Waves and Turbulence in the Stratosphere

D. K. Lilly

National Center for Atmospheric Research, Boulder, Colo. 80303

Peter F. Lester

San Jose State University, San Jose, Calif. 95192

(Manuscript received 10 September 1973, in revised form 13 December 1973)

ABSTRACT

Detailed stratospheric wind and temperature data were gathered by aircraft over the mountains of southern Colorado on 1 March 1970. In a unique operation, two instrumented RB-57F aircraft flew a total of twelve upwind and downwind legs at altitudes of 13 to 20 km, with an average separation of 600 m. The observational data show that the stratosphere was disturbed by sporadically-turbulent gravity waves of length 20–30 km, which were apparently generated directly or indirectly by the mountainous terrain. Wave amplitudes and turbulence frequency showed a general increase with height up to about 17 km, where they reached a maximum. The maximum amplitude layer was also characterized by a minimum in the mean wind speed. The horizontal and vertical velocity and temperature variances and covariances were evaluated and found to be generally consistent with predictions of linear gravity wave theory. The spectra of horizontal kinetic and potential energy were also calculated and appear to follow a \( -3 \) law over about a decade in wavenumber space. The covariance of the horizontal and vertical velocity perturbations is everywhere negative, with a mean downward momentum flux of about \( \sim 2 \) dyn cm\(^{-2}\). The correlation spectra show that most of this momentum flux is contributed by the long waves.

1. Introduction

On 1 March 1970, a coordinated investigation of expected mountain lee waves was conducted over the Sangre de Cristo Mountains of southern Colorado. Five aircraft measured the winds and temperature along 150–200 km tracks perpendicular to the mountain range at flight levels from about 6 km to 20 km MSL. The investigation was carried out as part of the 1970 phase of the Colorado Lee Wave Observational Program (Lilly et al., 1971).

Two instrumented RB-57F aircraft flew at altitudes above 13 km maintaining an average vertical separation of about 600 m, thus yielding the most detailed series of observations of the vertical structure of mesoscale waves in the stratosphere to our knowledge. In this investigation we concentrate on the results of these stratospheric flights.

2. Aircraft instrumentation and data reduction

Two different instrumentation and recording packages were carried by the participating RB-57F aircraft, one provided by the National Aeronautical Establishment of the National Research Council of Canada, and the other developed by the University of Dayton Research Institute for use by the Air Force Cambridge Research Laboratories. Henceforth the aircraft carrying the former instrumentation will be referred to as “Coldscan” (CS) while that with the latter will be known as “Coldtoy” (CT).

Detailed discussions of the instrumentation and of the preliminary data reduction have been given by Lilly et al. (1971) and by MacPherson and Lum (1969) for the CS system. The measured parameters utilized in the present study include static and dynamic-minus-static pressure, total temperature, Doppler ground speed and drift angle, and magnetic heading. All of these parameters except the CS Doppler data were recorded on analog tape and subsequently digitized to allow extraction of data at 1-sec intervals. The Doppler drift angle and ground speed on CS were recorded manually by the navigator at 1-min intervals. Navigation data were also recorded manually on both aircraft at certain fixed positions on the flight legs. The above parameters were reduced, mostly by digital methods, to produce time records of position, ambient pressure, ambient temperature and potential temperature, and wind speed and direction. Because the instrumentation and calibration procedures were by no means optimal for the purposes of this study, a number of adjustments were made and the final results have various remaining uncertainties as described below.
The position data were obtained by utilizing the navigator's notes on positions from standard navigational aids (principally DME—distance measuring equipment), and filling in between these by integrating the recorded Doppler ground speeds. Because of various equipment and manual recording errors these positions have an uncertainty of about \( \pm 3 \) km for CT, probably somewhat worse at the highest altitudes. The situation for CS was aggravated by an uncertainty of up to 1 min in timing between the navigator's notes and the CS data recording system time ticks. Subjective adjustments have been made that are believed to reduce the uncertainty to generally less than 30 sec, corresponding to \( \pm 6 \) km of distance along the track.

The ambient pressure was taken to be the same as the measured static pressure on one aircraft (CT), with a constant correction applied to the other. The accuracy is unknown but was originally believed to be within \( \pm 2 \) mb for each aircraft. From flight data comparisons between the two (on a different day) it was found that the CS static pressure was recorded as 3.5 mb higher than that from CT at an ambient pressure of 200 mb. In addition, it has been possible to make comparisons between the tape-recorded static pressure from CT and that measured by the pilot's altimeter and recorded on the flight log. These comparisons also show differences as large as 3 mb, but there is no way to ascertain whether, if either, is correct. This uncertainty is most significant in its effect on the vertical gradient of mean potential temperature, as determined by flight-leg averages of the two aircraft. The correction applied to the present data consists of decreasing the CS static pressure by 3.5 mb on the assumption that the error was constant with height and probably due to a recording bias. This led to an adjustment in potential temperature of about 2K for the lowest legs and about 6K for the highest.

It should be regarded as about equally probable that the error was one of recording amplification, which would be corrected by multiplying the observed pressures (and potential temperatures) by a constant ratio. The difference between these two possible corrections, and the implied uncertainty, increases with altitude, which is generally the case for all the parameters measured by external sensors. The pressure changes along a flight leg, however, can be regarded as considerably more accurate than the absolute uncertainties.

