Effect of a CISK Parameterization on Tropical Wave Growth

C.-P. CHANG AND T. M. PIWOWAR

Dept. of Meteorology, Naval Postgraduate School, Monterey, Calif. 93940

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ABSTRACT

The linear boundary layer solution which includes the effect of temporal acceleration is used to parameterize the CISK (Conditional Instability of the Second Kind) mechanism of tropical waves. By simplifying the first law to a statement of balance between adiabatic cooling and diabatic heating, which is usually valid for weak tropical motions, the model is formulated as a one-level primitive equation expressed at the top of the boundary layer and is solved numerically for its eigenvalues. The growth rates are generally scale-independent and are fairly small. The presence of a quasi-Stokes boundary layer near the equator and a transition zone between this layer and a quasi-Ekman layer poleward seems important only in the wave structures but not the growth rates.

1. Introduction

Thermal forcing in the form of CISK (Conditional Instability of the Second Kind) as a possible mechanism for synoptic-scale tropical wave growth has been studied by several investigators. Due to a major discrepancy, the results thus far seem to be inconclusive. Yamasaki (1969, 1971), Hayashi (1971) and Murakami (1972) found maximum growth rates at observed wave scales (several thousand kilometers), while Chang (1971) and Williams and Robertson (1973) did not find any preferred wavelength in the synoptic-scale range.

Most of these previous studies used either a geostrophic/Ekman boundary layer formulation (vertical motion proportional to vorticity at the top of the boundary layer) or a surface drag representation for the lowest level in the model. Recently, Holton *et al.* (1971) pointed out that, in the presence of synoptic-wave oscillations, a transition zone exists near the equator where the Ekman structure of the more poleward latitudes changes rather drastically into a Stokes-type boundary layer in the equatorial region. Generally, this transition of boundary layer flow has not been fully accounted for by the previous investigators. The purpose of this paper is to use an improved boundary layer formulation, which includes this boundary layer flow modification near the equator, to examine the role of CISK in tropical wave growth.

The linear boundary layer solution (Chang, 1973b) which includes the temporal acceleration is used for the CISK parameterization. By neglecting the small temperature fluctuations in the thermodynamic equation, which is generally valid for weak synoptic-scale disturbances in the lower tropical atmosphere, a one-level equation is derived for the top of the boundary layer. This equation is similar to the free atmosphere equation in Chang (1973a). An eigenvalue problem is then formulated and the growth rates of the lowest meridional modes are obtained numerically.

2. The model

The basic equations are the linearized, nondimensional Boussinesq equations on an equatorial beta-plane, with solutions expressed by zonal wavenumber λ and frequency ν:

$$i\nu u=-i\lambda p+yv, \tag{1}$$

$$i\nu v=-\frac{\partial p}{\partial y}-yu, \tag{2}$$

$$i\lambda u+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0, \tag{3}$$

$$wS=Q, \tag{4}$$

where u, v, w are the zonal, meridional and vertical velocities, respectively, p is the pressure divided by density, Q the diabatic heating rate, and S the static stability parameter. Eq. (4) is a simplified thermodynamic equation in which the local change and horizontal advection terms have been neglected compared to the leading terms (Holton, 1972). This implies that for the weak tropical motions of interest, there is no storage of available potential energy. The basic nondimensional units are time scale $\tau\equiv(c\beta)^{-\frac{1}{2}}$, horizontal length scale $L\equiv(c/\beta)^{\frac{1}{2}}$, and depth scale H. Here $c\equiv(gH)^{\frac{1}{2}}$ and $\beta\equiv2\Omega/a$, with g being the gravitational constant, H the scale height, Ω the angular velocity of the earth, and a the radius of the earth.

By eliminating u, v, w in (1)–(4), a single differential equation in p results:

$$\frac{\partial^2 p}{\partial y^2}-\left(\frac{2y}{y^2-\nu^2}\right)\frac{\partial p}{\partial y}+\left[\frac{\lambda}{\nu}\frac{(y^2+\nu^2)}{(y^2-\nu^2)}-\lambda^2\right]p=-\frac{i(y^2-\nu^2)}{\nu}\frac{\partial}{\partial z}\left(\frac{Q}{S}\right). \quad (5)$$

In this equation the only vertically differentiated term is the heating term on the right-hand side. This implies that the motion is vertically coupled only through diabatic heating. Hence, if the vertical profile of Q is known, (5) can be applied as an ordinary differential equation with y as the only independent variable at any level above the boundary layer.

