

Waves in the Jovian Upper Atmosphere

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We examine a propagating wave interpretation of the temperature profile features observed in the Jovian upper atmosphere by Veverka *et al.* Inertia-gravity waves with frequencies on the order of $3 \times 10^{-3} \text{ sec}^{-1}$ are consistent with the data. If the interpretation is correct, and if the waves carry energy upward, it implies 1) that there is excitation of such waves at lower levels, 2) that eddy diffusivities on the order of $10^6 \text{ cm}^2 \text{ sec}^{-1}$ are probably generated by the waves, and 3) that the energy carried by waves is important to the upper atmospheric heat balance.

1. Introduction

Inversion of the emission light curve from the occultation of β -Scorpii by Jupiter produces the temperature profile shown in Fig. 1 (from Veverka *et al.*, 1974). There are pronounced oscillations in the temperature profile. A spectrum of wavelengths ranging from a few kilometers to about 20 km appears to be present, with a large amplitude ($\sim 5\text{K}$) component with wavelength about 13 km.

In this paper we shall examine the possibility that the oscillations are propagating atmospheric waves. There may be other explanations, and we shall not attempt to discuss the alternatives. However, we shall argue that an inertia-gravity wave interpretation is consistent with all the information. There are good reasons to explore the possibility in some detail. If such waves exist, and if they are carrying energy upward, it tells us that there is a source of excitation lower down, with implications for length and time scales of

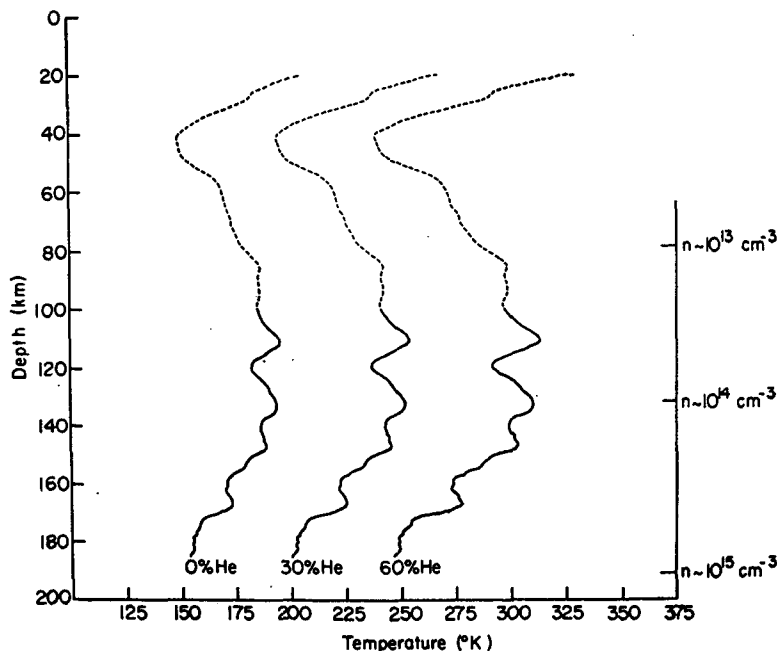


FIG. 1. Temperature profiles (from Veverka *et al.*, 1974). The zero of the altitude scale is arbitrary. Number densities are indicated. Temperature is uncertain where the profile is dashed. Flashes (light curve "spikes") did not occur at altitudes above the $n = 10^{13} \text{ cm}^{-3}$ level.

motions lower down. Also, Hodges (1969) has pointed out that vertically propagating waves can generate turbulence by producing instabilities, due to the increasing amplitude of wind and temperature perturbations with height. Finally, the energy carried by the waves could be important to the atmospheric heat balance.

Alternatively, the waves could be propagating downward from a source above. We regard this possibility as less likely, but cannot rule it out. The energy involved would be the same, but Hodges' arguments would not apply.