The ambient temperature was computed by use of the reduction formula

\[
T = T_{\text{r}} \left[ 1 - \frac{1}{R} \left( \frac{\rho}{\rho + \Delta \rho} \right)^{\frac{R}{\rho + \Delta \rho} - 1} \right],
\]

where \( T_{\text{r}} \) is the measured “total” temperature, \( \rho \) the static or ambient pressure, \( R \) and \( \rho_{\text{r}} \) the gas constant and specific heat at constant pressure, \( \Delta \rho \) the “pitot-static” or dynamic-minus-static pressure, and \( r \) the recovery factor, which differs from unity to the extent that the “total” temperature is less than that which would occur from adiabatic stagnation of the air flow relative to the sensor. Although the manufacturer's figures for \( r \) are close to unity (\( \sim 0.98 \) for CT, 0.995 for CS), some workers prefer to use values closer to 0.9 (personal communication with J. M. Nichols, British Meteorological Research Flight, and R. Braham, The University of Chicago). For this study, \( r \) was assumed to be unity for both aircraft. A reduction to 0.9 would increase the measured mean temperature by about 2K and would also lead to changes in the fluctuations along each leg tending toward a positive correlation between temperature and airspeed.

Flight data comparisons at 200 mb showed a difference of 0.5K between the CS and CT recorded “total” temperatures, with CS higher. This is in the same direction as the differences expected from the lower recovery factor of CT, though the magnitude is larger. Since the difference was relatively small compared to other possible errors, it was ignored in further processing. The reduction to ambient temperature through Eq. (1) is also sensitive to errors in static and pitot-static pressures. The uncertainty derived from static pressure errors is small, generally less than 0.5K, but that which arises from an error in pitot-static pressure (to be discussed next) may be as large as 3K at the highest levels. Thus, the total uncertainty of ambient temperature is 1-2K at the lowest levels, 3-5K at the highest. Possible errors in potential temperature are about double these values, but relative uncertainties along a flight leg are probably an order of magnitude smaller except at the smallest measured scales. The response time of the CT temperature sensor at the higher altitudes is several seconds, which is substantially slower than that of the static and pitot-static pressure sensors. This leads to spurious correlations of air speed and temperature at periods less than 10 sec. These effects were removed from the data when necessary by additional filtering, but the resultant spectral amplitudes should not be considered meaningful for frequencies >0.1 Hz.

The calculation of wind speed and direction involves combining four separate data outputs, and of these only the magnetic heading (corrected to true) is regarded as fully satisfactory. The true air speed (TAS) was calculated as an intermediate step to determining the wind vector from the relation

\[
TAS = \left[ 2 \rho T_{\text{r}} \left( \frac{\rho}{\rho + \Delta \rho} \right)^{\frac{R}{\rho + \Delta \rho}} \right]^{\frac{1}{2}},
\]

The principal error factor here is probably the recorded pitot-static pressure \( \Delta \rho \) which is proportional to the square of indicated airspeed. The comparison flight between CS and CT at 200 mb ambient pressure showed only rather small indicated air speed differences, of order 1 m sec\(^{-1}\). Comparisons with the pilot's indicated air speed, however, showed differences as large as 5 m sec\(^{-1}\) at the highest altitudes on CT.
The other elements of the wind vector calculation come from the Doppler navigation system, whose known or possible errors are numerous and somewhat unpredictable. First, it is believed, from comparison of Doppler and inertial systems on other aircraft, that very little useful information is present for periods ≤30 sec. Thus, for the present study the CT Doppler information was subjected to a fast-Fourier-transform ramp filter, with full cut-off at 30 sec and full cut-on at 60 sec. The CS data, not being available continuously, could not be treated this way, and presumably suffer errors of 1–3 m sec\(^{-1}\) from aliasing of high-frequency noise. In addition, the ground track integrated from the output of the Doppler systems was compared carefully with the navigator’s position record, not only for the flight legs on this day but for those on several other days in which the same instrumentation and recording system were utilized. From these comparisons it was deduced that the CT Doppler ground speeds were consistently 10 m sec\(^{-1}\) high, probably due to inaccurate recorder calibration. The CT data were then

Fig. 1. The location and topography of the study area in south central Colorado. The flight track (indicated by the diagonal line) traverses the Sangre de Cristo Range, with peaks up to 4 km MSL, and the Pueblo VORTAC navigational aid. The contour interval is 610 m (2000 ft). Heights above 12,000 ft are cross-hatched diagonally toward upper right, heights between 8000 and 12,000 ft toward upper left.

though they were substantially less than that below 150 mb. It is believed that the CS measurements may be somewhat more reliable at the higher altitudes.

Fig. 2. Cross section of topography and flight paths for the two stratospheric aircraft on 1 March, 1970. The twelve traverses were completed in about 2 hr 20 min. Times (GMT) at the eastern ends of the traverses are indicated at right. The abbreviations CS and CT stand for Coldscan and Coldtoy, the operational code names for the two aircraft. The altitude-pressure relationships are determined from the U. S. Standard Atmosphere.
corrected for this error. The momentum flux results discussed in Section 4 suggest that this correction may have been overdone, however. No comparable error was found in CS, where the navigator read the ground speed directly from his dials. Drift angle bias errors of order 2–3 deg were found on both aircraft, but these do not enter significantly into the present study, since the mean wind component perpendicular to the mean track is not utilized.

In summary, the parallel wind component is believed to be reliable to ±3 m sec⁻¹ in both its mean and in the somewhat shorter term fluctuation, except in the highest one or two levels where it may be in error by as much as 6 m sec⁻¹. For periods < 30 sec, where the Doppler information is removed entirely, the real ground speed fluctuations are probably small and the air speed fluctuations measure those of wind speed accurately. Thus, for the 10–30 sec periods for CT, and 5–30 sec periods for CS, wind speed fluctuations may be accurate to order 0.5 m sec⁻¹.

