Precipitation Characteristics at Vertical Incidence from Multiple Wavelength Doppler Radars

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ABSTRACT

A vertically looking, multiple-wavelength Doppler radar technique to estimate vertical velocity, drop size distribution and turbulence is presented. The ratio of the Doppler spectra, corresponding to the drop fall velocities at two different wavelengths, is uniquely related to the ratio of the radar scattering cross sections at the appropriate temperature. These ratios are used to estimate drop fall velocities and therefore drop size distributions and vertical wind. Turbulence, which broadens the velocity power spectrum, can be estimated by deconvolution and the drop size distribution subsequently derived.

1. Introduction

Estimation of drop size spectra and vertical wind velocity with Doppler radar historically has depended on either an assumed minimum detectable drop size or an approximation of the vertical velocity from the reflectivity (Atlas *et al.*, 1973). Generally, spectral broadening due to turbulence and cross-wind components has been ignored.

The technique presented here uses multiple-wavelength, vertically pointing Doppler radars to estimate vertical wind velocity, drop size spectra and turbu-

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lence in the sample volume. Fundamental to our approach is the unique ratio of the cross sections which exists when at least one of the radars is sampling outside the Rayleigh region. In theory, the drop size distribution, vertical velocity and turbulence are uniquely determined; however, accuracy is limited by statistical properties of radar echoes.

2. Doppler velocity spectra ratio

Neglecting statistical fluctuations, turbulence, and cross-wind components, the ratio of Doppler velocity spectra measured with two vertically pointing radars of different wavelength forms a unique curve.

The average power $S(v)dv$ scattered from precipitation particles with velocities between v and $v+dv$, measured with vertically pointing monostatic Doppler radar, is

$$S(v)dv=C\sigma(v)N_v(v)dv, \qquad (1)$$

where $\sigma(v)$ is the radar backscattering cross section and $N_v(v)$ the number density. The radar constants are contained in C.

The ratio of Doppler velocity spectra derived from a common volume from two radars at two different wavelengths ($\lambda=1, 2$), normalized by the ratio of the radar constants is

$$\frac{S_1(v)}{S_2(v)}=\frac{\sigma_1(v)}{\sigma_2(v)}. \qquad (2)$$

Since the number density seen by the two radars is the same, $N_v(v)$ does not appear in (2).

At any temperature, the spectral ratio is a unique function of the radar cross section. Ratios of three different wavelengths (1.25, 3.2 and 10.0 cm) as a function of drop size and terminal velocity are shown in Fig. 1 for 10C and −10C. The appropriate complex refractive index is given by Ray (1972). The drop size inferred from the terminal velocity was obtained from an empirical fit by Atlas *et al.* (1973) to data of Gunn and Kinzer (1949).

Vertical wind in the scattering volume linearly shifts the ratio along the abscissa. Downward wind increases the terminal velocity of all drops relative to the ground by a constant amount, equal to the vertical wind, and the ratio curves in Fig. 1 would be shifted to the right. The vertical wind is found by fitting the measured ratio with that expected for still air, a determination independent of drop size distribution and radar calibration. However, determination of drop size distribution requires accurate individual radar system calibration. In addition to standard calibration techniques, calibration may be made in a relative sense for the region of small drops (Walker *et al.*, 1964).

