier than the corresponding interpolated synoptic temperature field. This time difference is due to the fact that the Fourier amplitudes were determined using the actual temperature measurements, whereas the synoptic temperature fields shown in Figs. 2–6 were interpolated to the time of each speed measurement.

Inasmuch as the dominant mode is wavenumber 4, it is useful to examine the phase of this wave as a function of time. This phase relationship is shown in Fig. 16. We have also plotted in this figure the time variations of the amplitude of wavenumber 4 and of the zonally-averaged temperatures at \( R = 8.15 \) and 14.08 cm. From the slope of the phase curve we calculate that the average phase speed of wavenumber 4 at \( R = 11.54 \) cm is 0.184 cm sec\(^{-1}\). We also observe that the phase speed exhibits cyclic behavior, slowing down to about 0.140 cm sec\(^{-1}\) when the wave amplitude is large and speeding up to 0.282 cm sec\(^{-1}\) when it is small. This is a significant feature of amplitude vacillation which has also been detected in movies of the top surface flow taken by a camera rotating with the annulus.

7. Energetics

Studies of the atmospheric energy cycle in recent years have greatly improved our understanding of the general circulation of the earth's atmosphere (see, e.g., Lorenz, 1967; Starr et al., 1970; Saltzman, 1970; Winston and Krueger, 1961). Similar studies of the energetics of time-dependent motions in rotating, differentially heated fluids in laboratory experiments can be expected to aid our comprehension of the complex mechanisms which bring the about observed flow patterns, and to assess better the conditions under which valid analogies can be made between the behavior of such fluids and that of the atmosphere. Proper evaluation of the pertinent energy integrals requires measurements of flow direction, speed and temperature at hundreds of points at a number of depths in the fluid. As noted earlier, we are in the process of developing direction-measuring transducers and fabricating a more extensive synoptic network of velocity and temperature measurements in a much larger annulus. In the meantime, it is useful to examine energy integrals which can be evaluated with the present set of measurements. In this section we present estimates of the zonal and eddy available potential energies, the zonal and eddy kinetic energies, and the rates of energy conversion per unit mass at mid-depth in the main body of the fluid (outside the boundary layers). Mathematically, these integrals take the form
\begin{align}
A_Z &= \frac{g \alpha}{2} \frac{\int (\bar{T}'')^2 \bar{R} dR}{\int \bar{R} dR}, \\
A_E &= \frac{g \alpha}{2} \frac{\int (T'')^2 \bar{R} dR}{\int \bar{R} dR}.
\end{align}

\begin{align}
K_Z &= \frac{\int (\bar{u}^2 + \bar{v}^2) \bar{R} dR}{2 \int \bar{R} dR}, \\
K_E &= \frac{\int [(\bar{u'})^2 + (\bar{v'})^2] \bar{R} dR}{2 \int \bar{R} dR}.
\end{align}

\begin{align}
(A_Z \rightarrow K_Z) &= \frac{g \alpha \int \bar{w}' \bar{T}'' \bar{R} dR}{\int \bar{R} dR}, \\
(A_Z \rightarrow K_E) &= \frac{g \alpha \int \bar{w}' T'' \bar{R} dR}{\int \bar{R} dR},
\end{align}

\begin{align}
(A_Z \rightarrow A_E) &= -g \alpha \int \left( \frac{\partial \bar{T}''}{\partial R} + \frac{\partial \bar{T}''}{\partial z} \right) \bar{R} dR \frac{\partial \bar{T}}{\partial z} \int \bar{R} dR,
\end{align}

\begin{align}
(K_K \rightarrow K_Z) &= \frac{1}{\int \bar{R} dR} \int \left[ \frac{\partial}{\partial z} \left( \bar{w}' + \bar{u}' \bar{w}' \frac{\partial \bar{u}}{\partial R} + \frac{\partial \bar{w}}{\partial z} \frac{\partial \bar{u}}{\partial R} \right) \bar{R} + \frac{\partial}{\partial z} \left( \bar{w}' + \bar{v}' \bar{w}' \frac{\partial \bar{v}}{\partial R} + \frac{\partial \bar{w}}{\partial z} \frac{\partial \bar{v}}{\partial R} \right) \bar{R} \right] dR.
\end{align}

The bar and tilde represent averages around a latitude circle and over the entire horizontal area of the fluid, respectively, and the single and double primes represent corresponding departures from these averages.