3. Topographic and meteorological environment

The location of the study area is shown in Fig. 1. As one proceeds to the northeast along the flight track, the topography rises sharply from the San Luis Valley (~2.5 km MSL) to the crest of the mountains which offer a nearly continuous obstacle to southwesterly flow up to elevations of 4 km MSL. The high, but rather irregular crest of the San Juan Range is located about 100 km to the southwest of the Sangre de Cristo Range and the lower Wet Mountains are found about 30 km to the northeast.

Fig. 3a. Sea level pressure analysis over western United States at 0000 GMT 2 March 1970, the approximate end time for the flights. The flight track through the Pueblo VORTAC (PUB) is indicated. The topography (2000 and 3000 m contours) underlies the analysis.

Fig. 3b. Height (m) and temperature (°C) analysis at 150 mb at 0000 GMT 2 March 1970. Wind arrows are labeled with velocity in m sec⁻¹.

Fig. 3c. As in Fig. 3b except for 70 mb.

Fig. 2 illustrates topography and flight paths in an abbreviated cross section. Each aircraft flew a series of six upwind and downwind legs in about 1200 m altitude increments except in the upper portion of the cross section where waves and turbulence either reduced or increased separation. The twelve traverses took about 2 hr and 20 min.

During the flights the pilots, on several occasions, flew constant power settings (rather than constant altitude). The result of this procedure is clearly visible in the variations of flight altitude (Fig. 2) especially on
legs 3, 5 and 6 for CT, and leg 5 for CS. An aircraft flying a constant power setting will tend to have an altitude fluctuation in phase with streamlines for a downwind flight and will be 180° out of phase for an upwind flight. It is not possible to use this effect to deduce quantitative vertical velocity, however, without accurate pitch data. The mean wind direction is from left to right in Fig. 2 and in similar figures, and is very nearly parallel to the aircraft track at all altitudes.

The large-scale flow patterns which dominated the area during the period of aircraft investigations are illustrated in Fig. 3. These include the analyses for sea level (Fig. 3a), 150 mb, which is near the lowest flight leg (Fig. 3b) and 70 mb, which is close to the upper-most flight level (Fig. 3c). A west-east pressure gradient is present across the Sangre de Cristo Mountains at the lowest level and generally southwesterly flow prevails throughout the lower stratosphere. A relatively strong jet stream is indicated by 40-50 m sec⁻¹ wind speeds at 150 mb. The apparent wave pattern in the isotherms at 70 mb (Fig. 3c) follows the station data and has also been observed by other investigators in association with stratospheric CAT in mountain lee waves (Crooks et al., 1968). The initial analysis of the 70-mb height field showed similar waves. Although the data were subsequently corrected for radiation errors, the wave pattern persisted and the winds appeared to be strongly ageostrophic. It is assumed that the wave pattern resulted from the increased radiosonde error with height and contamination by gravity wave aliasing. Since these errors could not be separated, contours were finally smoothed to conform with the wind field.

Temperature (T), potential temperature (θ), and wind data based on averages along each flight leg and from rawinsondes launched at 0000 GMT 2 March 1970 (at the time of the conclusion of aircraft flights) at Denver, Colo. (DEN), and Albuquerque, N.M. (ABQ), are shown in Fig. 4. DEN and ABQ are located, respectively, 230 km north and 320 km south of the study area. Profiles of temperature and wind speed have been fitted by eye to the aircraft data.

The rawinsonde temperature and wind profiles both display a sawtooth pattern that is characteristic of a stratified atmosphere disturbed by mesoscale wave motion (Crooks et al., 1968). The somewhat smoother aircraft profiles, together with their large standard deviations, suggest that such soundings cannot be relied upon for mean state profiles but indicate the existence of mesoscale disturbances. Note from the standard deviations in Fig. 4 that there is a general increase in the variations of temperature and wind velocity with
altitude along the aircraft track. It will be seen that this increase in variance is associated with an upward increase in the intensity of mesoscale wave disturbances to a maximum just below the highest flight levels.

For purposes of construction of vertical sections along the flight tracks, potential temperature data were numerically filtered to eliminate motions on scales $\lesssim 15$ km. This additional filtering was applied to eliminate unrealistic gradients between aircraft tracks (e.g., superadiabatic lapse rates) probably caused by traveling and occasionally breaking short waves. Therefore, the analysis presented in the upper portion of Fig. 5 represents the longer (and stationary or slow-moving) wave structure of the lower stratosphere. Because the CS
recording system operated on the basis of "events," i.e., significant changes in temperature and/or the occurrence of turbulence, data were not available for the far downwind portions of those legs at lower levels where there was no significant activity. Also, data samples were truncated further because of the loss of end points in filtering.

Fig. 5 also includes (below the solid line) an analysis of the potential temperature field measured by three other aircraft in the troposphere. This analysis was done manually and with somewhat less care than that for the stratospheric data. It is presented here for completeness, but most of the detailed computations shown later have not been carried out in the lower levels, nor has as careful an attempt been made to verify or correct the data.

