The Tropical Mixed Layer and Cumulus Parameterization

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ABSTRACT

A model of the convectively driven mixed layer is used to interpret BOMEX Phase III data. It is concluded that the compensating downdrafts and the moisture transports due to trade cumulus must be included explicitly in any theory of the dynamics of the tropical mixed layer.

1. Introduction

The moist tropical boundary layer over the ocean tends to be well mixed, inversion capped, and overlaid by stable stratification (Malkus, 1962). Therefore, air in the mixed layer is stable to unsaturated ascent above the mixed layer inversion. But if a mixed layer parcel is forceably raised (see Fig. 1) at constant virtual moist static energy to saturation and then still further, it would have a positive buoyancy above z_c and would then rise under its own power—this is what is meant by conditional instability. Air in the mixed layer has the highest values of h_v in the lower troposphere by virtue of its high moisture content, so that mixed layer air has the lowest level of zero buoyancy and is consequently most potentially buoyant. It is therefore reasonable to assume that cloud air originates in the mixed layer.

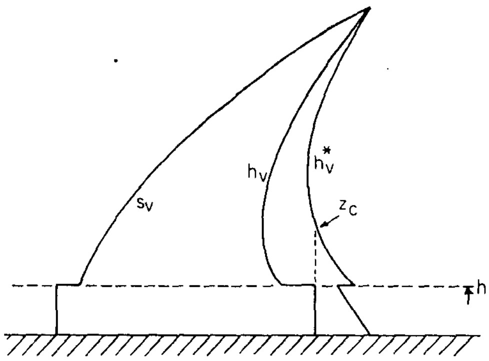

FIG. 1. The vertical structure of a model tropical atmosphere: $S_v = c_p T_v + gz$, where T_v is the virtual temperature; $h_v = S_v + Lq$; and $h_v^* = S_v + Lq^*$, where q is the specific humidity and q^* the saturation specific humidity. The virtual temperature is given by $T_v = T(1+\delta q)$, where $\delta = 0.61$.

Recent cumulus parameterization schemes of Arakawa and Schubert (1974) and Betts (1973), and a stripped-down scheme of Lindzen (1974) have considered just such a role for the mixed layer. Because the compensating subsidence due to the clouds' mass flux tends to suppress the top of the mixed layer, Arakawa and Schubert were led to consider the time variation of mixed layer height and temperature. Recent papers by Betts (1973) and Tennekes (1973) based on work of Deardorff (1972) and Lilly (1968) provide a more complete and easily applicable theory of mixed layer growth.

The difference between such a convectively driven mixed layer and a more commonly assumed neutral Ekman layer cannot be overemphasized. The vertical mixing time of a convectively unstable layer is much shorter than the eddy diffusion time f^{-1} of a neutral Ekman layer, so that buoyancy dominates and horizontal momentum becomes a passive contaminant. Ekman notions such as depth depending on eddy diffusion, wind turning with height, and frictional convergence and pumping, all become inoperative. Instead, turbulent quantities scale with the depth of the mixed layer, the wind is constant with height, and adjustment to a geostrophic interior occurs only *at* the inversion. The tropical mixed layer is mixed by buoyancy *not* wind friction.

It is the purpose of this note to apply an extended version of Betts' and Tennekes' theory to some real data for tropical mixed layers. The theory, including evaporation and subsidence, will be formulated in Section 2. Section 3 will look at BOMEX Phase III results to test the theory. In Section 4 some questions about cumulus parameterization schemes will be raised, and Section 5 will summarize our conclusions.

2. Convectively unstable mixed layers

Wind friction and buoyancy produce comparable turbulence in a Monin-Obukhov depth L, where

$$L = -u_*^3 [g\theta_v^{-1} k (\overline{w'\theta_v'})_0]^{-1}, \tag{1}$$

$g\theta_v^{-1}(\overline{w'\theta_v'})_0$ is the vertical flux of buoyancy from the surface, u_* the surface friction velocity, and k the von Kármán constant. If the buoyancy flux is upward, the layer is unstable and L is by convention negative. If $-L$ is small (say half) the mixed layer height, the region between $-L$ and h is in free convection, and the turbulence due to buoyancy dominates the turbulence due to friction.