The drop size distribution $N_D(D)$ may be obtained from

$$N_D(D)=N_v(v)\left(\frac{dD}{dv}\right)^{-1} \qquad (3)$$

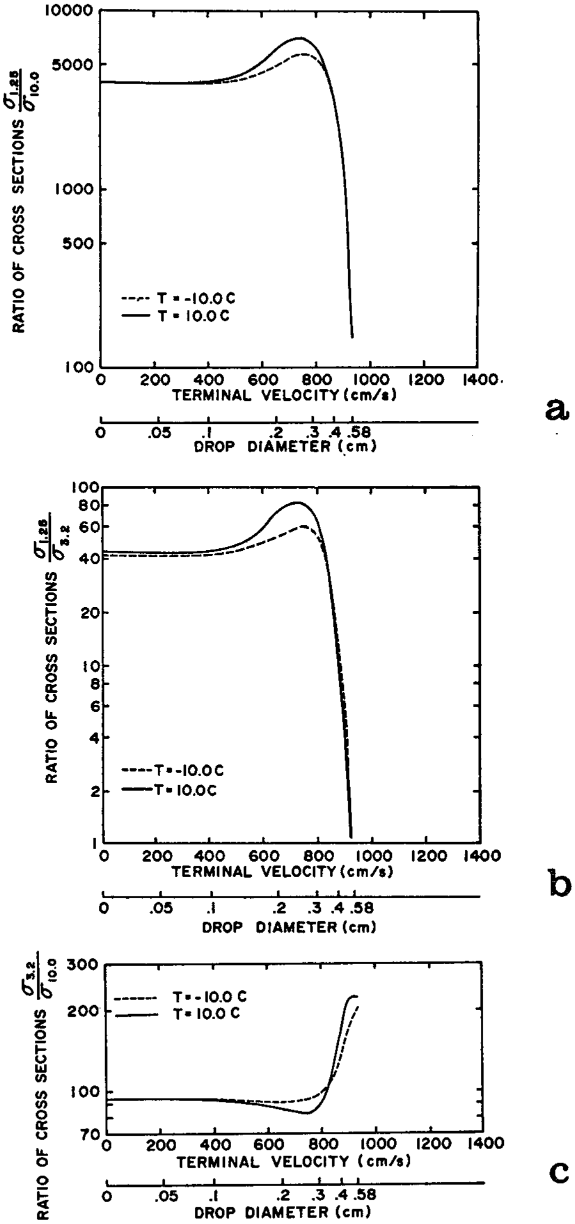

Fig. 1. Ratio of cross sections at 10C and −10C as a function of terminal velocity and drop diameter for wavelength ratios of (a) 1.25/10.0, (b) 1.25/3.2 and (c) 3.2/10.0.

for both radars, where $N_v(v)$ is obtained from (1) after the spectra have been corrected for the effects of vertical wind.

Kinzer and Gunn (1951) show that the thermal relaxation time for freely falling evaporating drops is about 4 sec. The thermal relaxation time for drops falling in a saturated environment is about 12 sec. Assuming a terminal velocity of 9 m sec^{-1} for the largest drops, the drops should be in near-thermal equilibrium in a time less than it would take them to fall through the depth of the sampling volume for expected environmental temperature gradients.