Several significant features can be noted on the potential temperature field shown in Fig. 5:

1. Although wave activity prevails throughout the stratosphere, the most intense waveness is found downwind of the crest of the Sangre de Cristo Range between altitudes of 17 and 20 km.
2. The dominant wave length is 20 to 30 km.
3. The double amplitude varies from a few hundred meters at lower stratospheric levels to 1 km in the zone of the most intense activity.
4. The waves have a tendency to tilt downwind with altitude. This phase change is probably due to a downstream progression of the waves which appears as a downwind tilt because the higher levels were flown last.
5. There are some indications of distinct laminae in the potential temperature field in the form of alternately weak and strong vertical gradients of \( \theta \). In particular, an adiabatic layer is found just downstream of the region of strongest waveness.
6. Clear air turbulence (reported or deduced from vertical acceleration records) occurred throughout the region of major wave activity but was most frequent near 17 km. In general it was reported as "light" but several episodes of "moderate" turbulence occurred.
7. The tropospheric analysis shows rapidly diminishing wave amplitude below the tropopause, with virtually no hint of activity near 300 mb, but with a simple forced mode pattern of detectable amplitude closer to the mountains.

The wind field based on CT continuous wind data, CS 1-min winds, and other recorded Doppler winds in the troposphere, are presented in Fig. 6. Since the flight tracks were approximately parallel to the mean wind vector at all levels, the data analyzed in Fig. 6 were wind magnitudes. Noteworthy features are:

1. The alternating layers of relatively strong and weak winds. These larger-scale patterns are reflected in the profile derived from the leg averages shown in Fig. 4 and further suggest the layered structure of the stratosphere. Instrumental uncertainties may account for part of the apparent layering, but there seems little doubt of the minimum near 17.5 km.
2. The wind speed minima in the upper portion of the diagram are generally associated with highly disturbed regions where the isentropes (Fig. 5) have steep slopes. Reference to the original data indicates that wind directions were occasionally easterly in this region.
3. Vertical shears reach a maximum above and below both the most active region of waving and the down- stream adiabatic layer. Maximum vertical shears are of the order of \( 3 \times 10^{-2} \) sec\(^{-1} \), measured over 500 m intervals. Even greater shears were found over smaller intervals (i.e., \( \sim 5 \times 10^{-2} \) sec\(^{-1} \)).
4. Tropospheric wind variations, like those of potential temperature, are insignificant in the upper troposphere but increase somewhat in the lower levels.

The apparent downstream progression of the waves, which otherwise resemble large-scale mountain waves, remains unexplained. The propagation rate appears to be a few meters per second, although quite variable. The results from the aircraft flights at lower altitudes have not been very helpful. In the upper troposphere the high mean flow and low stability are theoretically consistent with weak waves but not so weak as to essentially vanish. The wave activity apparently observed in the lower troposphere involved flights made in and out of clouds which had a distinctly cumuliform appearance. A possible conclusion is that the waves were actually induced by air flowing over moving cumulus clouds, rather than by the mountains directly, but the lack of strong shear near the cloud tops makes that mechanism dubious and the low-level wave field did apparently remain fairly steady during the flight period. The extremely weak activity in the upper troposphere suggests the possible existence of oblique propagation of wave energy, with the stratosphere in the flight area serving as a focus. The Sangre de Cristo Mountain Range in the flight area is not outstandingly threedimensional, however, and we have no other observational data to support the suggestion. A referee suggests the possibility of time variations of the flow over the mountains, but we have no evidence to support that suggestion either.

4. Wave energy, power spectra, and momentum flux results and interpretations

Statistical analyses were performed in both amplitude and frequency domains on data which had not been subjected to the 15-km filter. In general, probability distributions of \( \theta \) and VT (the wind component along the aircraft track) were not Gaussian. Most distributions showed significant skewness and/or large kurtosis. These features suggest, respectively, significant horizontal energy flux by the waves and the presence of horizontal inhomogeneities (Lester, 1972). A few distributions were bimodal, consistent with the presence of wave motions.
Fig. 6. Wind speed analysis corresponding to the isentropic analysis in Fig. 5. The mean wind direction is very nearly along the flight track at all levels, so that the profile is generally similar to one of wind component along the track. Note the alternating layers of relatively strong and weak winds, and the minima at about 17.5 km, associated with steeply sloping isentropes (Fig. 5).

Vertical velocities ($w$) were not measured directly by the aircraft, but for isentropic flow, we may write

$$w = -\left(\frac{\partial \theta}{\partial t} + \mathbf{V}_H \cdot \nabla \theta\right) \frac{\partial \theta}{\partial z}, \quad (3)$$

where $\mathbf{V}_H$ is the horizontal velocity vector. With the assumptions that wave fronts are two-dimensional, perpendicular to the flight path, and moving downstream with a speed $C$, Eq. (3) then becomes

$$w = \frac{(VT-C)\partial \theta/\partial s}{\partial \theta/\partial z}, \quad (4)$$
Table 1. Variances of potential temperature and wind components.