The CISK process is parameterized so that the diabatic heating function $Q(z)$ is related to the vertical velocity w_T at the top of the boundary layer ($z=z_T$):

$$Q(z)=\eta(z)w_T, \quad (6)$$

where $\eta(z)$ is a proportionality function which specifies the vertical profile of heating. Notice that (6) also implies negative heating when downward motion occurs at the top of the boundary layer. This is difficult to avoid due to the separation of the x-dependence of the solutions.

The vertical motion at the top of the boundary layer is expressed by

$$w_T=R_1\frac{\partial^2 p}{\partial y^2}+R_2\frac{\partial p}{\partial y}+\lambda R_3 p, \quad (7)$$

where the coefficients R_1, R_2, R_3 are given in the Appendix. Eq. (7) is derived by integrating the total boundary convergence, using the solutions in Chang (1973b). This boundary layer solution includes the temporal acceleration and therefore is frequency-dependent. If the boundary layer depth becomes infinite ($z_T\rightarrow\infty$), a singularity in w_T exists at the "critical latitude" where $y=\nu$ (Holton *et al.*, 1971). If z_T is finite, the flow may possess quasi-singular behavior around this latitude, depending on z_T and the mode of symmetry about the equator (Chang, 1973b). The critical latitude, in essence, represents the transition zone between the quasi-Ekman and the quasi-Stokes regimes.

The heating profile $\eta(z)$ is specified such that $\partial\eta/\partial z$ varies from S to $2S$ at the top of the boundary layer. Consequently (5) can be applied at z_T. [Notice that (4) and (6) require that $\eta=S$ at the top of the boundary layer.] It turns out that this specification of the gradient of heating at z_T does not change the qualitative characteristics of the growth rates; therefore, it is not a severe restriction in this problem.

In all cases considered, the resultant p-w phase relationship integrated over the meridional domain implies a growth requirement that $\partial\eta/\partial z>0$ at z_T. Alternatively, the condition for instability found by Ooyama (1969) that $\partial\eta/\partial z>0$ at some internal level above z_T is changed to $\partial\eta/\partial z>0$ at the lowest level where the simplified thermodynamic equation (4) is valid.

Substituting (6) and (7) into (5), we obtain a second-order homogeneous equation in p at z_T:

$$E\frac{d^2p}{dy^2}+F\frac{dp}{dy}+Gp=0, \quad (8)$$

where the coefficients E, F, G are functions of y and ν and are given in the Appendix. Eq. (8) is solved numerically for its eigenvalues ν using an iterative procedure described by Stone (1970), with the boundary conditions being that $p=0$ at 30N and 30S. Two external parameters are varied in the following ranges: z_T from 1 to 4 km and $\partial\eta/\partial z$ from S to $2S$. The solutions are obtained for both the symmetric (pressure symmetric about the equator) and the asymmetric (pressure asymmetric about the equator) cases.

3. Results

Solutions for the "free modes," i.e., when the right-hand side of (5) is set to zero, are obtained as a test for the model. Only real frequencies are obtained which are shown in Fig. 1. All synoptic-scale wavelengths (2000–10,000 km) have periods of ~3–7 days for the asymmetric case and ~5–10 days for the symmetric case. These frequencies correspond quite closely to those described by the Rossby wave formula $\nu=\lambda/(\lambda^2+l^2)$,

