Relative Dispersion of Constant-Level Balloons in the 200-mb General Circulation

Pierre Morel and Michèle Larcheveque

Laboratoire de Météorologie Dynamique, C.N.R.S., Paris, France

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ABSTRACT

The EOLE experiment with 480 constant-volume balloons distributed over the Southern Hemisphere at about the 200-mb level, has provided a unique set of trajectories of tagged particles in the general westerly circulation. All such data observed in the latitude belt 30 to 50S have been analyzed to estimate the mean square relative velocity \((dD/dt)^2\) and mean diffusivity \(d\Phi/dt\) of balloon pairs separated by the distance \(D\). It is found that the eddy dispersion process is homogeneous, isotropic and stationary up to scales \(D\sim1000\) km, and agrees well with the prediction of the two-dimensional turbulence model of atmospheric motions. The mean separation increases exponentially with time up to 6 days, and more slowly like \(t^2\) later. An estimate of the gross ausstausch coefficient of the general circulation from these data is \(1.6\times10^6\) m² sec⁻¹.

1. Introduction

The study of the relative dispersion of clusters or pairs of Lagrangian particles in atmospheric flow is not only basic to understanding the process of eddy diffusion but can also serve to characterize the energetics of two-dimensional eddy motions in a range of scales 100–1000 km which is not easily observed otherwise. Relative diffusion in the atmosphere was early investigated by Richardson (1926) and later by many investigators, including Sutton (1932), Batchelor (1950, 1952), Obukhov (1959), Corrsin (1962) and Lin and Reid (1963). These studies were primarily concerned with the rate of particle separation in relation with the particle distance, and were generally based on the current models of small-scale three-dimensional atmospheric turbulence. More recent investigations of Mesinger (1965), Kao and Al-Gain (1968), Murgatroyd (1969) and Deardorff and Peskin (1970) have been aimed at extracting Lagrangian statistics from numerically simulated two- and three-dimensional flows or from numerically computed particle trajectories using actual maps of the large-scale circulation. Attempts to obtain such statistics directly from the observed trajectories of constant-level balloons in the lower stratosphere (200 mb) have also been made by Kao and Hill (1970), Wooldridge and Reiter (1970), and Morel (1970).

The purpose of this paper is to describe statistics on relative dispersion, based on a much larger sample of Lagrangian observations obtained in the course of the EOLE experiment (1971–72), in fact the largest body of Lagrangian observations ever obtained on large-scale atmospheric circulation. The EOLE experiment consisted in the release of 483 constant-volume balloons at the 200-mb level in the Southern Hemisphere. The balloons were located periodically by the special purpose EOLE data collection and navigation satellite (Morel and Bandeen, 1973). Altogether, 12,239 individual estimates of the relative dispersion rate of balloon pairs, obtained during the period October 1971–January 1972, are analyzed in this study. The advantage of using a reasonably large ensemble of observations is to allow testing the obvious statistical characteristics of the atmospheric flow in respect to homogeneity, isotropy and stationarity (Sections 2–4). It is then possible to try and establish the eddy diffusion law for particles by medium-to-large-scale eddies in the atmospheric circulation (100–1000 km) and determine the energy spectrum of these components of the flow (Sections 5–7).

2. The problem

It is now accepted that the medium-to-large-scale circulation of the free atmosphere is a typical example of two-dimensional turbulent flow with a very large Reynolds number, according to the ideas of Kraichnan (1967), Leith (1968, 1971), Batchelor (1969) and Charney (1971). It is further understood that the dynamics in the large wavenumber end of the spectrum are governed by the inertial transfer of enstrophy or squared vorticity so that the one-dimensional energy spectrum follows a \(k^{-2}\) power law (Kraichnan, 1971). These theoretical predictions are well supported by numerical simulations of two-dimensional flows or general circulation models, and to some extent, by observational evidence such as presented by Wiin-Nielsen (1967), Kao and Wendell (1970) and Julian et al. (1970). These observational studies, based on
We can show, however, that an alternate indirect method, based on analyzing the instantaneous relative dispersion rate of randomly distributed particles moving with the fluid, can yield the same information. We are thus proposing here an experimental verification of the $k^{-4}$ spectral dependence of kinetic energy in the scale range 100 to 1000 km, based on about 12,000 measurements of the dispersion rate $dD/dt$ of particle pairs.

