Determinism in Mesoscale Wind Spectra at Columbia, Missouri

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ABSTRACT

Power spectra of mesoscale eddies with periods ranging from 10 min to 8 hr were computed for the months of January and July from three years of surface wind data obtained at Columbia, Mo. To educe the degree of determinism in the spectra, their variability with time of year, time of day, wind speed and direction, and with the presence or absence of thunderstorms was measured.

Diurnal spectra for January and July were found to be similar. A comparison of semidiurnal daytime and nighttime spectra showed that the former contained considerably more energy than the latter and that the difference was greater in July than in January. The eddy energy was shown to increase by a factor of approximately 7 at night in the presence of thunderstorms. No effect of wind direction could be found.

Over a large portion of the lower mesoscale range the diurnal power spectra followed a -1 law, similar to that for "wall turbulence" in the layer of strong shear in the boundary layer over a flat plate.

1. Introduction

Mesoscale atmospheric eddies, with periods ranging from 10 min to 8 hr, are of the utmost importance in both air pollution and atmospheric energetics problems. For example, the duration of sampling at a stationary array in atmospheric diffusion experiments usually corresponds to periods in the mesoscale range. Eddies in the mesoscale range have an overriding influence on the measured diffusion because of the rapid increase of eddy energy with eddy size (Kolmogoroff, 1941) and because of the decreasing effectiveness with time of the smaller eddies (Batchelor, 1949).

The degree to which mesoscale eddies link synoptic-scale and microscale eddies in phase space and the degree to which mesoscale eddies contribute to the generation and dissipation of the wind are not clearly understood. This topic is one of the major areas for investigation in the GARP Atlantic Tropical Experiment (GATE). Some authors consider the mesoscale gap to indicate very little linkage, especially in the dissipation process (Fiedler and Panofsky, 1970), whereas others consider mesoscale eddies to be of major importance in both processes, but especially in generation (McInnis and Kung, 1972; Gray, 1970). It can certainly be claimed that mesoscale turbulence is worthy of study.

It is the purpose of this paper to examine the power spectra of atmospheric eddies in the mesoscale range in order to ascertain the extent to which such eddies are deterministic. The relatively uniform mid-continental terrain near Columbia in central Missouri was chosen for the analysis because it provides a good contrast with more distinctive topography whose power spectra will be studied later. Eventually it will be desirable to accumulate spectra for a large number of sites in order to produce a spectrum climatology.

2. Determinism in power spectra of atmospheric turbulence

Studies of the power spectrum of the horizontal wind have been published by Panofsky and McCormick (1954), Van der Hoven (1957), Panofsky (1969), Oort and Taylor (1969), Vinnichenko (1970), and others. The time spectrum given by Vinnichenko (Fig. 1) shows general features common to most spectra: an annual cycle most prominent in the free atmosphere (above 1 km), a flat peak rising from the planetary wave scale near 60 days to a maximum near 5 days, a diurnal cycle, a semi-diurnal cycle near the ground indicating that this spectrum was measured near 100 m (Taylor, 1917), a broad spectral gap in the mesoscale range centered near a period of 1 hr, and a microscale peak near 1–2 min. The mesoscale region is characterized by Fiedler and Panofsky (1970) as being a region of negligible generation lying between the synoptic-scale peak created by barotropic and baroclinic instabilities and a microscale peak resulting from Kelvin-Helmholtz instabilities (vertical wind shear and thermal instability). Mesoscale turbulence can be naturally divided in two depending on whether or not the eddies exceed in size the depth of the troposphere. In the large-scale, low-frequency portion of the mesoscale region, where wavelengths exceed the depth of the troposphere, eddies are basically two-dimensional

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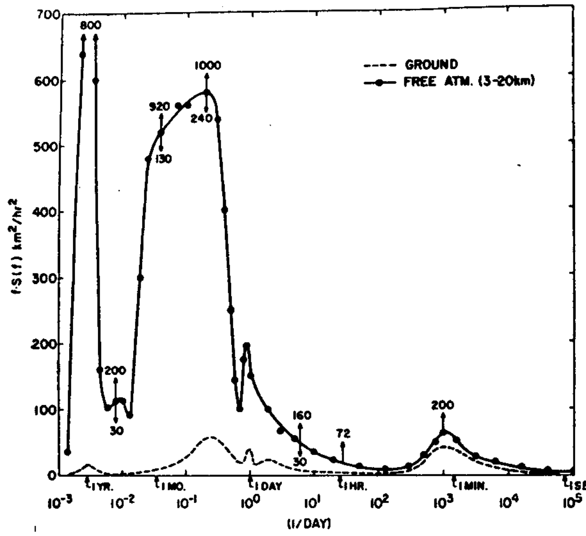

FIG. 1. Average kinetic energy of the east-west wind component in the free atmosphere and near the ground. Numbers indicate maximum and minimum values of kinetic energy at particular periods (Vinnichenko, 1970).

(Ogura, 1958), eddy energy is transferred to lower frequencies, and the spectrum follows approximately a K^{-3} law (Kraichnan, 1967; Hess and Clarke, 1973; and many others), This will be called the upper-mesoscale region. In the high-frequency portion of the mesoscale region, the portion we will principally be concerned with here, the turbulence is three-dimensional, eddy energy cascades to higher frequencies, and the spectrum follows the $K^{-5/3}$ law in the free atmosphere (MacCready, 1962; Vinnichenko, 1970; and many others). We will call this the lower-mesoscale region.

The region of strong shear in the lower part of the turbulent boundary layer might be expected to create rapid stretching of vertically oriented vortices similar to conditions close to a "wall." As will be shown later in this paper the spectra of mesoscale eddies near the ground exhibit the same K^{-1} law spectrum of eddy energy as is observed in two-dimensionally isotropic "wall turbulence" in the wind tunnel.