The above integrals were evaluated using the trapezoidal rule. The integrals \(A_Z\) and \(A_E\) were determined using only the temperatures measured at the thermistor locations. The remaining integrals require the determination of flow directions and the use of interpolated temperature and speed data. Flow directions were extracted from the photographs at 20 points around each of five latitude circles, \(R = 9.21, 10.15, 11.11, 12.07\) and 13.02 cm, representing \(2/8, 3/8, 4/8, 5/8\) and \(6/8\) of the gap width of the annulus. Speed and temperature data were interpolated to these points by least-squares polynomial fitting (Pfeffer et al., 1962). The quality of the streak photographs did not justify evaluations of the flow direction closer to the inner and outer walls.

In the expressions for \((A_Z \rightarrow A_K)\) and \((K_K \rightarrow K_Z)\), only the underlined terms were evaluated. In the case of the latter integral this is justified by a scale analysis which shows that the first term on the right is of order unity, \(<< 1\). Here \(R_0\) is the Rossby number and \(R_i\) the Richardson number. In the case of the integral \((A_Z \rightarrow A_E)\) meteorological evidence and theory suggest that the second term should be systematically of opposite sign and about half the magnitude of the first. Our only reason for not calculating this term is that we did not have measurements of the static stability at more than one latitude. Such measurements are required to calculate the latitudinal covariance of \(\bar{T}' \bar{w}'\) and \(\partial \bar{T}'' / \partial z\).

Before proceeding with a presentation of the energy variations with time, it is worthwhile to call attention to the degree of non-uniformity to be expected in the quality of the results based on the type of data we have available from this experiment. The most accurately determined integrals should be \(A_Z\) and \(A_E\) since they involve only measured temperatures at each of the 98 thermistor locations covering the space \(7.58 \lesssim R \lesssim 14.64\) cm. Less accurate will be the determinations of the integrals involving individual velocity components, \(u\) and \(v\), since flow directions and speeds as determined in the present investigation have larger relative errors than do temperature measurements. The greatest uncertainty can be expected in our estimates of the integrals which depend upon the vertical component of velocity, since \(w\) is inferred indirectly from \(u, v\) and \(T\) using the thermodynamic energy equation. Integrals evaluated at mid-depth in the fluid, moreover, cannot be regarded as being representative of the entire fluid depth. Since \(u\) and \(v\) reach their maximum magnitudes in the upper layers of the fluid (i.e., at jet-stream levels), integrals which involve double or triple products of mid-depth velocity components must be underestimates of the vertical mean. And, since \(T'\) and \(w\) attain their maximum values around mid-depth, integrals involving these quantities must be overestimates of the vertical mean. It is clear, therefore, that quantitative energy budgets and comparisons of energy levels and conversion rates are not feasible with these data. The calculations do, however, yield significant information about relative time phases and qualitative features which are important to interpretative studies. In the discussion following the presentation of Figs. 17–19 we will attempt to elucidate the physical processes involved in amplitude oscillation by cross-referencing various figures together with the sequence of photographs 125, 128, 131, 134 and 137.

The time variations of the zonal and eddy components of the available potential energy, \(A_Z\) and \(A_E\), respectively, and of the rate of energy conversion \((A_Z \rightarrow A_E)\) over approximately 23 cycles are shown in Fig. 17. The tick marks on the curves indicate the times at which the photographs in Figs. 2–6 were taken. The curves display a time phase lag between \(A_Z\) and \(A_E\) with the latter following the former in time by nearly 54 sec (14.8 revolutions, or 100° phase lag). This and
Fig. 17. Time variations of the zonal and eddy components of available potential energy, $A_Z$ and $A_E$, respectively, and of the rate of energy conversion from $A_Z$ to $A_E$, at mid-depth in the fluid, over approximately 2½ amplitude-vacillation cycles. The tick marks on the curves indicate the times at which the photographs in Figs. 2–6 were taken. Note the phase relationship between $A_Z$ and $A_E$ and the fact that the rate of energy conversion ($A_Z \rightarrow A_E$) is in phase with $A_E$.

other phase relationships obtained in the present investigation were anticipated from meteorological considerations. There is also a remarkably high correlation in the time variation of $A_E$ and the rate of energy conversion ($A_Z \rightarrow A_E$) with no measurable time lag, indicating that the eddy flux of heat down the gradient of the zonally-averaged temperature [see Eq. (7)] is directly proportional to the eddy available potential energy.