The line of reasoning starts with the idea that whenever two particles are separated by any distance $D$, the further statistical increase of their distance is governed by turbulent eddies of size comparable with $D$ (Corrsin, 1962). If $D$ is smaller than most of the turbulent structure, the two fluid material points have almost equal velocities and do not separate very much. If, on the other hand, $D$ is larger than most of the turbulent structure, the velocities of the two particles are uncorrelated and their variances add up in the mean square relative velocity; thus,

$$\langle (V_1 - V_2)^2 \rangle = \langle V_1^2 \rangle + \langle V_2^2 \rangle - 2 \langle V_1 \cdot V_2 \rangle \approx \langle V_1^2 \rangle + \langle V_2^2 \rangle. \tag{1}$$

For any particle separation, we can see that the eddies most efficient in dispersing are those smaller than the separation; the efficiency is measured here by the contribution to "diffusivity" per unit kinetic energy and varies as sketched in Fig. 1a. Although the smallest eddies are most efficient, they contain relatively little energy. Large eddies, on the other hand, contain most of the energy but they are very inefficient and contribute little to dispersion. Hence, we arrive at the conclusion that the main contribution to relative dispersion comes from a spectral window around scales of motion of the same order of magnitude as the particle separation $D$ (Fig. 1b). Sampling the relative dispersion statistics for increasing initial particle separations is therefore equivalent to moving the spectral window from $1/D_{\text{min}}$ to $1/D_{\text{max}}$. If we may further assume that $D$ lies in a similarity range, the $k$-dependence of the kinetic energy spectrum can be inferred from this information alone.

3. The data base

The purpose of the study was to investigate the medium-to-large-scale relative dispersion of clusters of constant-level balloons in the atmospheric flow. It turned out, however, that studying clusters of more than two balloons was impractical on account of the relatively small number of events when several balloons could all be located on one orbit and again on the next orbit of the satellite (Fig. 2). For this reason, we restricted our study to balloon pairs characterized by separation $D$.

It will be remembered that the EOLE balloon launching schedule called for the quasi-simultaneous...
release of three to four balloons per day, weather permitting. In practice, groups of four balloons were indeed deployed in the atmosphere within a 2- to 3-hr interval, thus forming small linear clusters along the direction of wind. On account of this group launching strategy, each set of three or four balloons provided about as many independent pairs which could then be tracked during several days. The statistics presented here are based on 463 such original pairings of EOLE balloons. But since the domain offered for dispersion is finite, the reverse event of two balloons being brought together by this random mixing process is not unlikely. We therefore looked for such chance pairings occurring in the course of the balloon network evolution and were able to select 1034 cases where two balloons came within 250 km of each other. The further dispersion of these pairs are also included in our statistics. A preliminary analysis indicated no systematic difference between the original pairs and the chance pairs (Fig. 3).

Because two different balloons \(i\) and \(j\) could not, naturally, be interrogated and located simultaneously, a (small) correction was applied to reduce the measured locations to one single time \(t_1\) for the two members of a pair. Since the time interval between interrogation \(i\) and interrogation \(j\) could be at most 300 sec, the maximum correction needed was \(\pm 15\) km and the error involved in this procedure is quite negligible with respect to random relative location errors of the order of 0.5 km in both the \(X\) and \(Y\) directions. Now, each pair of balloons could provide many nearly independent samples of the dispersion rate \(dD^2/dt\) [each being estimated over the time interval \((t_2-t_1)\) between two successive orbits of the spacecraft, i.e., \(\sim 103\) min or 6000 sec] from the finite-difference form

\[
\frac{dD^2}{dt} = \frac{D^2(t_2) - D^2(t_1)}{t_2 - t_1}.
\]  

(2)

Our statistics are based on 12,239 such estimates. Our statistical approach consisted in splitting these values in 14 equally populated classes of increasing separation \(D\), varying from 0–60 km in the first class to 4000–10,000 km in the last class.

The accuracy of balloon location measurements with respect to the EOLE satellite orbital track and with respect to each other has been discussed by Morel and Necco (1973). The relative position of two balloons is not sensitive to the rather large uncertainty in the instantaneous position of the spacecraft or satellite clock error, and its accuracy is restricted solely by random electronic errors to about 0.7 km in both directions. The resulting effect is of the order of 1 km² for \(D^2\) and 0.3 m² sec⁻¹ for \(dD^2/dt\), a very small error indeed compared to observed diffusivity values \(dD^2/dt \sim 10^3\) m² sec⁻¹.