If there were no means other than the cascading process for generating mesoscale energy, the problem of prediction would be a purely statistical one. There is, however, considerable lower-mesoscale energy generated by mesoscale thermal convective processes such as sea breezes, thunderstorms, squall lines, Bénard-like cellular convection, and horizontal roll vortices (LeMone, 1973; and many others). Topographic obstructions have also been shown to create an extensive range of circulation patterns including the Kármán-like vortices observed by satellite photography in the lee of ocean islands [see the Guadaloupe Island photograph in NASA (1967)]. There is some evidence for the production of mesoscale turbulence through resonance between the shear flow in the Ekman layer and internal gravity waves (Kaylor and Faller, 1972; Vinnichenko and Dutton, 1969). Shear in the turbulent boundary layer, although contributing greatly to the cascading process (Hinze, 1959, p. 263; Tennekes and Lumley, 1972, p. 256), contributes little to the direct generation of mesoscale turbulence because the shearing stress over the depth of the boundary layer is too small.

As far as is known no author has yet attempted to predict the variability of the power spectrum of eddies in the lower-mesoscale range on any basis, although the Pasquill stability categories used in pollution research make a successful use of such variability (Pasquill, 1961).

3. Data processing and analysis

Wind speed and direction were measured at a site 5 mi southeast of Columbia, Mo., in an area of gently undulating topography locally free of trees and other obstructions. A Belfort type L anemometer (starting speed 2.5 mph) was mounted on a small building at an elevation of 28 ft above the ground. The anemometer transmitted wind speed and direction to a chart recorder set to advance at 3 inches hr^{-1}.

Six months of data were chosen for this study: July 1968, 1969, 1970, and January 1969, 1970, 1971. Five minute averages of wind speed and direction were abstracted from the records and transformed to north-south, v, and east-west, u, components.

To obtain power spectral estimates of the lower-mesoscale eddy energy as a function of variables such as wind speed, wind direction, time of day, and the presence or absence of thunderstorms, it was necessary to break the observations into shorter intervals than the monthly periods over which they were abstracted. The shortest interval chosen was one-half day, allowing spectra to be computed out to periods of about 4 hr.

To achieve some measure of stationarity, the data were first smoothed by applying a normal distribution smoothing function using a standard deviation of 4 hr as shown in Fig. 2, and then filtered by subtracting the smoothed curve from the original data. The filtered data were then referred to as $\hat{u}$ and $\hat{v}$. Twelve days were missing out of the six months of record. For the purpose of smoothing, zeroes were substituted for the missing data. All values within 8 hr of any missing data point were omitted from the analysis.

A standardized program was used to compute the power spectra from the filtered data (Dixon, 1968). Daily power spectral estimates for both $\hat{u}$ and $\hat{v}$ were computed over 24-hr periods beginning at 0000 CST for 48 frequencies ranging from 1 cycle per $8\frac{1}{3}$ hr to the Nyquist frequency of 1 cycle per 10 min. Semi-diurnal estimates were computed for daytime (0600 to 1800 CST) and nighttime (1800 to 0600 CST) at 24 fre-

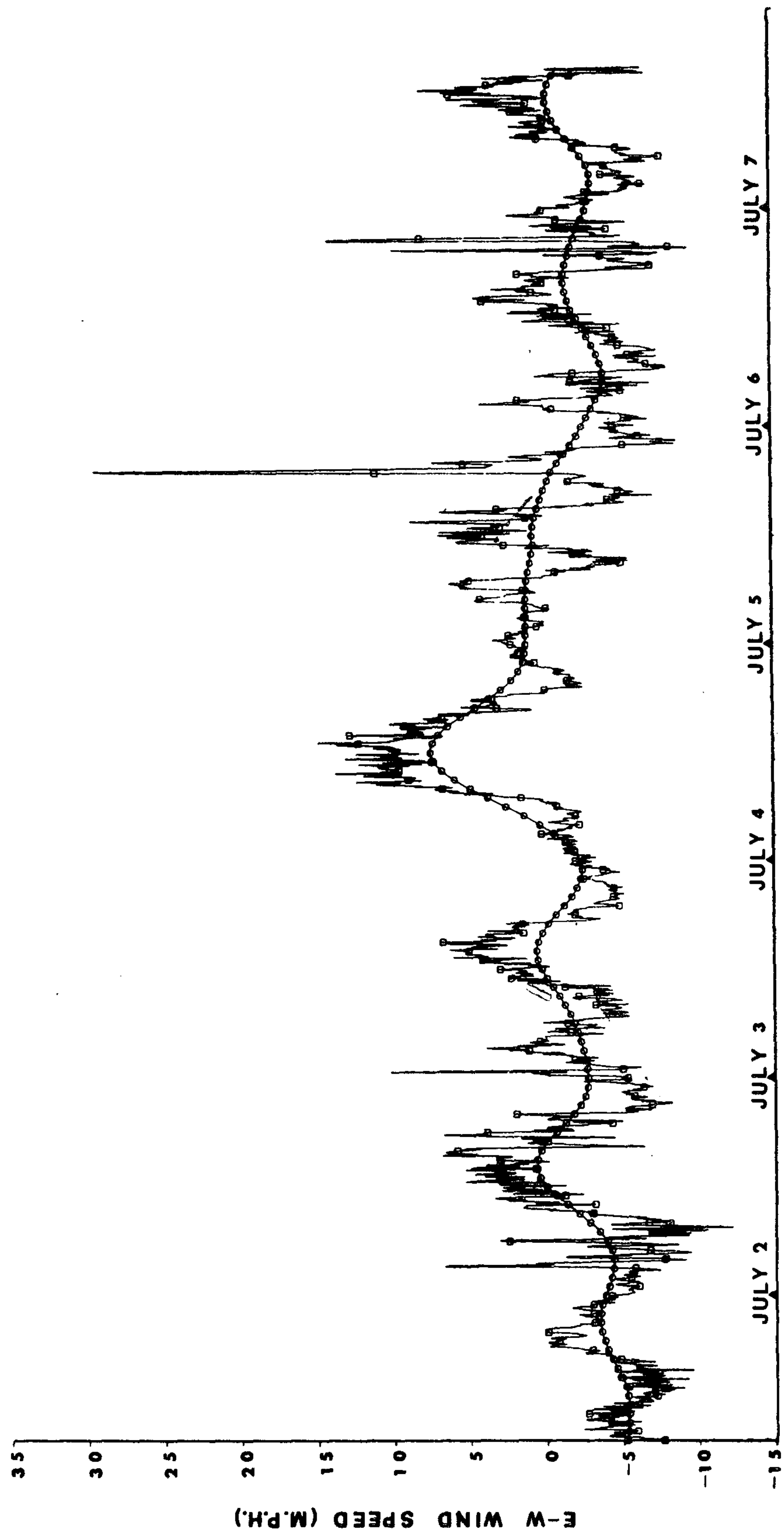

FIG. 2. Five-minute averages of the east-west wind component (squares) and the same data after normal distribution smoothing (circles) for July 1969. The circles and squares are plotted at hourly intervals. The date markers are located at 0000 CST.