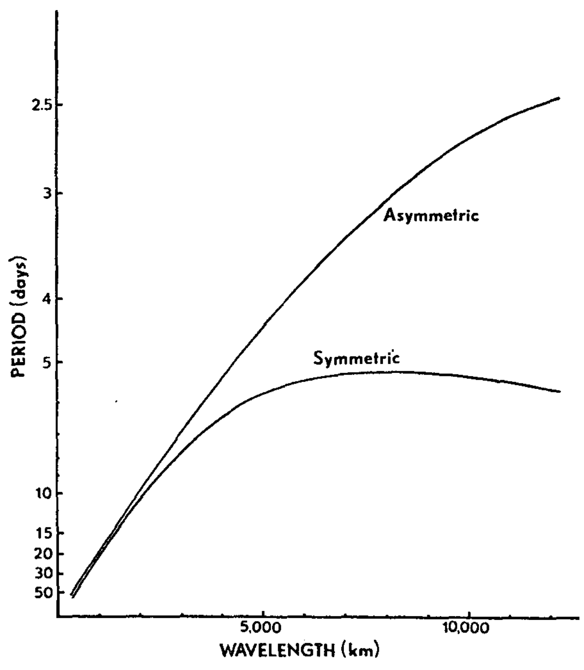

FIG. 1. Wave periods for free modes.

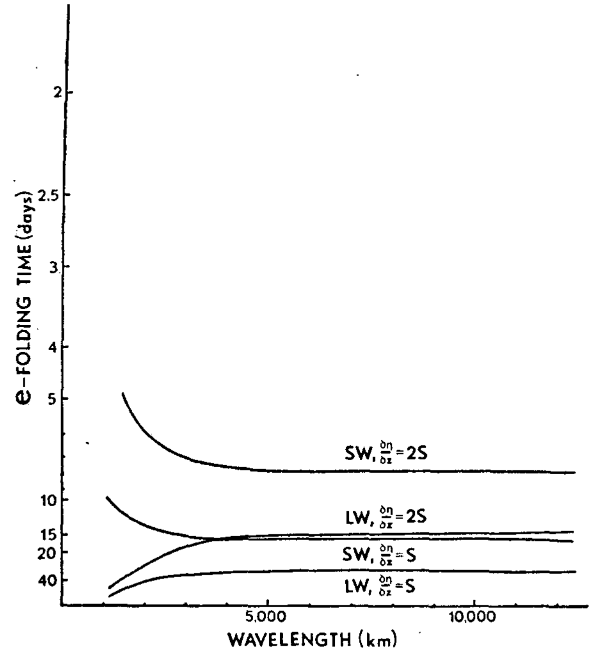

FIG. 2. Growth rates for asymmetric case and $z_T=1$ km.

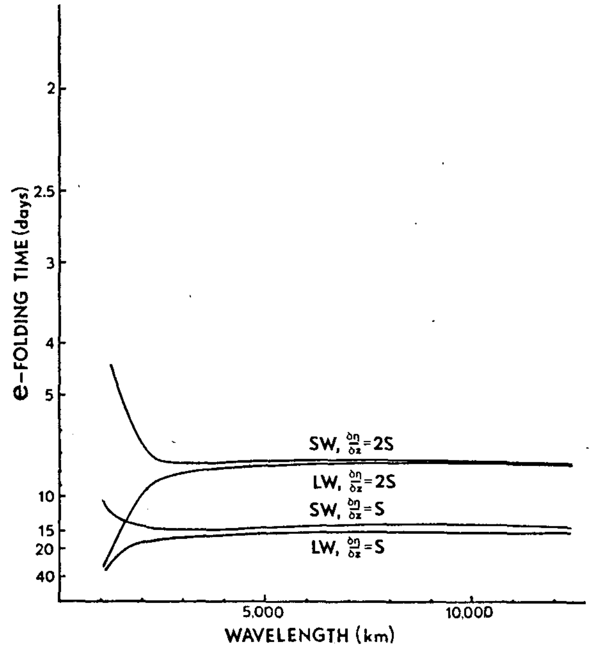

FIG. 4. Growth rates for symmetric case and $z_T=1$ km.

where l is the meridional wavenumber, and the approximated Matsuno's (1966) wave-frequency equation $\nu \approx \lambda/(\lambda^2+2n+1)$, where n is the meridional mode number. Forced modes are then computed by including the effect of CISK in the model.

a. Growth rates

Figs. 2–5 show the growth rates as a function of wavelength for both symmetry cases and $z_T=1$ and 2 km. The important result for all cases is that the synoptic-scale waves have essentially the same growth rate—a result which agrees with those of Chang (1971) and Williams and Robertson (1973) in quasi-geostrophic models.