A more serious problem is the question of the representativeness of a point velocity measurement (one single tracer) with respect to the mesoscale air parcel which surrounds it. It has been found that the layered vertical structure of the lower stratosphere circulation could induce significant differences between the horizontal velocity of a constant-level tracer and the mean flow velocity of the surrounding air parcel. This difference is due to possible strong vertical wind shears in excess of 1 m sec⁻¹ (100 m)⁻¹, and contributes an additional variance to the velocity and relative velocity.

![Fig. 2. Mutual visibility condition for measuring the dispersion of a balloon cluster.](image-url)
Diffusivity
\( \text{(m}^2 \text{s}^{-1}) \)

\[ \frac{d \bar{Y}^2}{dt} \]

\[ \frac{d \bar{X}^2}{dt} \]

\( \text{DISTANCE X OR Y (km)} \)

Fig. 4. Comparison of diffusion rates for three groups of balloon pairs originating from longitude 100–120E (dots), 20–40W (open circles) and 140–160W (crosses).

measurements. For relative velocity, then, we can estimate from Morel and Necco (1973):

\[ \Delta \left( \frac{dD}{dt} \right) \leq (3 \text{ m sec}^{-1})^2. \] (3)

It should be emphasized that this source of error does not, fortunately, affect the Lagrangian estimates of the mean diffusivity, because the extra relative velocity \( \Delta(dD/dt) \) resulting from the vertical shear is not correlated with the separation vector \( D \). We shall therefore expect to arrive at more precise and definite conclusions by considering the mean diffusivity rather than the mean square separation velocity of particle pairs.

4. Homogeneity, stationarity and isotropy

It is common practice to base theories of eddy diffusion in the atmosphere on the assumption that such turbulent processes are essentially homogeneous and isotropic in space, and stationary in time. Whether this is at all a valid hypothesis, particularly for the larger scales of motion, remains to be demonstrated, however. The uniquely dense and accurate data collected during the course of the EOLE experiment provide us with the first satisfactory opportunity of establishing these basic properties of large-scale atmospheric turbulence on a firm experimental evidence. The test we have chosen for this purpose is to compare estimates of the mean diffusivity in both zonal and meridional directions, computed with various subsets of the EOLE data.

Consider first Fig. 4 which shows the dependence of diffusivities \( d\bar{X}^2/dt \) and \( d\bar{Y}^2/dt \) vs separation \( X \) and \( Y \), respectively, for three groups of chance pairs observed during the month of October 1971 and originating from three longitude ranges (100–120E, 140–160W, 20–40W) in the latitude belt 30–50S. As expected, no systematic difference appears between these samples, each containing about 90 independent pairs of particles. On this basis, we may accept spatial homogeneity as an established property of medium-to-large-scale eddies at least within the general westerly circulation, and accordingly average over all longitudes.

Consider further Fig. 5 which shows the same information as Fig. 4, only for three different 1971 time intervals (22 October–1 November; 15–26 November, 29 November–20 December). Again, no systematic difference is found between these three samples in-

Fig. 5. Comparison of diffusion rates of balloons during three successive unconnected periods between 22 October and 20 December, 1971.
cluding 178, 185 and 231 pairs, respectively. We have therefore accepted that eddy diffusion was a stationary process during the time span of the experiment and accordingly averaged all observations in the period 11 October 1971 through 11 January 1972.