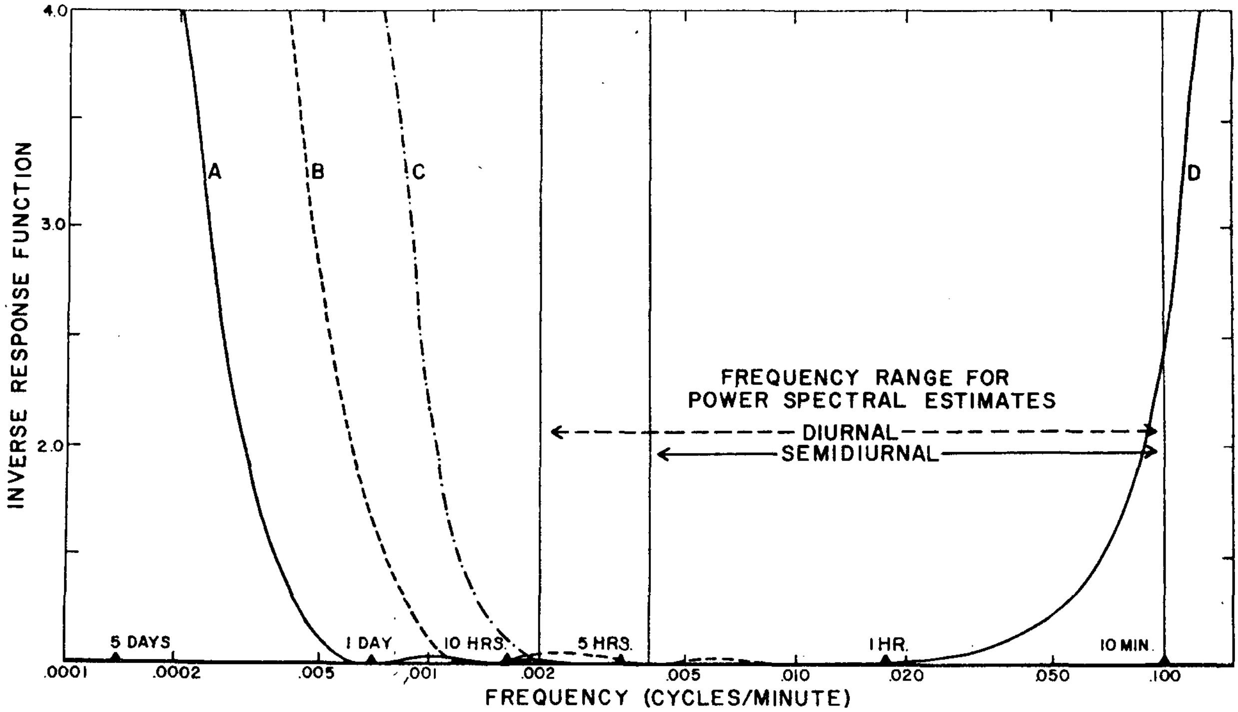

FIG. 3. Inverse frequency response of the measured spectrum due to a finite observation interval of 24 hr, A, a finite observation interval of 12 hr, B, normal distribution smoothing, C, and 5-min averaging, D.

quencies ranging from 1 cycle per $4\frac{1}{6}$ hr to the Nyquist frequency.

Distortions in the power spectra generated by each of the operations performed on the data were reduced by applying appropriate inverse response functions. The effect of a finite sampling period is corrected by the approximate relation given by Ogura (1957):

$$S_\infty(\omega)=S_T(\omega)\left[1-\frac{\sin\left(\frac{\omega T}{2}\right)}{\frac{\omega T}{2}}\right]^{-1},$$

where $S_\infty(\omega)$ is the corrected spectral density and $S_T(\omega)$ the spectral density for a finite period T. The effect of 5-min averaging is corrected by a relation first used in the meteorological literature by Griffith *et al.* (1956):

$$S_u(\omega)=S_{\bar{u}}(\omega)\left[\frac{\sin\left(\frac{\Delta t\omega}{2}\right)}{\frac{\Delta t\omega}{2}}\right]^{-2},$$

where Δt is the averaging interval (5 min) and $S_{\bar{u}}(\omega)$ the measured spectral density of the averaged wind. The effect of using the normal distribution function is corrected by a relation computed by the authors to be (see Bracewell, 1965, p. 130)

$$S(\omega)=S_f(\omega)\left[1+\exp(-\omega^2\sigma^2)-2\exp\left(-\frac{\omega^2\sigma^2}{2}\right)\right]^{-1},$$

where $S_f(\omega)$ is the spectrum of the filtered data. The inverse response functions for each of the three processes (Fig. 3) indicate that only the 5-min averaging significantly affects the spectra over the range of frequencies considered.

Two types of aliasing error occur in power spectra. The first, observed in one-dimensional spectra when wave fronts are not aligned normal to the axis of measurement, does not change the shape of the spectrum for isotropic turbulence in the "inertial-subrange" (Tennekes and Lumley, 1972, p. 265) and therefore affects none of the comparisons to be made later. The same conclusion is drawn for two-dimensional isotropic wall turbulence (Hinze, 1959, p. 268).