For each set of external parameters, two solutions are generally found: a long-wave (LW) mode, one that possesses greater growth rates in the long-wave portion of the synoptic scale; and a short-wave (SW) mode, one that favors short-wave growth. Both modes of the symmetric case generally show larger growth rates

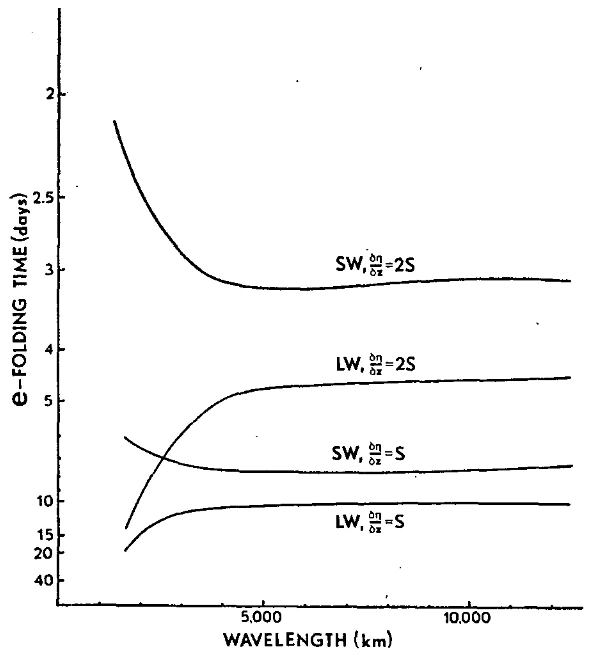

FIG. 3. Growth rates for asymmetric case and $z_T=2$ km.

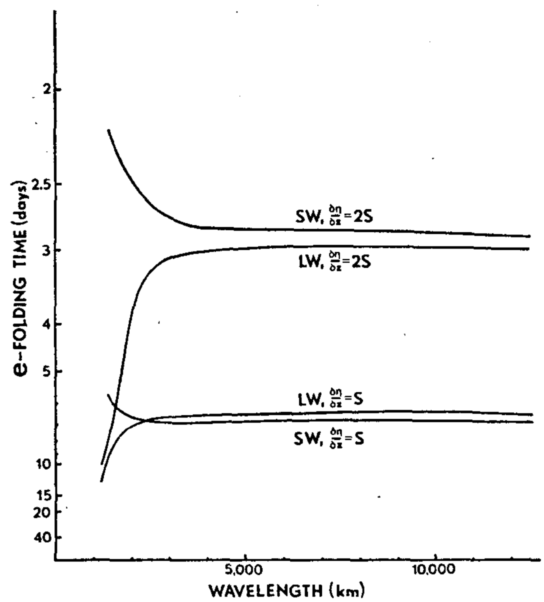

FIG. 5. Growth rates for symmetric case and $z_T=2$ km.

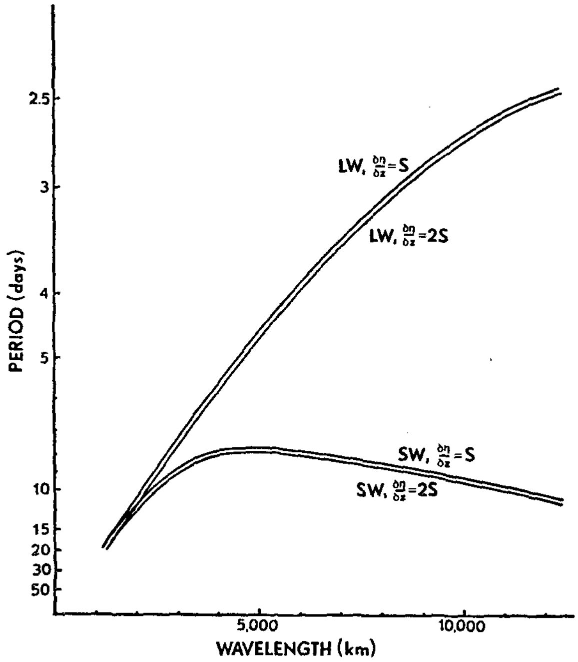

FIG. 6. Wave periods for asymmetric case and $z_T=2$ km.