As far as isotropy is concerned, one can first notice that neither Fig. 4 nor Fig. 5 indicate a significant difference between the zonal ($X$) and meridional ($Y$) directions. A more quantitative statement can be made by comparing the mean square relative separations, $\bar{X^2}$, $\bar{Y^2}$, and the cross correlation of the zonal and meridional components of this separation $\bar{XY}$ (Fig. 6). We see then that the cross correlation $\bar{XY}$ is at least one order of magnitude smaller than $\bar{X^2}$ or $\bar{Y^2}$, as befits an isotropic random process. A similar result was also found with simulated particle trajectories by Kao and Al-Gain (1968). Finally, a more sensitive test is found by considering the ratio of the mean square separation observed in the two orthogonal directions $X$ and $Y$. We have plotted in Fig. 7 the ratio of the rms zonal dispersion to the rms meridional dispersion of chance pairs found initially with a separation smaller than 250 km. The original pairs of balloons released from one particular launching site have been excluded from these statistics because they exhibit an initial bias favoring dispersion in the $X$ direction. The striking result indicated by Fig. 7 is that the further dispersion process is essentially isotropic for distances as large as 1500 km but becomes strongly anisotropic in favor of the zonal direction for larger scales. This, of course, was to be expected on account of the fairly narrow latitude range (essentially $40^\circ$ from 20–25S to 60–65S) offered for dispersion. Nevertheless, we are satisfied that the isotropic random turbulence model holds remarkably well in the range of scales 100–1000 km, but not at all for the larger scales of motion or smaller wavenumber range $k=10–20$ where experimental evidence regarding the $k^{-3}$ energy spectrum has been sought so far. For our part, we shall restrict the theoretical interpretation of our data to this 100–1000 km range within which the isotropic two-dimensional turbulence model of atmospheric motions can be expected to be a precise approximation of the real flow.

5. Relative separation of particles as a function of time

The diffusion of tagged particles can be most simply described by considering the relative zonal and meridional particle separations as functions of time, starting from the initial time $t_0$ of their release with a standard initial separation $D_0$. Now, the chance pairs which formed in the course of the experiment do not provide useful data for this purpose, due to their excessively diverse initial separations. On the
other hand, the separation of original pairs or clusters of balloons released from one single site exhibits a marked original bias along the zonal direction. Nevertheless, the average separations of original pairs plotted in Fig. 6 does show general agreement with the findings of Kao and Al-Gain (1968) derived from artificial trajectory statistics.

Lin (1971) suggested that the relative dispersion of particle clusters in the enstrophy-cascading range of two-dimensional turbulence, should increase exponentially with time. His argument, based on similarity considerations, postulates that the eddy diffusivity \( dD^2/dt \) depends solely upon the scale parameter \( D \) and the enstrophy dissipation rate \( \beta \). For dimensional consistency, one must then have

\[
\frac{dD^2}{dt} = A\beta^3 D^3 = \frac{2D^3}{T}, \tag{4}
\]

where \( A \) is an unknown numerical constant independent of both \( D \) and \( \beta \), or alternately \( T \) is an unknown time constant. Hence, on integrating, we have the exponential separation law

\[
D^2 = D_0^2 \exp\left(\frac{2t}{T}\right). \tag{5}
\]

It is striking that the mean separation rate of the balloon pairs released quasi-simultaneously, during the first five and six days of flight, fits very well this semi-empirical exponential law (Fig. 8), with an rms initial separation \( D_0 = 80 \) km and a time constant \( T = 2.7 \) days.

Further diffusion does not proceed exponentially, as one would expect. The observations of the dispersion of original pairs of EOLE balloons 10–20 days after their release would rather precisely fit the \( t^1 \) law:

\[
\frac{D^2 \tau}{2} \approx 1.6 \times 10^4 \text{ m}^2 \text{ sec}^{-1}. \tag{6}
\]

One could venture from this an estimate of the "eddy diffusion coefficient" for the large-scale motions of the atmosphere, which would be significantly smaller than the commonly quoted value \( 10^7 \text{ m}^2 \text{ sec}^{-1} \).

6. Power spectrum of the relative velocity

The further separation of two particles already separated by a distance \( D \) is due essentially to eddy motions of scale smaller than or about equal to \( D \). The power spectrum of the relative velocity \( dD/dt \) must then be closely similar to the eddy velocity spectrum for a single particle. If similarity holds over the range of wavenumbers considered in this study, it may not be necessary to determine the exact contribution of various eddies to diffusion, per unit kinetic energy (Fig. 1a); instead, one can approximate this using a sharp cutoff at wavenumber \( k \sim 1/D \). Then

\[
\left( \frac{dD}{dt} \right)^2 = 2\bar{u}^2 + 2\bar{v}^2 = 4 \int_{k=1/D}^{\infty} E(k)dk, \tag{7}
\]

where \( E(k) \) is the unidimensional power spectrum of the essentially two-dimensional mesoscale-to-large-scale motions of the atmosphere. Assuming a \( k^{-\alpha} \) spectral dependence, one readily finds

\[
\left( \frac{dD}{dt} \right)^2 \sim \int_{k=1/D}^{\infty} k^{-\alpha} dk \sim D^{\alpha-1}. \tag{8}
\]