<table>
<thead>
<tr>
<th>Flight leg</th>
<th>VT (m sec⁻¹)</th>
<th>σ₁ (°K)²</th>
<th>σ_T (m² sec⁻²)</th>
<th>σ_N (m² sec⁻²)</th>
<th>σ_w(C=0) (m² sec⁻²)</th>
<th>σ_1² (m² sec⁻²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CS6</td>
<td>21.6</td>
<td>33.0</td>
<td>36.4</td>
<td>28.3</td>
<td>1.96</td>
<td>66.7</td>
</tr>
<tr>
<td>CT6</td>
<td>21.6</td>
<td>64.6</td>
<td>36.4</td>
<td>28.3</td>
<td>1.96</td>
<td>66.7</td>
</tr>
<tr>
<td>CS5</td>
<td>39.3</td>
<td>33.0</td>
<td>20.4</td>
<td>7.7</td>
<td>2.44</td>
<td>30.5</td>
</tr>
<tr>
<td>CS4</td>
<td>39.3</td>
<td>33.0</td>
<td>20.4</td>
<td>7.7</td>
<td>2.44</td>
<td>30.5</td>
</tr>
<tr>
<td>CT4</td>
<td>18.8</td>
<td>66.6</td>
<td>24.2</td>
<td>9.0</td>
<td>2.51</td>
<td>35.5</td>
</tr>
<tr>
<td>CT3</td>
<td>17.3</td>
<td>11.6</td>
<td>13.2</td>
<td>9.8</td>
<td>7.67</td>
<td>30.7</td>
</tr>
<tr>
<td>CS3</td>
<td>28.0</td>
<td>23.9</td>
<td>16.9</td>
<td>11.7</td>
<td>0.92</td>
<td>29.5</td>
</tr>
<tr>
<td>CT2</td>
<td>36.9</td>
<td>12.5</td>
<td>16.9</td>
<td>11.7</td>
<td>0.92</td>
<td>29.5</td>
</tr>
<tr>
<td>CS2</td>
<td>21.1</td>
<td>12.9</td>
<td>16.9</td>
<td>11.7</td>
<td>0.92</td>
<td>29.5</td>
</tr>
<tr>
<td>CT1</td>
<td>21.1</td>
<td>12.9</td>
<td>16.9</td>
<td>11.7</td>
<td>0.92</td>
<td>29.5</td>
</tr>
</tbody>
</table>

where \( \partial \theta / \partial z \) is the derivative downstream. Since \( \partial \theta / \partial z \) could only be estimated from leg averages, this equation was not practical to use for calculation of \( w \) from the data. Instead, the continuity equation was used to deduce that \( \rho \dot{V}_T = \partial \psi / \partial z \) and \( \rho \dot{w} = - \partial \psi / \partial \theta \), with \( \psi \) a streamfunction. Since \( \psi \) should be a function of \( \theta \) alone (neglecting effects of turbulence), then \( \rho \dot{V}_T / \partial \psi / \partial \theta \). This function was evaluated from leg-averaged data, allowing \( w \) to be computed from

\[
w = - \frac{\partial \psi}{\partial \theta} \quad \rho \quad \partial \theta / \partial z,
\]

(5)

Results were obtained for \( C = 0 \), 4 and 8 m sec⁻¹.

The mean of VT and variances (\( \sigma^2 \)) of \( \theta \), VT, VN (the wind component normal to the aircraft track), \( w \) (as computed for \( C = 0 \), the steady-state assumption), and of the total wind vector \( \mathbf{V} \), are shown in Table 1. The flight legs are listed in descending order. CT5 is not included because a large fraction of the \( \theta \) record was contaminated by a spike in the static pressure data. The Doppler radar was also inoperative for an extensive period during that leg, resulting in the loss of wind data. As noted earlier, continuous wind data were not available for CS.

The variances for \( \theta \) and VT in Table 1 increase irregularly with height up to the region of greatest wave activity (CT4, CS4, CS5 and CT6). The vertical velocity variance has a maximum at CT2 but does not otherwise show a clear trend.

The total energy \( E_T \) of a two-dimensional wave packet may be written as the sum of kinetic and potential energies, i.e.,

\[
E_T = \frac{1}{2} \rho \sigma^2_{\theta} + \frac{1}{2} \rho \sigma^2_{\psi} N^2 / (\partial \theta / \partial z)^2,
\]

(6)

where \( \rho \) is the mean density of the air and \( N^2 \) the Brunt-Väisälä frequency, \( g (\partial \theta / \partial z) / \theta \). Eq. (6) is adapted from Bretherton (1969) with the mean squared particle displacement replaced by \( \sigma^2_{\psi} / (\partial \theta / \partial z)^2 \).

Energy computations were performed for the flight legs flown by CT. The values of \( N^2 \) and \( (\partial \theta / \partial z) \) were computed from the vertical profiles constructed from averages along the aircraft tracks (Fig. 4). Density \( \bar{\rho} \), potential energy \( P \), and kinetic energy \( K \) are shown in Table 2. The computed values of potential and kinetic energies vary over a rather wide range, but their averages over the five legs are nearly equal. Linearized gravity wave theory predicts this result for energies averaged over a vertical wavelength. It is interesting to note that the vertical distance between CT1 and CT6 is approximately equal to a vertical wavelength of a stationary mode of long horizontal wavelength.

We note from Table 1 that the ratio of vertical to total velocity variance is of order 0.05 for all legs except CT2, where it is about 0.25. These ratios also appear to be consistent with predictions of linear theory. From the continuity equation the ratio of vertical to total motion amplitude for each Fourier wave mode is equal to the ratio of total to vertical wavenumber. Thus, for a given horizontal wavenumber, \( |w| / |u| \) increases as the vertical wavelength. The wave equation is approximately

\[
k^2 + l^2 = \frac{N^2}{(\bar{\theta} - C)^2},
\]

(7)

where \( k \) and \( l \) are the horizontal and vertical wavenumbers, \( l^2 \) the Doppler-shifted Sverdrup parameter, and \( \bar{\theta} \) is identified with VT. Therefore, the ratio of vertical to total energy should be

\[
\frac{\sigma^2_w}{\sigma^2_\theta} = \frac{k^2}{l^2},
\]

(8)

If a horizontal wavelength of 25 km is assumed, along

Table 2. Mean density, stability and wave energy.