The second aliasing error occurs in spectral analysis when discrete samples are drawn from continuous data. Energy from all frequencies higher than the Nyquist frequency is folded into the measured spectrum. The effect for random turbulence is unpredictable (Griffith *et al.*, 1956; Oort and Taylor, 1969). Five-minute averaging serves to eliminate most of the aliasing problem but no estimate of the magnitude of the remaining error is available.

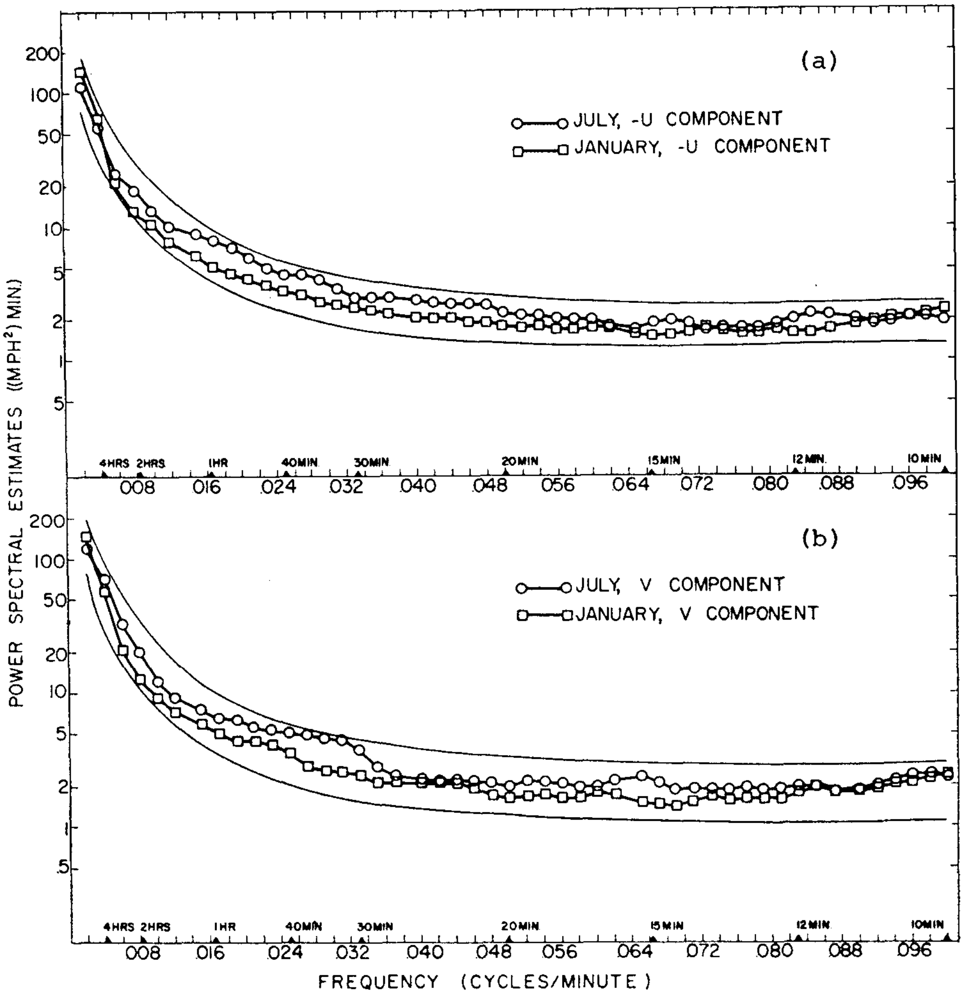

FIG. 4. Ensemble averages of diurnal power spectral estimates for July and January for both u and v components. The solid curves are obtained by excluding the three highest and lowest spectral estimates from the estimates of the six monthly averages.

4. Results

Ensemble averages of daily power spectral estimates of $\hat{u}$ and $\hat{v}$ were computed for the months of January and July (Fig. 4). No significant difference can be discerned between the January and July spectra despite observed differences in the mean wind speed (Table 1). These results strongly suggest that eddy energy in the lower-mesoscale range is not produced by shearing stress in the boundary layer, at least for the reported wind speeds. No difference was observed between the spectra for the u and v components in any of the data selected for this study (Fig. 4), a result which would be expected in isotropic turbulence only if the average deviation of the wind from the north-south axis lay at approximately 45° (Hinze, 1959, p. 167).

TABLE 1. Observed average wind speeds (mph) at Columbia, Mo., for January 1969, 1970, 1971 and July 1968, 1969, 1970.

	Days	Nights
January	11.6	10.7
July	9.4	7.4

The effect of stability on the energy in the lower-mesoscale range is clearly demonstrated by the day-night comparisons shown in Figs. 5a–c. During July a comparison of the daytime power spectral estimates with those for the nighttime reveals that the former contain almost an order of magnitude more energy than the latter for periods <20 min, gradually decreasing to about a factor of 2 difference at a period of 4 hr. It is obvious that the addition of turbulent energy through convective instability is not confined to eddies with periods near the microscale peak, but occurs in significant quantities throughout the lower-mesoscale range. The anomalous curve for July 1969 will be discussed later.

A comparison of the January daytime and nighttime spectra (Fig. 5b) also reveals more energy in the former, although, as might be expected considering the reduced solar radiation in January, the effect is much