We shall use a simple set of equations to discuss wave characteristics. We neglect the curvature of the planet and adopt the beta-plane approximation (Lindzen, 1967). The basic state will be assumed to be isothermal and motionless. There is not enough information to justify anything more complicated. The neglect of wind shear could introduce serious error, since it is known that vertical propagation can be greatly affected if wind velocities approach the wave phase speed. We comment further on this after evaluating phase speeds. Unfortunately, there is no alternative to simple assumptions at this stage, and we must hope that conclusions will be qualitatively correct in spite of the errors.

In the next section the equations and assumptions are stated, and conditions for vertical propagation are derived. The eddy diffusivities associated with amplitude-limited waves are derived and evaluated, following Hodges (1969) and Hines (1970). Energy fluxes are evaluated. Then in the last section, conclusions are presented.

2. Equations and assumptions

We consider small-amplitude motion in an isothermal atmosphere on a beta plane. Symbols are defined as follows:

- x, y, z eastward, northward and upward directions
- p, ρ, T pressure, density and temperature
- t time
- u, v, w eastward, northward and upward velocities
- f Coriolis frequency
- β [= df/dy]
- ν diffusivity
- c_p heat capacity at constant pressure,
- R gas constant
- γ ratio of specific heats
- g acceleration of gravity.

All dependent fields are written in the form of a basic state plus a perturbation; for example, $\rho = \rho_0 + \rho_1$. The basic state is motionless. Since it is isothermal, p_0 and ρ_0 decay exponentially with height with the atmospheric scale height $H = RT_0/g$. For molecular hydrogen, $R = 4.2 \times 10^7$ ergs gm⁻¹ (°K)⁻¹, and with $T_0 = 200$ K, $g = 2600$ cm sec⁻², the scale height $H = 32$ km.

The equations are as follows:

$$\rho_0 \frac{\partial u_1}{\partial t} - \rho_0 f v_1 + \frac{\partial p_1}{\partial x} = \rho_0 \nu \nabla^2 u_1 \tag{1}$$

$$\rho_0 \frac{\partial v_1}{\partial t} + \rho_0 f u_1 + \frac{\partial p_1}{\partial y} = \rho_0 \nu \nabla^2 v_1 \tag{2}$$

$$\rho_0 \frac{\partial w_1}{\partial t} + \frac{\partial p_1}{\partial z} = \rho_0 \nu \nabla^2 w_1 - g \rho_1 \tag{3}$$

$$\frac{\partial \rho_1}{\partial t} + \rho_0 \frac{\partial u_1}{\partial x} + \rho_0 \frac{\partial v_1}{\partial y} + \rho_0 \frac{\partial w_1}{\partial z} + w_1 \frac{\partial \rho_0}{\partial z} = 0 \tag{4}$$

$$\rho_0 c_p \frac{\partial T_1}{\partial t} - \frac{\partial p_1}{\partial t} - w_1 \frac{\partial p_0}{\partial z} = c_p \rho_0 \nu \nabla^2 T_1 - \nu \nabla^2 p_1 \tag{5}$$

$$\frac{p_1}{p_0} = \frac{\rho_1}{\rho_0} + \frac{T_1}{T_0} \tag{6}$$

The diffusivity terms have been approximated. There should also be terms involving the gradient of ρ_0 and terms proportional to the flow divergence, if the diffusion is to act analogously to molecular diffusion. Neglect of these terms is valid when the dissipation is weak (small damping per wavelength) and when the wavelength is small compared to the scale height. These conditions are satisfied in this application.

In the energy equation (5) it is assumed that the diffusive energy flux is proportional to the entropy gradient rather than the temperature gradient. A turbulent eddy diffusivity presumably would act in this manner. The difference is not important in this application.

It is convenient (see, for example, Lindzen, 1967) to define new variables:

$$\left. \begin{aligned} (u', v', w', T') &= \rho_0^{1/2} (u_1, v_1, w_1, T_1) \\ (\rho', p') &= \rho_0^{-1/2} (\rho_1, p_1) \end{aligned} \right\} \tag{7}$$

In the present case it is also useful to define

$$\frac{\hat{\partial}}{\partial t} = \frac{\partial}{\partial t} - \nu \nabla^2 \tag{8}$$