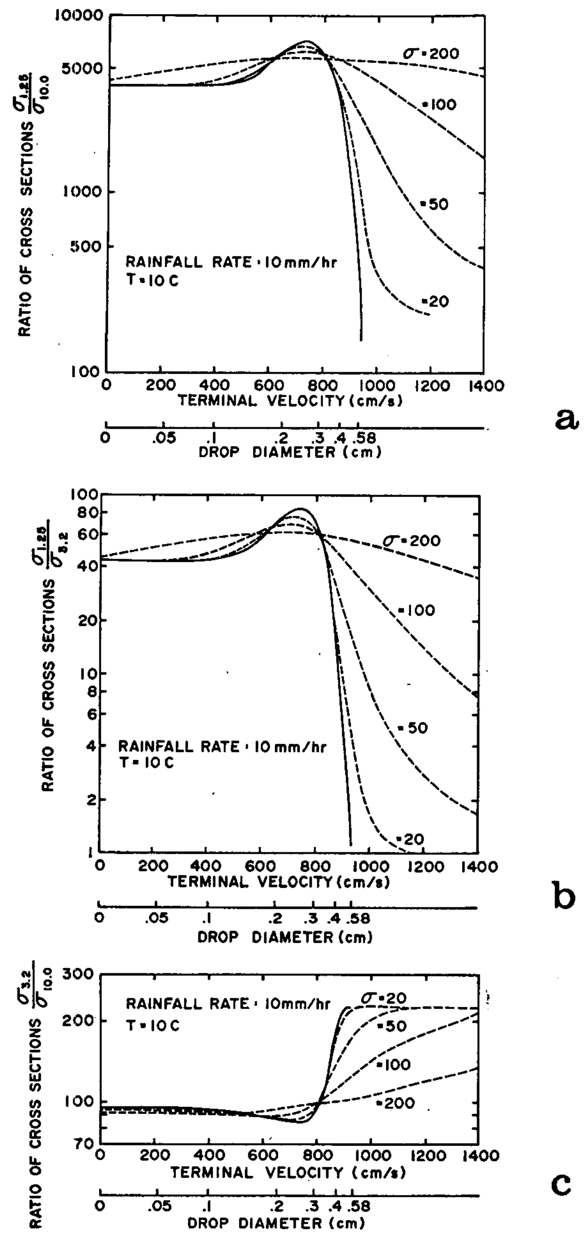

FIG. 2. Ratio of power density spectra for wavelength ratios of (a) 1.25/10.0, (b) 1.25/3.2 and (c) 32./10.0, including the effect of Gaussian turbulence, with $\sigma = 20$, 50, 100 and 200 cm sec^{-1}.

3. Doppler spectrum broadening

Atlas (1964) lists four factors which broaden the Doppler spectrum: particle fallspeed spectrum, wind shear, cross-wind component and turbulence.

The particle fallspeed spectrum is a consequence of the velocity dependence on drop diameter and is central to this technique.

Wind shear, or the variation across the beam of the wind along the beam axis, has been treated by Atlas *et al.* (1969). The increase in spectral variance for most regions of a storm would be modest and, in practice, indistinguishable from turbulence effects.

Cross-wind effects were considered by Sloss and Atlas (1968) and they too are manageable; a cross wind of 30 m sec^{-1} produces a cross-wind variance contribution of only about 5 cm^2 sec^{-2} for a 0.5° half-beamwidth.

Turbulence, i.e., fluctuations about the radial wind component, typically can be expected to have a variance contribution of 1.0 m^2 sec^{-2} with extreme values of 0.5 to 2.0 m^2 sec^{-2} (Nathanson, 1969). To demonstrate the effect of turbulence, a Gaussian modeled turbulence spectrum with standard deviations of 0.2, 0.5, 1.0 and 2.0 m sec^{-1} is imposed on the ratios of $\sigma_{1.25}/\sigma_{10.0}$, $\sigma_{1.25}/\sigma_{3.2}$ and $\sigma_{3.2}/\sigma_{10.0}$, and is illustrated in Figs. 2a, 2b and 2c, respectively. These curves are for a temperature of 10C and a Marshall-Palmer (1948) drop size distribution with a rainfall rate of 10.0 mm hr^{-1}. The ratios containing the effects of turbulence were obtained by the indicated convolution

$$\frac{S_1'(v)}{S_2'(v)} = \int_0^\infty S_1(\lambda)T(\lambda-v)d\lambda \bigg/ \int_0^\infty S_2(\lambda)T(\lambda-v)d\lambda, \quad (4)$$

where T represents the turbulence spectrum. The effect of turbulence is clearly to smooth the ratio curve. Although, neglecting turbulence, the ratios are independent of drop size distribution, the ratios are a function of drop size distribution when turbulence is included. In practice, turbulent spreading is a function of drop size. The effect of turbulence is more pronounced at the large drop size end of the ratios, even though small drops are more responsive to turbulence. The result is that turbulence may be underestimated. The effects of turbulence and other spectral broadening components must be removed. Eq. (4) may be expressed as

$$\frac{F^{-1}[S_1'(v)]}{F^{-1}[S_2'(v)]} = \frac{\rho_1(\tau)\rho_T(\tau)}{\rho_2(\tau)\rho_T(\tau)}, \quad (5)$$

where $F^{-1}[\ ]$ denotes the inverse Fourier transform, and $\rho_{1,2}$ and ρ_T are the inverse Fourier transforms of S and T, respectively. Here T includes all the effects of spectral broadening. To estimate this broadening spectrum, the $\rho_T(\tau)$ is selected which minimizes the difference:

$$\sum_{i=1}^{N} \left\{ \frac{S_1(v_i)}{S_2(v_i)} - \frac{F\left[\dfrac{F^{-1}[S_1'(v_i)]}{\rho_T(\tau)}\right]}{F\left[\dfrac{F^{-1}[S_2'(v_i)]}{\rho_T(\tau)}\right]} \right\}^2 = R. \quad (6)$$

Here, $F[\ ]$ represents the Fourier transform.