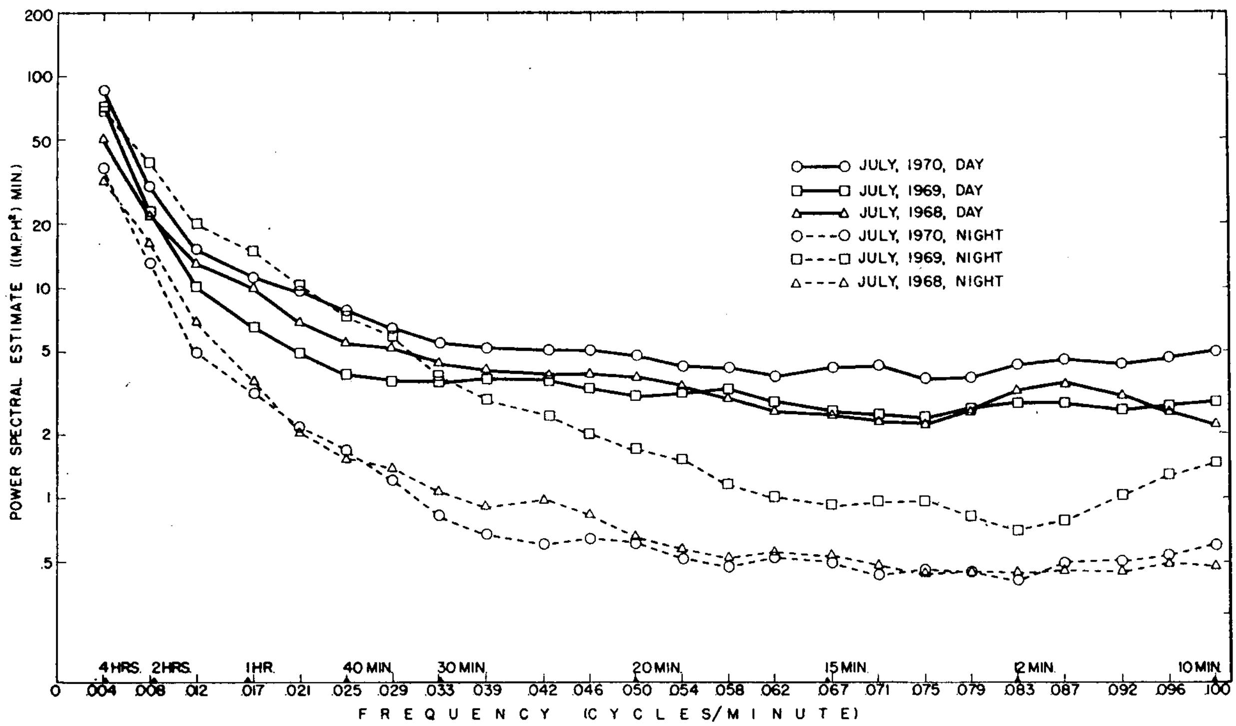

FIG. 5a. Ensemble averages of daytime and nighttime power spectral estimates for three July months.

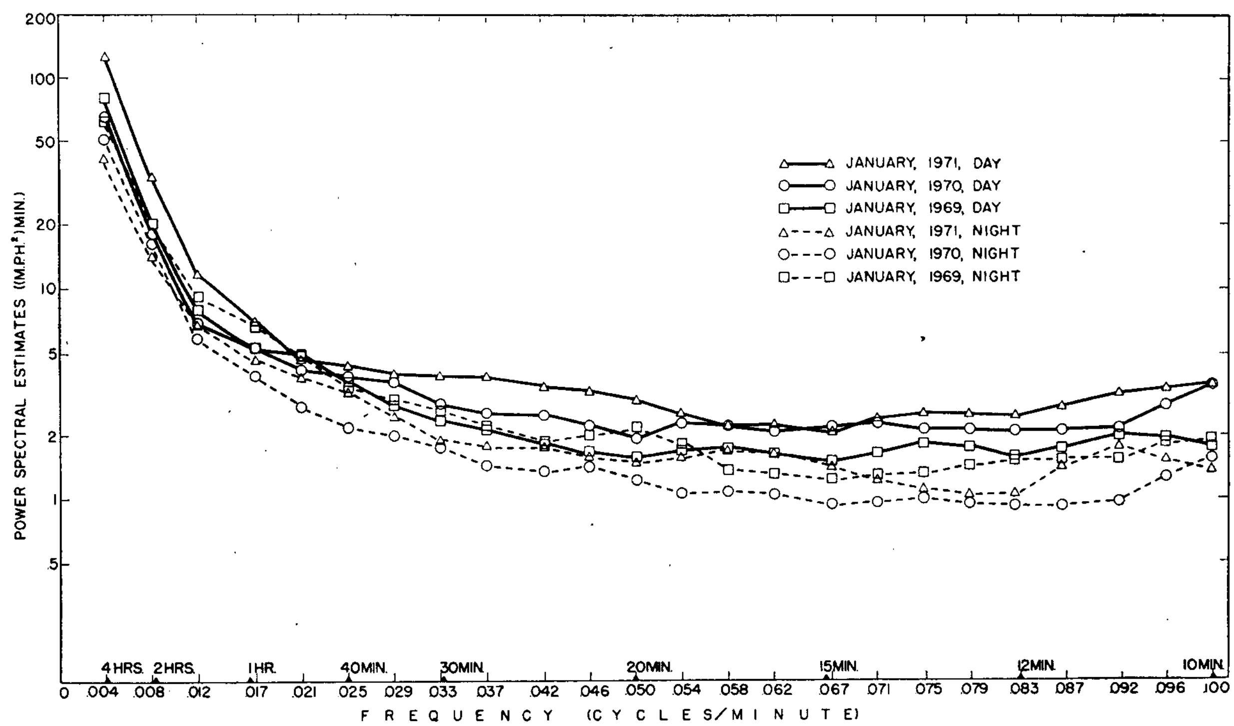

FIG. 5b. As in Fig. 5a except for three January months.