The time variations of the zonal and eddy components of the kinetic energy, $K_Z$ and $K_E$, respectively, and of the rate of energy conversion ($K_E \rightarrow K_Z$) over the cycle encompassing photos 125–137 are shown in Fig. 18. The phase lag between $K_Z$ and $K_E$ is roughly the same.

Fig. 18. Time variations of the zonal and eddy components of kinetic energy, $K_Z$ and $K_E$, respectively, and of the rate of energy conversion from $K_E$ to $K_Z$, at mid-depth in the fluid. The tick marks on the curves indicate the times at which the photographs in Figs. 2–6 were taken. Note that ($K_E \rightarrow K_Z$) is always positive and that its peak occurs at a time when $K_Z$ is decreasing and $K_E$ is increasing in magnitude.
as that between $A_Z$ and $A_E$, shown in the preceding figure. Careful comparison of the two figures reveals some evidence of a small phase lag between $A_Z$ and $K_Z$ and between $A_E$ and $K_E$. The quantity and quality of the present data are not, however, sufficient to give us confidence in such small phase differences. Of greater significance is the fact that the rate of energy conversion ($K_E \rightarrow K_Z$) is positive over the entire cycle and that it reaches its peak value at a time when $K_Z$ is decreasing and $K_E$ is increasing in magnitude. Fultz et al. (1959) have presented evidence which suggests that during tilted-trough vacillation the zonal current varies in response to the eddy flux of momentum accomplished by the macroscale waves in the fluid. In the present example of amplitude vacillation the curves in Fig. 18 reveal that the time variations of $K_Z$ and $K_E$ are not responsive to the energy conversion ($K_E \rightarrow K_Z$) which depends upon the eddy flux of angular momentum up the gradient of the zonally-averaged angular velocity [see Eq. (8)]. Although further study is required, this might be one of the more important characteristics which distinguishes amplitude vacillation from tilted-trough vacillation. During amplitude vacillation, $K_Z$ and $K_E$ vary more in response to the conversions from potential to kinetic energy, ($A_Z \rightarrow K_Z$) and ($A_E \rightarrow K_E$), and to the respective rates of dissipation of these energies. The relative magnitudes of these processes are shown more clearly in Fig. 19. Although the quantity and quality of the present data do not permit us to assign any significance to the difference in magnitude between ($A_Z \rightarrow A_E$) and ($A_E \rightarrow A_Z$), we cannot escape the conclusion that these two processes are roughly one order of magnitude greater than the process ($K_E \rightarrow K_Z$).

The average positive value of ($K_E \rightarrow K_Z$) is significant because it is characteristic of the experiments we have conducted in the Rossby wave regime and is consistent with atmospheric measurements in all seasons (Starr et al., 1970). Also significant is the reversal of the direction of the energy conversion ($A_Z \rightarrow K_Z$) once the eddy (wave) processes become sufficiently large. This is brought about by the strong development of an indirect "Ferrel" cell in the main body of the fluid (Fig. 13) which is driven by the eddy fluxes of heat and momentum. As in the atmosphere, the Ferrel cell serves to convert energy from kinetic to potential by raising cold, dense fluid at sub-arctic latitudes and lowering warm, light fluid at subtropical latitudes. The area under the curve ($A_Z \rightarrow K_Z$), averaged over the vacillation cycle, is negative indicating that this process dominates in the "climatological average."