Eqs. (1)-(6) can then be reduced to one equation for v' :

$$\begin{aligned} & \frac{\gamma}{\gamma-1} N^2 \left(\frac{\hat{\partial}^2}{\partial t^2} + N^2 \right) \left[\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \frac{\hat{\partial}}{\partial t} + \beta \frac{\partial}{\partial x} \right] v' \\ & + \left(\frac{\hat{\partial}^2}{\partial t^2} + f^2 \right) \left(\frac{\hat{\partial}^2}{\partial t^2} + N^2 \right) \frac{1}{H} \left(\frac{\partial}{\partial z} - \frac{1}{2H} \right) \frac{\partial}{\partial t} v' \\ & + \left(\frac{\hat{\partial}^2}{\partial t^2} + f^2 \right) \left[\frac{\gamma}{\gamma-1} N^2 \left(\frac{\partial}{\partial z} - \frac{1}{2H} \right) \right. \\ & \left. - \frac{1}{H} \frac{\hat{\partial}}{\partial t} \frac{\partial}{\partial t} \right] \left[\left(\frac{\partial}{\partial z} + \frac{1}{\gamma H} - \frac{1}{2H} \right) \frac{\hat{\partial}}{\partial t} \right] v' = 0, \tag{9} \end{aligned}$$

where N is the Brunt-Vaisala frequency, given by $N^2 = g^2 / (c_p T_0)$.

Solutions can be written in the form

$$v' = \rho_0^{1/2} v_1 = (\text{constant}) \exp[i(kx + \lambda y + \mu z + \omega t)]. \quad (10)$$

Thus if μ is real, the velocity and temperature amplitudes vary with height inversely with the square root of the basic state density.

We shall proceed in two steps to investigate solutions for the conditions of interest. First, to set the general scene, we neglect friction and inquire about the criteria for existence of vertically propagating waves. We substitute (10) into (9) and assume that $\partial/\partial t = \partial/\partial t$. This leads to a quadratic equation for μ , whose solution is

$$i\mu = \pm \frac{1}{H} \left[\frac{1}{4} - \frac{\omega^2(\gamma-1)}{\gamma^2 N^2} - \frac{N^2 - \omega^2}{f^2 - \omega^2} \left(\frac{\beta k}{\omega} - k^2 - \lambda^2 \right) \right]^{1/2}. \quad (11)$$

For vertically propagating waves, μ must be real and the quantity under the radical must be negative. The regions in the ω, k plane where the condition is satisfied are indicated in Fig. 2. We arbitrarily assume that $\lambda = k$ in plotting the figure. The ω, k plane is divided into three regions where propagation is possible: acoustic (with $\omega \gg f$), inertia-gravity ($f < \omega < N$), and Rossby ($\omega \ll f$). The numerical values of the parameters are $N^2 = 2.6 \times 10^{-4} \text{ sec}^{-2}$, $f = 3 \times 10^{-4} \text{ sec}^{-1}$, $\beta = 2.5 \times 10^{-14} \text{ sec}^{-1} \text{ cm}^{-1}$, $\gamma = 1.47$. These are appropriate for hydrogen at 200K, and a beta-plane centered at 60° latitude, the occultation latitude.

Next, let us turn to the specific solutions of interest. For purposes of a numerical example, we shall adopt a vertical wavelength of 13 km, although other components are also present. The corresponding value of μ is $2\pi/(13 \text{ km})$, or $4.8 \times 10^{-6} \text{ cm}^{-1}$. Again assuming $\lambda = k$, the three solutions for $\omega(k)$ are plotted in Fig. 2, representing propagating waves with this wavenumber. They are given, approximately, by

$$\omega = \frac{\beta k}{\frac{f^2}{k^2 + \lambda^2} + \frac{\mu^2}{N^2}} \quad (\text{Rossby}), \quad (12)$$

$$\omega^2 = \frac{N^2(k^2 + \lambda^2) + f^2 \mu^2}{k^2 + \lambda^2 + \mu^2} \quad (\text{inertia-gravity}), \quad (13)$$

$$\omega^2 = c^2(k^2 + \lambda^2 + \mu^2) \quad (\text{acoustic}), \quad (14)$$

where c is the speed of sound, given by $c^2 = H^2 N^2 \gamma^2 / (\gamma - 1)$. Eqs. (12)–(14) are obtained from (11) by neglecting terms of order ω/f , $\beta/(k\omega)$ and N/ω , respectively. All three kinds of waves can propagate vertically with the observed wavelength.