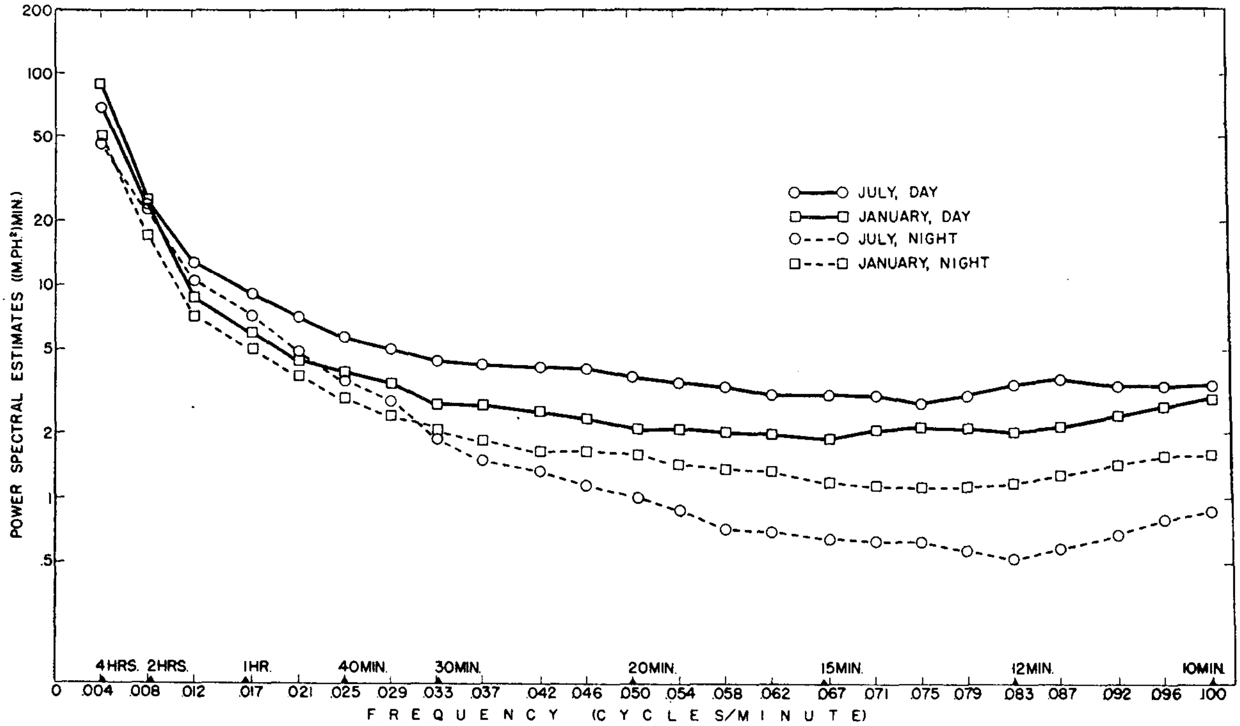

Fig. 5c. As in Fig. 5a except for July and January.

smaller than in July. An average of all the daytime and nighttime spectra for January and July (Fig. 5c) clearly shows the greater July range.

The anomalous nighttime spectrum of July 1969 contains more energy between periods of 1 to 4 hr than even the daytime spectra (Fig. 5b). A check of the

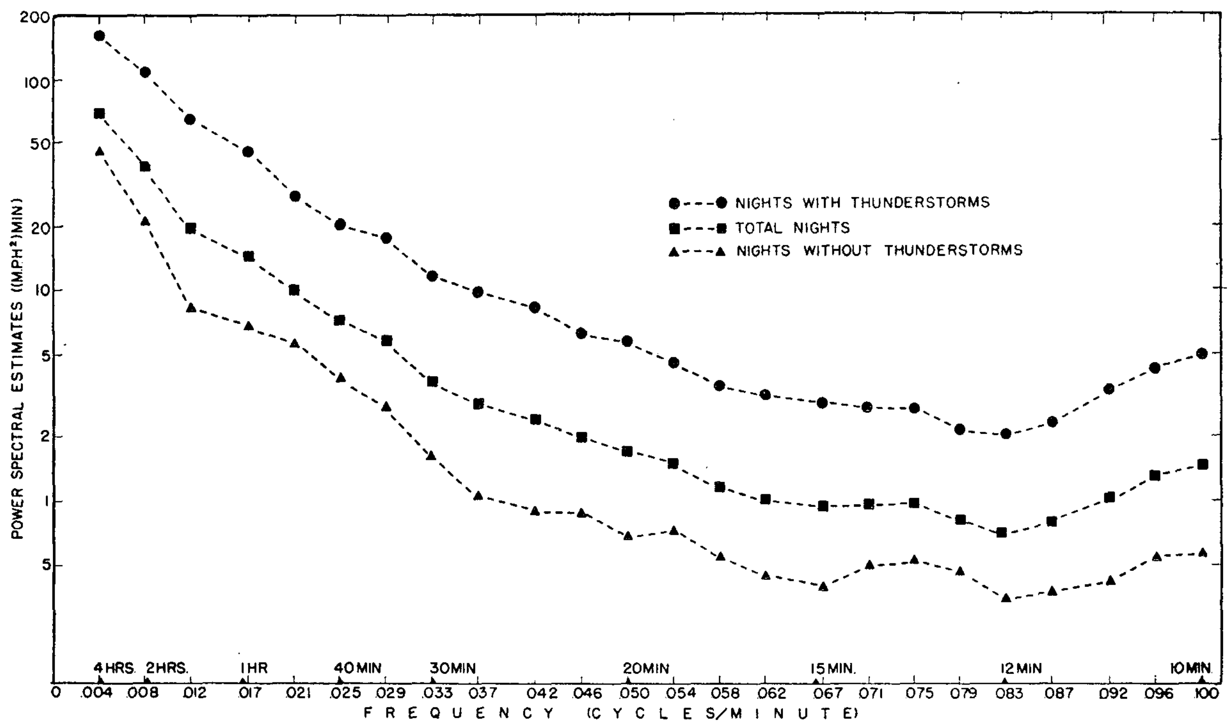

Fig. 6. Ensemble averages of nighttime power spectral estimates for January 1969 for nights with and without thunderstorms.