<table>
<thead>
<tr>
<th>Flight leg</th>
<th>( \bar{\rho} ) (10⁻³ g cm⁻³)</th>
<th>( \sigma^2_{\theta} ) (°K²)</th>
<th>( N^2 ) (10⁻⁶ sec⁻²)</th>
<th>( P ) (dyn cm⁻²)</th>
<th>( K ) (dyn cm⁻²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CT6</td>
<td>1.05</td>
<td>29.8</td>
<td>6.2</td>
<td>23.6</td>
<td>35.0</td>
</tr>
<tr>
<td>CT4</td>
<td>1.37</td>
<td>16.2</td>
<td>3.6</td>
<td>62.5</td>
<td>20.9</td>
</tr>
<tr>
<td>CT3</td>
<td>1.56</td>
<td>21.8</td>
<td>3.1</td>
<td>9.7</td>
<td>27.7</td>
</tr>
<tr>
<td>CT2</td>
<td>2.02</td>
<td>20.4</td>
<td>5.1</td>
<td>29.6</td>
<td>33.0</td>
</tr>
<tr>
<td>CT1</td>
<td>2.54</td>
<td>22.1</td>
<td>5.1</td>
<td>29.8</td>
<td>36.0</td>
</tr>
<tr>
<td>Average</td>
<td>1.94</td>
<td>19.2</td>
<td>5.1</td>
<td>29.8</td>
<td>36.0</td>
</tr>
</tbody>
</table>
with $C=0$, the ratio on the right of (8) is also of order 0.05 at all levels except CT2, where it is $\sim 0.17$ due to the large horizontal wind speed at that level.

Spectra of unfiltered (except for removal of high-frequency Doppler noise) CT wind components appear in Figs. 7–9. The horizontal axes are labeled in both the sampling frequency (Hz) and wavelength, as determined from the mean true air speed of the aircraft ($\sim 200 \text{ m sec}^{-1}$). Spectra of VT are characterized by peaks at wavelengths $>20 \text{ km}$ and by spectral slopes which approach $-3$ in the wavelength range from 3 to 20 km. The spectra of VN are cut off at wavelengths $<6 \text{ km}$ due to the filtering of the Doppler drift angle. The spectra of $w$ in Fig. 9 [computed from Eq. (5), $C=0$], are necessarily proportional to the potential temperature spectra multiplied by the square of the wavenumber.

Potential temperature spectra are shown in Fig. 10. The spectral slopes in the 3–20 km range also average about $-3$ for the lowest and highest flight legs (CT1, CS1, CS2, CS5, CS6) and appear to be steeper and somewhat irregular (especially for CS4 and CT3) at the intermediate levels. These flight legs show a large excess of spectral density over the $\theta$ spectra of flight legs immediately below, particularly at scales $>4 \text{ km}$. For example, compare CS1 and CS2 with CS3 and CS4. Spectral amplitudes should be disregarded for wavelengths $<2 \text{ km}$ because of inadequate instrument response for CT and digitizing errors for CS.

Plots with area proportional to the spectral density of $\theta$ (not shown) indicate the presence of several large $\theta$-variance peaks at scales of 10–35 km for all spectra.

Fig. 7. Spectra of CT wind tangential to the aircraft track (VT), obtained from winds measured by the Doppler system, but with the ground speed low-pass filtered by a 30–60 sec fast-Fourier transform ramp filter. Horizontal axis is labeled in both sampling frequency and wavelength, using a mean true air speed of the aircraft ($\sim 200 \text{ m sec}^{-1}$). The diagonal line has a slope of $-3$. Leg 5 is omitted due to recording difficulties on the aircraft.

Fig. 8. Spectra of CT wind normal to the aircraft track (VN). The drift angle is low-pass filtered (similar to the ground speed filtering on Fig. 7), which results in a cutoff for frequencies $>1/30 \text{ Hz}$.

There are minor peaks at smaller scales in the spectra for CS3 and CS4.

The tendency for both VT and $\theta$ to exhibit a substantial range with power spectra proportional to $k^{-3}$ is of some interest. Lumley (1964) and Shur (1962) predicted a buoyancy range with $E(k) \sim N^2 k^{-3}$ in the case of gravity waves produced by a turbulent shear embedded in a stratified layer. It has been suggested (Rhines, 1972) that a region of high-amplitude gravity waves sporadically breaking down into turbulence would also produce such a spectrum. The assumption that leads to this prediction seems to be that each spectral region is separately "saturated," i.e., at marginal stability with respect to turbulent breakdown,
then

\[ Z(k) = 2Ak^{-3}, \quad (10) \]

and

\[ \Theta(k) = \frac{A}{2} (\partial \Theta / \partial z)^2 k^{-3}, \quad (11) \]

where \( Z(k) \) and \( \Theta(k) \) are displacement and potential temperature variance spectra of the horizontal wavenumber \( k \). Eq. (10) implies a wave slope spectrum equal to \( 2Ak^{-1} \), so that each octave contributes equally to slope variance. The most probable mechanisms of wave breakdown are, however, either shearing instability or convective instability by wave overturning, and both of these seem to lead to limitations on the amplitude spectra of the vertical wavenumber. The relationships between vertical and horizontal spectra must depend on the predominant mechanism of exchange of wave energy between modes, whether by direct resonant interaction or by local turbulent breakdown followed by re-emission of wave energy.

We have estimated the dimensionless constant \( A \) of Eqs. (9)–(11) for the Coldtoy potential temperature and VT velocity spectra and for the Coldscan potential temperature spectra. Values range from \( 3 \times 10^{-4} \) to \( 3 \times 10^{-3} \), with the largest values occurring in the regions of large-amplitude waves, flight legs 3–5. The mean value of \( A \) is near \( 10^{-5} \). At some small-scale limit the spectra should merge into the inertial range form, with \( \Theta(k) \) and \( Z(k) \) becoming proportional to \( k^{-5/3} \). Our observational data are unable to provide evidence of this effect because of their inaccuracy at smaller scales.