The Rossby mode is probably unrealistic. Frequencies and phase velocities are very small, and even an ex-

remely small wind shear would affect the structure of the wave. We shall return to this point.

Next, let us turn to the dissipative effects, following Hodges (1969) and Hines (1970). In the absence of damping, the velocity and temperature perturbations display an amplitude growth with height proportional to $e^{z/(2H)}$ [Eq. (10)]. One can ask for the value of the diffusivity ν just sufficient to suppress this amplitude growth. This critical value may be an indication of the wave-generated eddy diffusivity if the wave amplitude is constrained by internal instabilities (breaking, so to speak) which are triggered by a Richardson number or static instability criterion. Also, this value of ν gives an indication of the molecular diffusivity which would begin to affect the wave importantly. Since the molecular diffusivity increases with height, this provides an estimate of the maximum propagation height.

To solve for the critical value of ν , we return to the full equation (9) and substitute the solution

$$v' = (\text{constant}) \exp[i(kx + \lambda y + \mu z + \omega t)]. \quad (15)$$

We then let

$$\mu = \mu_0(1 + \delta), \quad \omega = \omega(1 + \epsilon). \quad (16)$$

The growth is suppressed if we require that $\delta = i/(2H\mu_0)$. From (8), it follows that

$$\epsilon = -i\nu(k^2 + \lambda^2 + \mu^2)/\omega. \quad (17)$$

Since $\delta = 0.0033i \ll 1$ for the wavelength observed, a perturbation expansion can be employed (Hines, 1970). It must be checked *a posteriori* that ϵ is also small. This turns out to be the case.

Substitution of (15) into (9), gives after neglect of terms in ϵ^2, δ^2 ,

$$\frac{\gamma}{\gamma-1} N^2 (N^2 - \omega^2) \left(k^2 + \lambda^2 - \frac{\beta k}{\omega} \right) + (f^2 - \omega^2) \left(\frac{\gamma N^2}{\gamma-1} \mu_0^2 + \frac{2\gamma N^2}{\gamma-1} \mu_0^2 \delta - \frac{\omega \omega}{\gamma H^2} \right) = 0. \quad (18)$$

The leading order terms yield the dispersion relations (12)–(14). The first-order terms yield the equations for ν :

$$\nu = \frac{f^2 \mu_0}{HN^2 \beta k} \frac{\omega^2}{(k^2 + \lambda^2 + \mu_0^2)} \quad (\text{Rossby}), \quad (19)$$

$$\nu = \frac{\mu_0}{2H\omega} \frac{\omega^2 - f^2}{(k^2 + \lambda^2 + \mu_0^2)^2} \quad (\text{inertia-gravity}), \quad (20)$$

$$\nu = -\frac{\mu_0 c^4}{H \omega^3} \quad (\text{acoustic}). \quad (21)$$

The negative sign on the acoustic value tells us that the phase velocity must be upward ($\omega < 0$) for upward