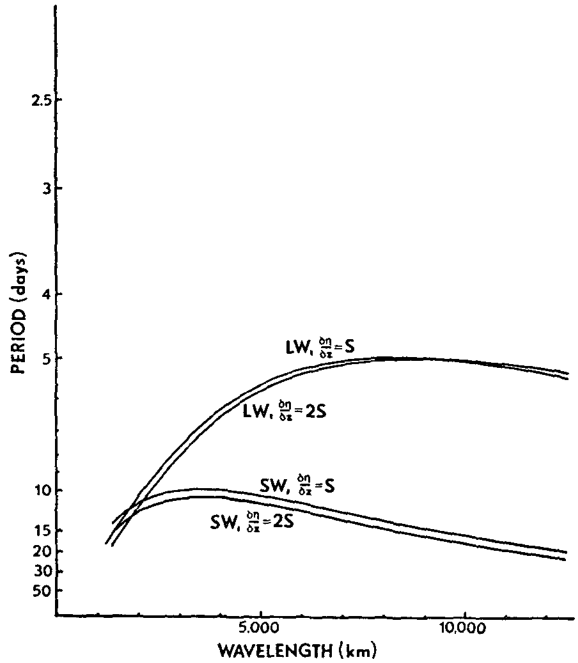

FIG. 7. Wave periods for symmetric case and $z_T=2$ km.

(faster e-folding times) than those of the asymmetric case, although for shorter wavelengths this difference rapidly decreases.

An increase in the slope of the heating function η from S to $2S$ approximately doubles the growth rates for all cases shown. Likewise, a deeper boundary layer also increases growth rates. This effect is most pronounced when z_T changes from 1 to 2 km, especially for the asymmetric case. However, when z_T is increased to >2 km (results not shown here), the increase in growth rate lessens and the growth rates appear to approach constant values with deeper boundary layers.

b. Periods

Figs. 6 and 7 contain the wave periods for $z_T=2$ km for both symmetry cases. The curves for the LW mode in both cases closely resemble those of the corresponding free modes, which is expected because the LW mode generally possesses smaller growth rates compared to the SW mode. Other results (not shown) indicate that varying $\partial\eta/\partial z$ and z_T produce little effect in the wave period.

c. Eigenfunctions

The p and w_T profiles for 2000-km wavelength, $z_T=2$ km and $\partial\eta/\partial S=S$ are shown in Figs. 8 (asymmetric case) and 9 (symmetric case). Particularly noted are the following:

1) The waves generally exhibit a NE-SW tilt. Such a tilt implies poleward transport of westerly momentum and agrees with observations summarized by Wallace (1971).

2) Vertical motion maxima are generally located equatorward of the pressure maxima.

3) The amplitude maximum of the LW mode is usually nearer the equator than the SW mode.

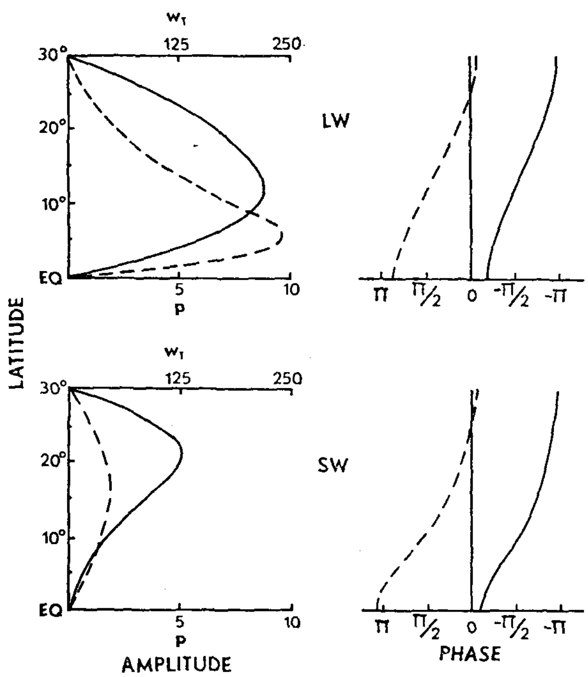

FIG. 8. Eigenfunctions for asymmetric case: wavelength $=2000$ km, $z_T=2$ km and $\partial\eta/\partial z=S$. Solid lines are pressure perturbation p and dashed lines are vertical motion w_T.