4. Statistical considerations

The statistical nature of precipitation echoes and power density estimates are neglected in the above section. Each spectral component (two degrees of freedom) is exponentially distributed with a variance equal to the mean square value (Blackman and Tukey, 1958). When n independent spectra are averaged, the variance (standard deviation) is reduced by n ($n^{\frac{1}{2}}$), and the mean estimates will have Gaussian distributions in accordance with the central limit theorem. The problem in averaging a large number of spectra is the required sampling time; the data collection period must be short enough that major changes do not take place in the radar sampling region.

The average spectral component variance for the ith radar is

$$\overline{\sigma_{im}^2}=\frac{\sigma_{im}^2}{n}, \tag{7}$$

where $\overline{\sigma_{im}^2}$ is the variance at the mth spectral point, averaged over n spectra. Since the square of the mean is numerically equal to the variance at each spectral point, (7) may be rewritten

$$\overline{\sigma_{im}^2}=\frac{\mu_{im}^2}{n}, \tag{8}$$

where μ_{im}^2 are the squared expected mean values of the mth spectral component. Since the distribution of the mean estimate for the mth spectral component is Gaussian, the variance for the ratio at two wavelengths of the mth spectral component mean estimates is simply (Topping, 1956):

$$\sigma^2=\frac{\mu_{1m}^2}{\mu_{2m}^2}\left[\frac{\overline{\sigma_{1m}^2}}{\mu_{1m}^2}+\frac{\sigma_{2m}^2}{\mu_{2m}^2}\right], \tag{9}$$

which reduces to

$$\sigma^2=\left(\frac{2}{n}\right)\left(\frac{\mu_{1m}}{\mu_{2m}}\right)^2. \tag{10}$$

The variance of the ratio of the average mean estimates then becomes $2n^{-1}$ multiplied by the ratio squared as given in Fig. 1, for each spectral component.

5. Summary

Previous estimates of drop size spectra and vertical wind ignore possible spectral broadening effects and either assume a drop size spectral form or an approximate vertical velocity. We obviate the need to make assumptions about the drop size spectra or vertical velocity. The unique character of the ratio of radar scattering cross sections and deconvolution of the spectra available from each radar is used to estimate turbulence. The vertical velocity may then be found. From the Doppler velocity power spectrum, the drop size distribution follows an appropriate conversion from velocity to drop size. This technique is applicable over a wide range of rainfall rates where broadening effects are not too severe and where a sufficient number of large (where the ratio changes dramatically) drops are present.

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REFERENCES

Atlas, D., 1964: Advances in radar meteorology. *Advances in Geophysics*, Vol. 10, New York, Academic Press, 192–228.

——, R. C. Srivastava and P. W. Sloss, 1969: Wind shear and reflectivity gradient effects on Doppler radar spectra: II. *J. Appl. Meteor.*, **8**, 384–388.

——, ——, and R. S. Sekhon, 1973: Doppler radar characteristics of precipitation at vertical incidence. *Rev. Geophys. Space Phys.*, **11**, 1–35.

Blackman, R. B., and J. W. Tukey, 1958: *The Measurement of Power Spectra*. Dover, New York, 16–22.

Gunn, R., and G. D. Kinzer, 1949: The terminal velocity of fall for water droplets in stagnant air. *J. Meteor.*, **6**, 243–248.

Kinzer, G. D., and R. Gunn, 1951: The evaporation, temperature and thermal relaxation-time of freely falling waterdrops. *J. Meteor.*, **8**, 71–83.

Marshall, J. S., and W. Palmer, 1948: The distribution of raindrops with size. *J. Meteor.*, **5**, 165–166.

Nathanson, F. E., 1969: *Radar Design Principles*. New York, McGraw-Hill, 68–107, 192–228.

Ray, P. S., 1972: Broadband complex refractive indices of ice and water. *Appl. Opt.*, **11**, 1836–1844.

Sloss, P. W., and D. Atlas, 1968: Wind shear and reflectivity gradient effects on Doppler radar spectra. *J. Atmos. Sci.*, **25**, 1080–1089.

Topping, J., 1956: *Errors of Observation and Their Treatment*. London, Unwin, 82 pp.

Walker, G. B., L. S. Lamberth and J. J. Stephens, 1964: Dual frequency observations of precipitation. *J. Appl. Meteor.*, **3**, 430–438.