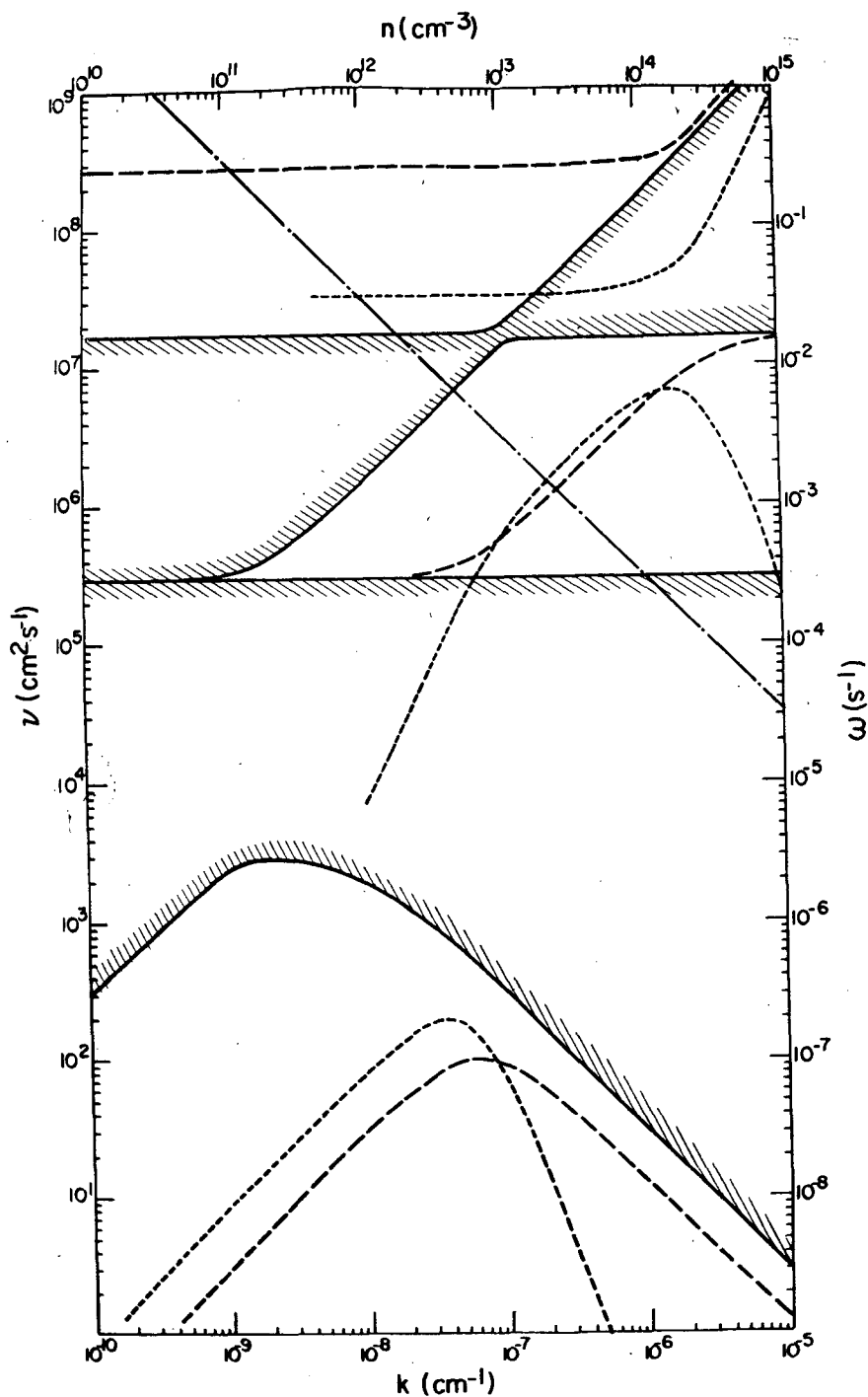


FIG. 2. Theoretical results. Frequency (ω) and diffusivity (ν) are plotted as functions of the east-west wavenumber k . The north-south wavenumber is assumed equal to k . In regions on the hatched side of solid lines no vertical propagation is possible. The long-dashed lines show solutions with the observed vertical wavenumber; from top to bottom they represent acoustic, inertia-gravity, and Rossby waves. The short-dashed lines are diffusivities calculated as explained in the text. They correspond, from top to bottom, to the same three solutions. The dash-dot line shows the kinematic viscosity of H_2 , as a function of number density (n) plotted across the top.

energy propagation. For the other two modes, the phase velocity must be downward.

Values are plotted in Fig. 2. For all three modes, the order of magnitude is roughly $v \sim \omega/\mu_0^2$. These results will be discussed in the next section.