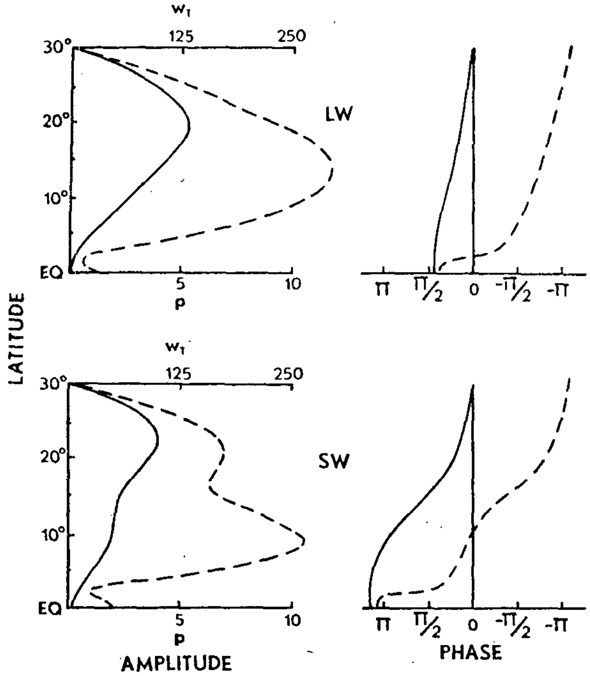

FIG. 9. As in Fig. 8 except for symmetric case.

4) For the given boundary layer depth, the neutral boundary layer solutions (Chang, 1973b; Marht, 1973) indicate that w_T possesses quasi-singular behavior near the transition zone between the quasi-Ekman and the quasi-Stokes flow regimes. Under the CISK-forcing these features are still evident except for the asymmetric SW mode. However, the sharp maxima in the asymmetric modes become more smoothed.

5) Deepening of the boundary layer depth (not shown) generally results in a sharpening and equatorward shift of the LW mode's maximum amplitude toward the critical latitude. This is similar to the neutral flow structures found by Chang (1973b).

6) The phase difference between the pressure and vertical motion fields generally remains $\frac{1}{2}$ cycle for both symmetry cases under the given conditions. The only exception appears with the symmetric case. In the vicinity of the critical latitude and equatorward, a rapid phase shift in w_T occurs such that the two fields are nearly in phase at the equator. Whereas an out-of-phase relationship implies a gain of kinetic energy from the source region immediately above the boundary layer, in-phase implies the reverse transfer of energy. However, the relative amplitudes of both p and w_T are small near the equator, thereby minimizing this local damping effect within the near-equatorial region.

4. Concluding remarks

The primary conclusion of this study is that there is no preferred synoptic-scale wavelength for CISK-forced waves. This agrees with the results of Chang (1971) and Williams and Robertson (1973), but differs from those of Yamasaki (1969, 1971), Hayashi (1971) and Murakami (1972). Our result tends to indicate that CISK probably is an energy source for the maintenance of tropical waves, but it is not the initiation mechanism. This point can also be supported by the smallness of the growth rate. The e-folding times for the more realistic vertical heating gradient ($\partial\eta/\partial z = S$) are found to be of the order of 5 days or slower, even for the deeper boundary layer depths. This magnitude of growth rate is quite small compared to baroclinically unstable waves in the mid-latitudes. Consequently, even if one scale is slightly preferred among the others, its relatively faster growth would be at such a slow rate that the competition for energy among the waves may not be strong enough to produce a dominant wavelength.

No explanations can yet be offered about the existence of two modes within the relevant time-scale range in our model. For the LW mode, although there is no preferred scale in the 2000–10,000 km range, it nevertheless exhibits decreasing growth rate with decreasing wavelength in the shortwave end. This seems to agree with the HB mode of Yamasaki (1969) and the findings of Murakami (1972). In addition, their results are similar to the LW mode in that the maximum amplitudes exist near the equator, and the wave periods are around 5 days. On the other hand, the SW mode, especially for the shallower boundary layers, has longer periods and its maximum amplitude is farther from the equator. Since the SW mode also prefers zero scale for growth, it may be identified as a hurricane (or typhoon) mode.