In order to gain some understanding of the physical processes responsible for amplitude vacillation let us
As a result of these processes the fluid becomes baroclinically unstable and the waves begin to grow. At the time of photograph 128 the eddy energies $A_E$ and $K_E$ and the energy conversion processes $(A_Z \rightarrow A_B)$ and $(A_E \rightarrow K_B)$ are rising rapidly (Figs. 17, 18 and 19), reflecting the baroclinic instability process. The zonal available potential energy $A_Z$ has just reached its peak (Fig. 17) and is beginning to decline under the influence of the increasing poleward eddy flux of heat (Fig. 11). Around the time of photograph 131 the waves are fully developed (Fig. 4) and the eddy fluxes of heat and momentum (Figs. 11 and 12), the eddy energies (Figs. 17 and 18), and the energy conversion processes $(A_Z \rightarrow A_B)$ and $(A_B \rightarrow K_B)$ (Fig. 19) are all near their peak values. The phase relationships among the streamlines, isotherms and vertical velocity contours (Fig. 7) display the classical meteorological pattern observed in the middle troposphere during rapid cyclone development. These relationships are predicted by all baroclinic instability theories. It is at this time that the adiabatic eddy fluxes of heat and momentum take control of the general circulation mechanism, creating a thermally indirect Ferrel cell (Fig. 13) and reversing the direction of the energy conversion process $(A_Z \rightarrow K_Z)$ (Fig. 19).

The zonal available potential energy $A_Z$ is decreasing rapidly in response to the energy conversion process $(A_Z \rightarrow A_B)$ (Fig. 17), and $K_Z$ and $K_B$ are changing in directions opposite to those implied by the peak value of the energy conversion $(K_Z \rightarrow K_B)$ (Fig. 18). The latter changes are in response to much larger magnitude conversions between potential and kinetic energy (Fig. 19). The advective processes involved and the energetics are shown schematically in Fig. 21.

At the time of photograph 134 (time $t_4$) the mid-latitude temperature gradient and the related zonal available potential energy have reached their lowest values (see Figs. 5, 8, 9 and 17). The eddy kinetic energy dissipation rate $D_E$, which is roughly proportional to the square of the fluid speed, must now strongly exceed the energy conversion rate $(A_B \rightarrow K_B)$ so that the eddy kinetic energy is in a state of rapid decline (Fig. 18). With the eddy processes no longer extracting large amounts of zonal available potential energy, $A_Z$ once again increases (Fig. 17) in response to the zonal available potential energy generation rate $G_Z$, and the symmetric regime again becomes reestablished several rotations after the time of photograph 137 (Figs. 11, 15, 17, 18, 19).

8. Probe effects

A multi-probe network creates substantial drag on the fluid to be measured. Since the probes form an integral part of the experimental apparatus and method, their effect on the flow field cannot be realistically eliminated or avoided. One cannot, therefore, expect the results of an experiment with many probes in the fluid to be quantitatively the same as those of an experiment.
at the same externally imposed conditions when there are few or no probes in the fluid. If the probe network is properly designed, however, the differences between experiments with and without large numbers of probes are no greater than the differences between experiments conducted in cylinders or annuli of different dimensions (e.g., a tall annulus and a shallow dishpan). The important point to realize is that regions of dimensionless parameter space exist within each experimental geometry or arrangement of probes in which phenomena related to atmospheric variability can be found and studied. The obvious advantage of conducting experiments in fluids instrumented with networks of sensors is that field distributions of the pertinent variables can be measured rapidly and uniformly throughout the fluid (even in irregular, non-periodic regimes), with the result that the time-dependent behavior of the cyclone waves and their interactions with the zonally-averaged temperature and velocity fields can be investigated as a function of changes in rotation rate, imposed temperature difference, and fluid properties. To be of maximum use, multi-probe networks must be designed carefully and the system being used must be defined clearly.

Great care has been exercised in the design of the probe system used here. It was realized early by us, as well as by other investigators (Fultz and Kaiser, 1971), that under certain conditions a single probe or a small number of probes could create undesirable non-uniform effects and distortions of the flow field. On the other hand, we discovered that a large number of probes distributed uniformly in the fluid did not suppress or destroy meteorologically significant flows. The probe effect appeared uniformly distributed without strong localized effects or distortions, as may be seen, for example, from the series of streak photographs in Figs. 2-6.