Besides the power spectra discussed above, we have also computed, from the CT data, correlation spectra of the various components of velocity with each other and with potential temperature. Most of these were un-revealing and apparently random, but the quadrature correlation between VT and \( \theta \) and the co-spectrum between VT and \( w \) produced definite and interesting results.

![Fig. 10a. Spectra of potential temperature for CT. No filtering is done, but the response of the thermometer limits the validity to periods \( \geq 10 \) sec.](image)

![Fig. 10b. Spectra of potential temperature for CS. The spectra are believed to be valid down to periods affected by digitizing errors, \( \sim 3-10 \) sec.](image)

but the reasoning is not altogether clear. A horizontal spectrum of kinetic energy proportional to \( k^{-3} \) implies an identical wave potential energy spectrum and also displacement and potential temperature spectra proportional to \( k^{-3} \), i.e., if

\[ E(k) = AN^2k^{-3}, \quad (9) \]

then

\[ Z(k) = 2Ak^{-3}, \quad (10) \]

and

\[ \Theta(k) = \frac{A}{2} (\partial \Theta / \partial z)^2 k^{-3}, \quad (11) \]

where \( Z(k) \) and \( \Theta(k) \) are displacement and potential temperature variance spectra of the horizontal wavenumber \( k \). Eq. (10) implies a wave slope spectrum equal to \( 2Ak^{-1} \), so that each octave contributes equally to slope variance. The most probable mechanisms of wave breakdown are, however, either shearing instability or convective instability by wave overturning, and both of these seem to lead to limitations on the amplitude spectra of the vertical wavenumber. The relationships between vertical and horizontal spectra must depend on the predominant mechanism of exchange of wave energy between modes, whether by direct resonant interaction or by local turbulent breakdown followed by re-emission of wave energy.

We have estimated the dimensionless constant \( A \) of Eqs. (9)–(11) for the Coldtoy potential temperature and VT velocity spectra and for the Coldscan potential temperature spectra. Values range from \( 3 \times 10^{-4} \) to \( 3 \times 10^{-3} \), with the largest values occurring in the regions of large-amplitude waves, flight legs 3–5. The mean value of \( A \) is near \( 10^{-5} \). At some small-scale limit the spectra should merge into the inertial range form, with \( \Theta(k) \) and \( Z(k) \) becoming proportional to \( k^{-5/3} \). Our observational data are unable to provide evidence of this effect because of their inaccuracy at smaller scales.

Besides the power spectra discussed above, we have also computed, from the CT data, correlation spectra of the various components of velocity with each other and with potential temperature. Most of these were un-revealing and apparently random, but the quadrature correlation between VT and \( \theta \) and the co-spectrum between VT and \( w \) produced definite and interesting results.

![Fig. 11. Cospectra of tangential wind (VT) and vertical wind (w), showing negative correlations (hence, downward momentum flux) at all levels for nearly all large-scale components.](image)
Table 3. Velocity variances and momentum flux.

<table>
<thead>
<tr>
<th>Level</th>
<th>$\langle V^2 \rangle$ (m$^2$ sec$^{-2}$)</th>
<th>$\bar{\rho}$ (kg m$^{-3}$)</th>
<th>$\bar{\rho} \bar{V} \bar{W}$ ($\bar{m}^3$ sec$^{-2}$)</th>
<th>$\bar{\rho} \bar{V} \bar{w}$ (dyn cm$^{-2}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CT6</td>
<td>36.4</td>
<td>0.108</td>
<td>1.96</td>
<td>-1.88</td>
</tr>
<tr>
<td>CT4</td>
<td>20.4</td>
<td>0.142</td>
<td>2.44</td>
<td>-2.50</td>
</tr>
<tr>
<td>CT3</td>
<td>24.2</td>
<td>0.163</td>
<td>2.31</td>
<td>-1.22</td>
</tr>
<tr>
<td>CT2</td>
<td>13.2</td>
<td>0.215</td>
<td>7.67</td>
<td>-2.05</td>
</tr>
<tr>
<td>CT1</td>
<td>16.9</td>
<td>0.250</td>
<td>0.92</td>
<td>-1.58</td>
</tr>
</tbody>
</table>

The rather large scatter of the momentum flux estimates and the lack of any clear trend from level to level is somewhat inconsistent with theoretical predictions of constant momentum flux in the case of small-amplitude laminar waves (Eliassen and Palm, 1960) or of decreasing flux amplitude in the presence of turbulence (Bretherton, 1969). A similar scatter was observed (Lilly and Kennedy, 1973) during other flights during the 1970 program, however, and also from preliminary analysis of flights of more optimally instrumented aircraft during the Wave Momentum Flux Experiment in 1973.

If instrumental error is a significant element, the most probable contributing factor is the uncertainty in mean wind speed. We note that the method of computation of vertical motion applied here [Eq. (5)] leads to an estimate of momentum flux in the form

$$\bar{\rho} \bar{\nu} \bar{W} = \bar{\rho} (\bar{V}^2 \bar{C}) \left( \frac{\partial \nu}{\partial \bar{z}} \right) \left( \frac{\partial \bar{\bar{w}}}{\partial \bar{z}} \right).$$  \hspace{1cm} (12)

Our estimates of $\bar{\nu}$ at the highest two levels, $\sim 20 \pm 6$ m sec$^{-1}$, lead to an uncertainty of momentum flux at those levels of nearly a factor of 2. It is also notable that all the eastbound (odd-numbered) flight legs have lower magnitudes of momentum flux than the westbound legs. This would be consistent with a positive error in our computed true air speeds or a negative error in the ground speeds, where a 10 m sec$^{-1}$ reduction was applied uniformly. If the wind speeds are in error as indicated, the corrected values of CT mean wind speed (see Fig. 4) would be higher on legs 1 and 3 and lower on legs 2, 4, and 6. The profile would thus be somewhat smoother and probably no less consistent with the CS averages.