A final quality of interest is the energy carried by the waves. The energy equation for the atmosphere takes the form

$$\frac{\partial}{\partial t} \left(\rho \frac{V^2}{2} + \rho c_v T + \rho g z_1 \right) + \nabla \cdot \left[\rho v \frac{V^2}{2} + \rho v c_v T + v p + \rho v g z_1 \right] = (\text{dissipative terms}), \quad (22)$$

where c_v is the heat capacity at constant volume, $V^2 = u^2 + v^2 + w^2$, and z_1 is the Lagrangian vertical displacement of particles. It follows that the vertical energy flux is given to leading order by the second and third terms in the brackets, since the first term is cubic in the perturbation amplitude and the fourth term has a negligible time average, since $w = dz_1/dt$. Thus, the vertical energy flux, after using the ideal gas equation, is

$$F = \frac{\gamma}{\gamma - 1} \overline{w p}, \quad (23)$$

where the overbar denotes a time average. After use of the linear relations (1)–(6), neglecting friction, and expressing the result in terms of the temperature amplitude ΔT_1 , we obtain

$$F = \frac{\gamma}{\gamma - 1} \frac{\omega}{\mu} \frac{\rho_0 c_p (\Delta T_1)^2}{T_0} \frac{1 - \frac{\omega^2}{N^2}}{1 + \frac{1}{\mu^2 H^2} \left(\frac{1 - \gamma - 1}{\gamma} \frac{\omega^2}{N^2} \right)^2}. \quad (24)$$

We discuss numerical results in the next section.

3. Results and discussion

A Rossby wave interpretation is not consistent with the observations. The molecular diffusivity of hydrogen (plotted in Fig. 2 as a function of number density) is too large to permit propagation. It is about $3 \times 10^4 \text{ cm}^2 \text{ sec}^{-1}$ at the $n = 10^{15} \text{ cm}^{-3}$ level. A diffusivity of $2 \times 10^2 \text{ cm}^2 \text{ sec}^{-1}$ is sufficient to damp Rossby waves with the observed vertical wavelength. We conclude that Rossby waves cannot penetrate to the heights where a wave is observed ($10^{15} > n > 10^{13}$).

Furthermore, there are two difficulties associated with the extremely low frequencies of the Rossby waves. The phase velocities are small ($\sim 1 \text{ m sec}^{-1}$) and the waves will be affected by wind shear. The outcome cannot be predicted with certainty, but downward reflection at critical levels is likely. Second, the frequency is lower than radiative decay rates [these are $\sim 10^{-6} \text{ sec}^{-1}$ according to Wasserman (1974)] and radiative damping will occur.

Inertia-gravity waves are consistent with the observations. A molecular diffusivity sufficient to cause attenuation is attained at the $n = 10^{13} \text{ cm}^{-3}$ level for the wave with $k = 2.0 \times 10^{-7} \text{ cm}^{-1}$, for example. The value of the diffusivity is about $10^6 \text{ cm}^2 \text{ sec}^{-1}$. The $n \approx 10^{13} \text{ cm}^{-3}$ level is where the transition to flashes or spikes occurred during the occultation (Veverka *et al.*, 1974), with no spikes at smaller number densities. The spikes may be indicators of fluctuations due either to the waves themselves, or turbulence induced by instabilities associated with the waves. The observations are thus consistent with damping of the wave at altitudes higher than the $n = 10^{13} \text{ cm}^{-3}$ level, and the theory suggests that this should happen for the inertia-gravity mode. The absence of spikes at higher levels supports an interpretation of upward energy propagation, with energy source lower down.

The phase speed of the inertia-gravity mode is large enough so that the waves probably can exist in essentially the form described in this paper. For the case $k = 2.0 \times 10^{-7} \text{ cm}^{-1}$, the frequency is $\omega = 10^{-3} \text{ sec}^{-1}$ and $\omega/k = 50 \text{ m sec}^{-1}$. While we do not know the wind shear magnitude, it is unlikely that differential velocities of this magnitude exist near the levels of interest.

The acoustic waves are inconsistent with the observations in that there is no reason to expect damping at the $n \approx 10^{13} \text{ cm}^{-3}$ level. According to Fig. 2, the molecular diffusivity would need to be $\sim 5 \times 10^7 \text{ cm}^2 \text{ sec}^{-1}$ in order to affect propagation. This is not the case until $n < 6 \times 10^{11} \text{ cm}^{-3}$.