It should thus be straightforward to determine the spectral law by plotting \((dD/dt)^2\) as a function of \( D \) (Fig. 9). This is not so simple in practice, however, because the measured relative velocities \( dD/dt \) do not reflect solely the horizontal shear associated with the quasi-geostrophic, quasi-two-dimensional eddies but also respond to the large persistent shear of the horizontal wind observed over small vertical distances in the lower stratosphere (Sawyer, 1961; Weinstein et al., 1966). We have no direct estimate of these vertical shears nor any precise determination of their contribution to the relative velocity variance as computed from the EOLE balloon trajectories. An indirect estimate could be inferred by Morel and Necco (1973) from the spurious rms divergence value found for
very small balloon clusters. Correcting the measured relative velocity variance \((dD/dt)^2\) for this effect amounts to a fairly large and uncertain modification of the original data, as can be seen in Fig. 9. For this reason we may only conclude that the mean square relative velocity of constant-level balloons, as measured during the EOLE experiment, are not incompatible with the \(k^{-3}\) power spectrum predicted by the two-dimensional turbulence model of quasi-geostrophic atmospheric motions. But this cannot constitute a definite proof.

7. Particle diffusivity as a function of distance

A more useful approach lies in considering the statistics of the dispersion rate, \(\overline{dD^2}/dt\), for various classes of particles separation, as given in Table 1 for the 12,239 individual pair balloon measurements. As pointed out in Section 3, there is no correlation between the vertical shear \(dV/dz\) due to the laminated structure of the lower stratosphere circulation, and the separation \(D\) of two randomly distributed tracers. Accordingly, the mean diffusivity

\[
\frac{\overline{dD^2}}{dt} = 2D \frac{d\overline{D}}{dt}
\]

is only due to the horizontal wind shear associated with quasi-geostrophic eddies of the general circulation. For such motions, of course, a finite correlation exists between the relative velocity \(dD/dt\) and the separation \(D\).

Assuming similarity of the atmospheric flow in the range of scales of the eddies which effectively contribute to the balloon dispersion process, we can postulate that the diffusivity depends only upon the eddy kinetic energy spectral density \(E(k)\) and a scale parameter \(k\) of the order of \(D^{-1}\). Then

\[
\frac{\overline{dD^2}}{dt} = f[E(k), k].
\]

By dimensional arguments, we see that this function is necessarily proportional to

\[
\frac{\overline{dD^2}}{dt} \sim [k^{-1}E(k)] \sim [DE(D^{-1})].
\]

Eq. (11) thus relates the wavenumber dependence of the energy spectrum to scale dependence of the diffusivity. If \(k^{-3}E(k)\) is assumed to decrease as \(k^{-\alpha-1}\), we then have

\[
\frac{\overline{dD^2}}{dt} \sim D^{(\alpha+1)/2}.
\]

A precise estimate of the \(k\) dependence of the energy spectrum can thus be inferred from the scale dependence of diffusivity. Diffusivity data based on all EOLE measurements (Table 1) have been plotted on a log-log diagram (Fig. 10) and indicate excellent agreement with (12) in the range \(D = 100-1000 \text{ km}\) where atmospheric motions best fit the model of homogeneous, isotropic, two-dimensional turbulence. The exponent \(\alpha\) determined by linear regression is found to be 3.0±0.1. This constitutes the most precise determination of the slope of the kinetic energy spectrum of mesoscale-to-large-scale motions, which could be extracted from the EOLE data set.
8. Conclusion

An extensive set of unusually consistent and precise 200-mb wind data, provided by the EOLE constant-level balloon experiment in the Southern Hemisphere, has been analyzed from the point of view of finding the scale dependence of eddy diffusion of pairs of balloons. These data are contaminated by a fair amount of noise, due to the fact that a discrete velocity measurement may differ from the true mean velocity of a representative air parcel around one particular tracer by as much as 1–2 m sec⁻¹. This noise can be eliminated by taking the correlation product of relative velocity dD/dt and separation D. One can therefore be confident that the measured scale dependence of diffusion dD²/dt is precisely related to the spectral dependence of eddy kinetic energy in the range of scales D = 100–1000 km. Excellent agreement was found with the k⁻³ spectrum predicted by the two-dimensional turbulence of models of the lower stratosphere circulation.

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