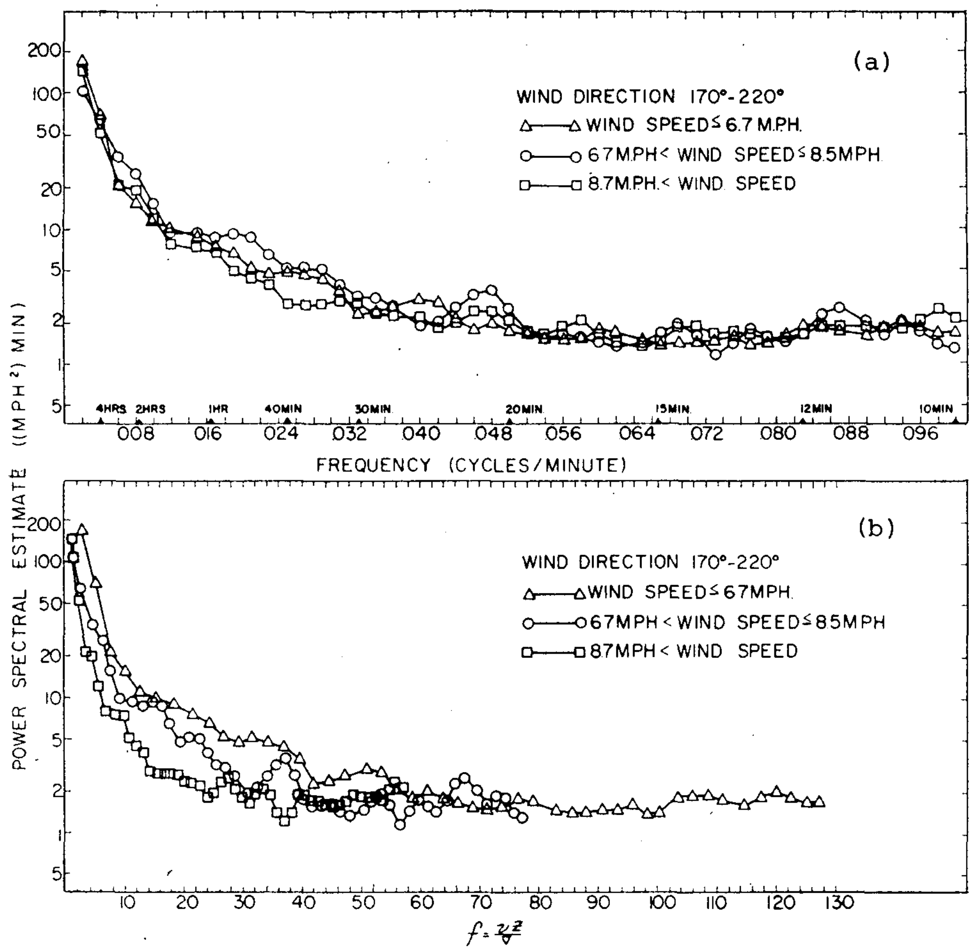

FIG. 7. Ensemble averages of diurnal power spectral estimates for three wind speed categories in the wind direction range 170°–220° as a function of the regular frequency range (a) and the dimensionless frequency f (b).

Local Climatological Data disclosed that eight nighttime thunderstorms occurred during July 1969 as compared to one and two for July 1968 and 1970. Nighttime power spectra prepared for July 1969 with the eight thunder-

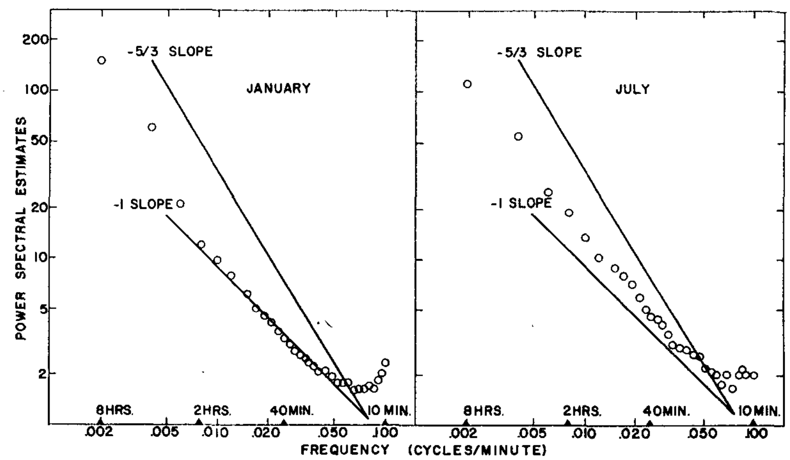

FIG. 8. Power spectra of the u components for ensemble averages of all daily January and July power spectral estimates plotted on a log-log scale. Lines with slopes of $-5/3$ and -1 are shown.