The presence of a slow-rotation, quasi-Stokes layer near the equator with its inefficient pumping does not have much influence on the wave growth, as can be inferred from the growth rates found in this study which is very similar to those of the quasi-geostrophic models of Chang (1971) and Williams and Robertson (1973). The inclusion of the temporal acceleration in the boundary layer equation thus seems important only in modifying the wave structures.

The problem of determining the initiation mechanism for the observed synoptic-scale waves over the tropical oceans still remains. Barotropic instability seems to be a possible mechanism due to the presence of horizontal mean wind shear near the ITCZ. In this event the NE-SW tilt of waves caused by CISK-type heating would enhance the barotropic energy conversion after the wave is initiated.

Our results, which show a relatively smooth profile of the vertical motion out of the boundary layer, indicate that for a growing boundary layer, the quasi-singular behavior around the flow transition zone is somewhat relaxed. This is due to the constraining effect on boundary-layer depth, which can be readily seen from the w_T solution (Chang, 1973b). The magnitude of w_T is partially proportional to a factor $(y-\nu)^{-\frac{1}{2}}$.

For growth conditions, ν is complex and this factor may be written as $(y-\nu_r-i\nu_i)^{-\frac{1}{2}}$ where y is always real. Thus, the so-called "critical-latitude mechanism" for ITCZ development (Holton *et al.*, 1971; Chang, 1973a), which depends on the deepening of the boundary layer depth for a concentration of boundary layer pumping, would be subjected to a constraining influence during the growing stage of the waves.

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APPENDIX

Coefficients R_1, R_2, R_3 and E, F, G

The coefficients for Eq. (7) are as follows:

$$R_1=B_+C_+-B_-C_-,$$

$$R_2=\frac{i}{2K\gamma}(B_+D_++B_-D_-)+B_+C_+[(y+\nu)^{-1}-2i\gamma z_TB_+A_+^2]+B_-C_-[(y-\nu)^{-1}+2i\alpha z_TB_-A_-^2],$$

$$R_3=\frac{i}{2K\gamma}(B_+D_+-B_-D_-)+B_+C_+[(y+\nu)^{-1}-2i\gamma z_TB_+A_+^2+\lambda]+B_-C_-[(y-\nu)^{-1}-2i\alpha z_TB_-A_-^2-\lambda],$$

where

$$\left.\begin{aligned}
A_+&=\exp(-\gamma z_T)\\
A_-&=\exp(-\alpha z_T)\\
B_+&=\frac{i}{2}(y+\nu)^{-1}(1-A_+^2)^{-1}\\
B_-&=\frac{i}{2}(y-\nu)^{-1}(1-A_-^2)^{-1}\\
C_+&=z_T-\frac{1}{\gamma}+\frac{2A_+}{\gamma}-A_+^2\left(z_T+\frac{1}{\gamma}\right)\\
C_-&=z_T-\frac{1}{\alpha}+\frac{2A_-}{\alpha}-A_-^2\left(z_T+\frac{1}{\alpha}\right)\\
D_+&=\frac{1}{\gamma^2}(2A_+-1-A_+^2)+2z_TA_+\left[\frac{1}{\gamma}(1-A_+)-z_TA_+\right]\\
D_-&=\frac{1}{\alpha^2}(2A_--1-A_-^2)+2z_TA_-\left[\frac{1}{\alpha}(1-A_-)-z_TA_-\right]
\end{aligned}\right\},$$

and

$$\gamma^2=\frac{i(\nu+y)}{K},$$

$$\alpha^2=\frac{i(\nu-y)}{K}.$$

The coefficients for (8) are

$$E=1+\frac{i(y^2-\nu^2)}{\nu S}R_1\frac{d\eta}{dz},$$

$$F=-\frac{2y}{(y^2-\nu^2)}+\frac{i(y^2-\nu^2)}{\nu S}R_2\frac{d\eta}{dz},$$

$$G=-\frac{\lambda}{\nu}\frac{(y^2+\nu^2)}{(y^2-\nu^2)}-\lambda^2+\frac{i(y^2-\nu^2)}{\nu S}\lambda R_3\frac{d\eta}{dz}.$$

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