In other respects, however, the acoustic modes are acceptable, and if the disappearance of spikes for $n < 10^{13} \text{ cm}^{-3}$ is not, in fact, an indication of wave damping, then we are left with acoustic modes as possibilities. It is somewhat difficult to imagine how such modes would be excited. High frequencies and large source velocities are necessary. The waves would carry a large amount of energy (see below).

Finally, the energy fluxes and heating rates associated with the waves are of interest. In Table 1, values

TABLE 1. Numerical results for three examples (L is the horizontal wavelength; the other symbols are defined in the text).

Wave type	k (cm^{-1})	L (km)	ω (sec^{-1})	ω/k (m sec^{-1})	μ (m sec^{-1})	ν ($\text{cm}^2 \text{ sec}^{-1}$)	F ($\text{ergs cm}^{-2} \text{ sec}^{-1}$)	dT/dt ($^{\circ}\text{K day}^{-1}$)
Rossby	8.0×10^{-8}	7.8×10^3	10^{-7}	1.3	2.0×10^{-4}	1.0×10^2	3.4×10^{-4}	2.2×10^{-4}
Inertia-gravity	2.0×10^{-7}	314	10^{-3}	50	2.1	2.8×10^6	3.4	2.2
Acoustic	2.5×10^{-6}	25	0.5	2.0×10^3	1.0×10^3	6.5×10^7	4.1×10^3	2.6×10^3

of the energy flux, from (24), are presented for an example of each kind of wave. Also presented are heating rates which would obtain if the wave amplitude were constrained to be independent of height. An amplitude $\Delta T = 5K$ is assumed, and fluxes are evaluated at $n = 10^{14} \text{ cm}^{-3}$. The heating rate, given by $dT/dt = -(\partial F/\partial z) \times (\rho c_p)^{-1}$, is independent of height.

Also presented in Table 1 are horizontal and vertical phase speeds, and the diffusivity value necessary to prevent amplitude growth.

There are two interesting points concerning the energy fluxes and heating rates. The acoustic wave would carry a very large flux (about 10% of the solar constant at Jupiter) and lead to heating rates of $\sim 2600K$ per day. It clearly could not be a continuous phenomenon. The inertia-gravity mode would produce heating rates of $\sim 1K$ per day at the levels where its amplitude is constrained. This would be important to the heat balance. The solar EUV flux absorption above the $n = 5 \times 10^{13} \text{ cm}^{-3}$ level is given by Strobel and Smith (1973) to be $1.3 \times 10^{-2} \text{ erg cm}^{-2} \text{ sec}^{-1}$. This is to be compared to $3.4 \text{ ergs cm}^{-2} \text{ sec}^{-1}$ for the inertia-gravity wave flux.

Our conclusion is that the inertia-gravity wave interpretation of the observation is consistent with all the information we have. If the interpretation is correct, the following points are important: 1) some source for excitation exists, probably at lower levels; 2) the diffusivity associated with the absorption of the wave is probably on the order of $10^6 \text{ cm}^2 \text{ sec}^{-1}$; and 3) the energy deposition associated with absorption is important to the heat balance.

In the earth's upper atmosphere, heating rates due

to absorption of waves may also be on the order of a few degrees per day (Hines, 1963). Radiative contributions to heating and cooling are somewhat larger than this, however, and the waves do not dominate the heat balance. It appears likely that on the outer planets, where radiative heating is weaker, the situation is different.

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CORRIGENDUM

The following corrections should be made in the note entitled "On the Analysis of the Condensational Growth of a Stationary Cloud Droplet in the Vicinity of Activation," by J. C. Carstens *et al.* (*J. Atmos. Sci.*, **31**, 592-596):

1. On p. 593 the first sentence in column 2 should read "... the value $s=0$ implies that both the sticking and thermal accommodation coefficients must take on the absurd values of infinity."
2. Eq. (A3) should read:

$$s = \left[\frac{l_{\alpha} b L}{K} + \frac{l_{\beta}}{D} \right] D_{\text{eff}}$$