If, instead of accepting the observational results indicating no momentum flux gradient, we assume that the true magnitude of downward momentum flux decreased by an amount $\Delta \tau \lesssim 1$ dyn cm$^{-2}$, we can follow the consequences of this assumption and try to determine whether they are consistent with other observations.

If all of the lost momentum were provided directly by the mean flow, its magnitude would decrease downstream according to the relation

$$\int \frac{\partial}{\partial \bar{s}} (\bar{\rho} \bar{V}^2 \bar{C}) d\bar{z} = -\Delta \tau.$$  \hspace{1cm} (13)

Taking the length of the flight legs to be nominally 180 km, $g \bar{\rho} d \bar{s} = 70$ mb, and $\Delta \tau \lesssim 1$ dyn cm$^{-2}$, we estimate a reduction of mean flow $\bar{V}^2$ of $\lesssim 25$ m$^2$ sec$^{-2}$. This magnitude of reduction would be nearly imperceptible if it occurred uniformly. If most of the reduction were concentrated in the one or two most turbulent levels, it might be more noticeable and could possibly account for some of the observed decrease in wind speed downstream of the most active regions of turbulence. However, this analysis neglects the fact that the mean flow is probably almost geostrophic initially and will attempt to return to geostrophy after a deceleration by drifting toward lower pressure. Since no pressure force measurements are available in our data, we cannot realistically evaluate the mean momentum balance that occurs from the effects of wave drag.

In regions of momentum flux emission, mean flow kinetic energy is also removed, being first transformed into wave energy which is in turn broken down into turbulence. The turbulence is then either dissipated by molecular viscosity or transformed into mean potential energy by development of stratification steps, with nearly adiabatic layers sandwiched between strong inversions. The net mean flow kinetic energy removed is

$$\Delta (K.E.) = \int \frac{\partial \tau}{\partial \bar{z}} d\bar{z},$$  \hspace{1cm} (14)

which is of order 2 W m$^{-2}$ if $\Delta \tau = 1$ dyn cm$^{-2}$. If all of
this energy were transformed into turbulence and then dissipated directly by viscous processes, the kinetic energy dissipation rate would be about 30 cm$^2$ sec$^{-3}$ averaged throughout the stratospheric flight layers. The maximum turbulent intensity reported was "moderate," probably corresponding to a dissipation rate of $\sim 100$ cm$^2$ sec$^{-3}$, according to Trout and Panofsky (1969), but most of the turbulence was "light" and turbulence only occurred about 10% of the time. Thus, the above maximum estimate of mean dissipation rate seems excessive by a factor of 2–10, suggesting that the decrease in momentum flux is substantially less than 1 dyn cm$^{-2}$.

Of the mean flow energy removed by wave breakdown processes, a significant amount is probably returned as potential energy in the form of changes in stratification. Laboratory experiments on Kelvin-Helmholtz wave breakdown by Thorpe (1973) indicate that the final equilibrium state is one in which one-fourth of the kinetic energy removed from the mean flow is so returned. Fig. 5 suggests the development of nearly-adiabatic layers downstream of the principal turbulent zones. If a region of depth $\delta z$ and initial temperature gradient $\partial \theta / \partial z$ is completely mixed to an isentropic state, its net change in potential energy per unit mass is approximately

$$\Delta (P.E.) = -\frac{g}{\partial} \left( \int_{z}^{z+\delta z} \frac{\partial \theta}{\partial z} dz \right) = \frac{g \partial \theta}{\partial} (\delta z)^2 / 12. \quad (15)$$

If we assume the existence of a turbulent breakdown region with an energy input of 50 cm$^2$ sec$^{-3}$ for a trajectory time of 10$^6$ sec, with one-fourth of that energy being transformed into mean potential energy, Eq. (15) predicts that it will produce a mixed layer about 180 m deep. Fig. 5 shows several apparently mixed regions of that depth or greater near 17 km.

5. Conclusions

Although the instrumentation used in obtaining the data for this study was far from ideal, and significant unremovable errors remain in both the vertical profiles and the fluctuation data, we retain confidence in the representation of the large wavelength modes, based on the internal consistency of their measured energetics and covariance structure. Our analysis of the data indicates that the observed waves were forced gravity modes, similar to long mountain waves in essentially all respects except for their apparent slow downstream propagation. The amplitude of the wave forcing, as measured by downward momentum flux of $\sim 2$ dyn cm$^{-2}$, was not unusually large by tropospheric stand-

ards. Because of the decreasing density and decreasing wind speed with height, and the lack of absorptive or reflective layers in the troposphere, the response in the lower stratosphere was strong and unusually pervasive, with large-amplitude waves everywhere below 20 km and some turbulence observed in most flight legs.

Perhaps the most definite result of the spectral analysis is the strong indication of a temperature and downstream velocity variance spectra proportional to $k^{-3}$. If these results can be verified by further examples, they seem to require a better theoretical explanation than has yet been provided.

Acknowledgments. We wish to acknowledge the cooperation of the 58th Weather Wing of the Air Weather Service for making available the B-57F aircraft and the National Research Council of Canada and Dayton University for the data recording systems and preliminary reduction. Mr. P. J. Kennedy of the National Center for Atmospheric Research performed most of the data compilation and reduction work.

REFERENCES