Several important criteria were used in the design of this probe network. The thermistor bead and wire assembly size was selected to be the smallest practical size consistent with high strength for mounting purposes, low drag, short response times for maximum time resolution, and low heat generation. The distribution of thermistors and return leads had to be quite uniform to distribute the momentum sinks and sources as evenly as possible and yield the maximum information from the network; hence the staggered pattern shown in Fig. 1.

Several approaches have been taken to study the influences of the probes on the behavior of the fluid. Our earliest efforts involved observation of the flow field in the annulus with and without probes under the same externally imposed conditions. A sequence of photographs corresponding to an experiment without probes performed at the same conditions as in the present experiment also revealed an amplitude-vacillating flow field (Fowlis et al., 1971). The flow field without probes showed a more sharply defined, larger amplitude jet stream which underwent a shorter period of amplitude vacillation (approximately 40 as compared with 53.4 revolutions). Although much more work must be done to establish quantitative relationships between experiments with and without probes in the fluid, preliminary results suggest that amplitude vacillation is more probable and has a longer period cycle in fluids with greater numbers of probes and in more viscous fluids with no probes. In general, these experiments suggest that the presence of probes has an effect on the flow similar to that of using a fluid of higher viscosity without probes.

A more quantitative indication of the influence of the probes was obtained by subjecting them to a flow field in the annulus which is relatively well understood, i.e., spin-up (the response to a change in the rotation rate of the container) of a homogeneous fluid. Relative speed as a function of time during the spin-up process was measured at mid-radius in the annulus, first with only one thermistor probe in the fluid, and then with all 98 thermistor probes present. The spin-up time (the time required to reach within 1/e of the imposed relative speed) at mid-radius with one thermistor and its return lead in the fluid was measured to be 44±2 sec; with 98 probes and 14 return leads in the fluid it was measured to be 26±2 sec. (The substantial reduction in the spin-up time implies an additional drag exerted on the fluid by the wires which is estimated to be of the same order of magnitude as the drag exerted by the boundaries.) The experiment was performed at $\Omega = 1.75$ rad sec$^{-1}$ with a spin-up of 2%. The calculated spin-up time (Greenspan and Howard, 1963), including the effect of the side-wall boundary layers (which had to be included in the calculation due to the narrow gap of the annulus), was 43 sec.

It is tempting to relate the smaller spin-up time at mid-radius in the presence of probes to the response of a fluid without probes but with viscosity higher by a factor of 2.9, for which the spin-up time at mid-radius is also 26 sec. It is recognized, of course, that such an interpretation is not strictly correct, since one cannot assume the viscous boundary layers to have as broad a reach as one obtains through a distribution of thermistor probes in the interior of the fluid. Nevertheless, it might be helpful, in a qualitative sense at least, when comparing the experimental results with either numerical or analytical results, to consider the presence of probes as giving an effectively larger viscosity to the fluid. Using the factor 2.9 we might, therefore, interpret our results as being closest to those which would be obtained in the same geometry without probes but at $Ta = 1.9 \times 10^6$ (instead of $1.6 \times 10^6$), $Pr = 165$ (instead of 57), and $Re = 0.900$.

In addition to the drag effect of the probes, there is also a thermal effect due to ohmic heating of the thermistors. The heating rates during the temperature and
speed modes of operation are approximately $0.7 \times 10^{-3}$ and $0.7 \times 10^{-1}$ cal sec$^{-1}$, respectively. The heat generated during the temperature mode is therefore negligible in comparison with that generated during the speed mode. Since the speed mode takes up approximately 40% of the temperature-speed sequence, due to the pulsing technique employed, the actual heat generation averaged over the time of the experiment is only 0.03 cal sec$^{-1}$. This value is an order of magnitude smaller than the heat transfer by conduction (0.37 cal sec$^{-1}$, assuming a conduction temperature profile and a temperature difference of 10°C) and approximately 50 times less than the heat flow through the annulus if we assume an average Nusselt number of 5 (Bowden, 1961).

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