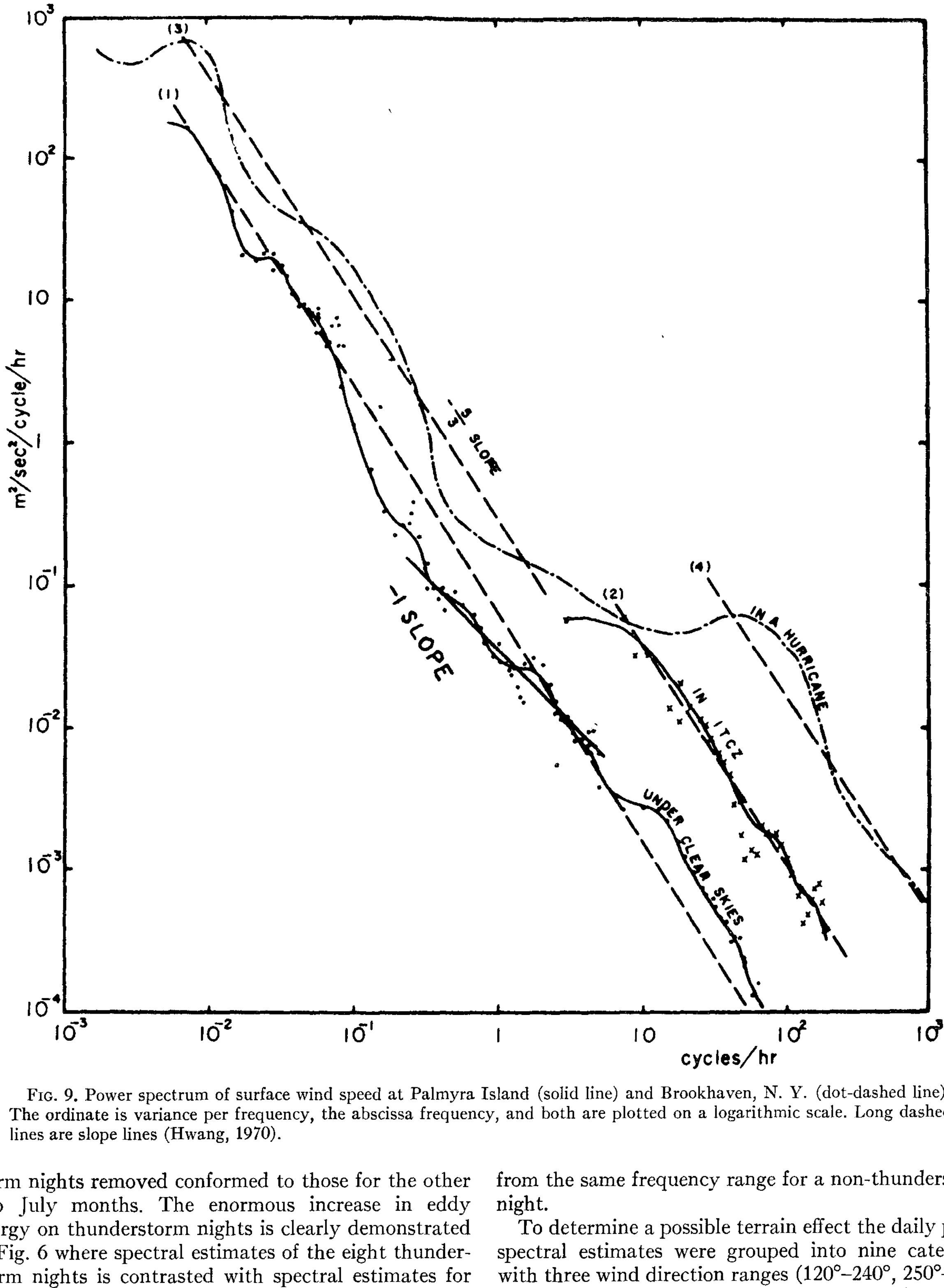

FIG. 9. Power spectrum of surface wind speed at Palmyra Island (solid line) and Brookhaven, N. Y. (dot-dashed line). The ordinate is variance per frequency, the abscissa frequency, and both are plotted on a logarithmic scale. Long dashed lines are slope lines (Hwang, 1970).

storm nights removed conformed to those for the other two July months. The enormous increase in eddy energy on thunderstorm nights is clearly demonstrated in Fig. 6 where spectral estimates of the eight thunderstorm nights is contrasted with spectral estimates for the remaining nights of that month. From these data one can infer that energy dissipation due to the cascade from the mesoscale range on one thunderstorm night in Missouri is about seven times greater than that from the same frequency range for a non-thunderstorm night.

To determine a possible terrain effect the daily power spectral estimates were grouped into nine categories with three wind direction ranges (120°–240°, 250°–350°, 360°–110°) and three wind speed ranges (<5 mph, 5–10 mph, >10 mph. Ensemble averages in each category were computed and the results compared. No consistent peaks could be associated with distinctive terrain features such as the Ozark mountains and only minor differences between the categories were observed.

A breakdown into twelve categories with six wind direction ranges (170°–220°, 230°–280°, 290°–340°, 350°–40°, 50°–100°, 110°–160°) and two wind speed ranges (<6.7 mph, >6.7 mph) also failed to reveal any directional effects. Plotting the spectra against the dimensionless frequency $f=\nu z/\bar{u}$ failed to reveal any outstanding peaks [Fig. 7; for a complete display see Heddinghaus (1972)].

One must conclude that in central Missouri there is no observable effect of wind direction on the power spectral estimates in the lower mesoscale range. This may be attributed in part to the uniformity of terrain roughness in all directions from Columbia, in part to the inability of the scale of roughness elements present to influence turbulence in the lower-mesoscale region, and possibly in part to the turbulent destruction of any large-scale eddies produced by distant topographic features such as the Ozark mountains. It is also possible that more data and narrower wind speed and direction categories would have revealed some topographical effects.

The frequency dependence of the energy spectrum for three-dimensional isotropic turbulence in the inertial subrange follows the $-5/3$ law as proven theoretically by Kolmogoroff (1941) and demonstrated both for the lower-mesoscale region and for the high frequency side of the microscale peak (Vinnichenko and Dutton, 1969; MacCready, 1953; Pond *et al.*, 1963; and others).

In the strong layer of shear near the ground, turbulence is not isotropic except for the smallest eddy sizes. This nonisotropy is a consequence of preferential generation of turbulence in the direction of the mean flow and of the presence of a solid lower boundary. Near the ground, turbulence should be more typical of what Hinze (1959, p. 501) has characterized as "wall turbulence." "Wall turbulence" is characterized by a rapid cascade of energy from larger to smaller scales brought about by the action of strong boundary layer shear on eddies which extend for a significant distance through the shear layer. In this region both theory[2] (Hinze, 1959, p. 268) and observation (Klebanoff, 1954; Laufer, 1954) indicate a -1 law for both the three-dimensional and the one-dimensional spectra.

Ensemble averages of the u component of all daily power spectra for January and July are shown plotted in Fig. 8 on a log-log scale. The slopes conform to a -1 law over a range of periods from 20 min to 2 hr. Corroboratory evidence can be seen in data presented by Hwang (1970) for Palmyra Island (Fig. 9). It can be concluded that three-dimensional turbulence in the layer of strong shear near the ground exhibits the same spectral characteristics as "wall turbulence." For periods >2 hr (depending on the wind speed) the eddies begin to occupy the full depth of the troposphere and become, more or less, two-dimensional. The slope of the profiles in Fig. 8 at periods >2 hr becomes indeterminate.

[2] The theory applies to homogeneous turbulence and therefore requires a linear shear in the boundary layer; however, the observations of Klebanoff and Laufer demonstrate that the law holds quite well in the logarithmic boundary layer.

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