For such a boundary layer, growth at the turbulent upper boundary is accomplished by entrainment ("penetrative convection"). The cooler turbulent boundary layer entrains hotter laminar fluid down through the inversion. If the air is moving vertically with velocity $\tilde{w}$ at the top of the mixed layer, the heat balance at the inversion is (see Fig. 2)

$$\Delta\theta_v\left(\frac{dh}{dt} - \tilde{w}\right) = -(\overline{\theta_v' w'})_i, \tag{2}$$

where both θ_v and $\Delta\theta_v$ are virtual temperatures. We have assumed horizontal homogeneity so that horizontal advections are neglected.

In general, we expect the mixed layer to grow relative to its environment so that $(\overline{\theta_v' w'})_i < 0$.

The inversion strength $\Delta\theta_v$ changes according to

$$\frac{d\Delta\theta_v}{dt} = \gamma_v\left[\frac{dh}{dt} - \tilde{w}\right] - \frac{d\theta_{mv}}{dt}, \tag{3}$$

where $\gamma_v = d\theta_v/dz > 0$ is the stable stratification and θ_{mv} the mixed layer virtual temperature, assumed constant with height (i.e., $-L \ll h$). The mixed layer virtual temperature changes as

$$\frac{d\theta_{mv}}{dt} = \frac{(\overline{\theta_v' w'})_0 - (\overline{\theta_v' w'})_i}{h}. \tag{4}$$

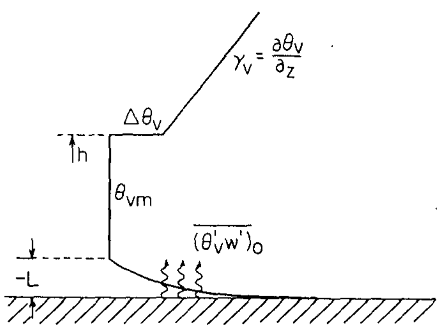

FIG. 2. Model structure of the mixed layer. All potential temperatures are virtual, $-L$ is the Monin-Obukhov length, $\Delta\theta_v$ the temperature discontinuity at the mixed layer inversion, and θ_{mv} the mixed layer virtual temperature.

Finally, the system is closed by a relation between the surface flux and the downward heat flux at the inversion (Deardorff, 1972), i.e.,

$$(\overline{\theta_v'w'})_i = -0.2(\overline{\theta_v'w'})_0. \qquad (5)$$

Eqs. (2)–(5) are basically Tennekes's (1973) model of the mixed layer, but here a mean vertical velocity $\bar{w}$ has been included. The constant 0.2 in (5) has been recently verified in the extraordinary calculations of Deardoff (1974).

The equations can be arranged in a particularly convenient form for numerical computation:

$$\frac{dh}{dt} - \bar{w} = \frac{0.2(\overline{\theta_v'w'})_0}{\Delta\theta_v}, \qquad (6a)$$

$$\frac{d\Delta\theta_v}{dt} - \frac{0.2\gamma_v}{\Delta\theta_v}(\overline{\theta_v'w'})_0 = \frac{-1.2(\overline{\theta_v'w'})_0}{h}. \qquad (6b)$$

These equations can be simply stepped forward in time if $(\overline{\theta_v'w'})_0$ is known as a function of time, and $\bar{w}$ is known as a function of time and height. Recall that since θ_v is a virtual temperature, the flux $(\overline{\theta_v'w'})_0$ contains both the sensible and latent heat fluxes. This point will become crucial in the next section.

The interesting thing about the system (6) is that an equilibrium solution exists when $\bar{w}<0$ (subsidence), namely

$$h = \frac{1.2(\overline{\theta_v'w'})_0}{\gamma_v(-\bar{w})}, \quad \Delta\theta_v = \tfrac{1}{6}\gamma_v h. \qquad (7)$$

When the mean velocity is upward, no such equilibrium solution exists since there is nothing to balance the turbulent growth of the mixed layer. Eq. (7) was used by Betts (1973) in his theory of trade cumulus convection.

Finally, some idea can be gotten of how long it takes the system to adjust to the equilibrium state (7). For this purpose, consider an initial state at $t=0$ whose height h is at equilibrium, but whose inversion strength Δ is *not* equal to $\Delta_0 = \frac{1}{6}\gamma_v h$, Eq. (6b) can be solved implicitly for t as a function of Δ at constant h:

$$|\tfrac{1}{6}\gamma_v h - \Delta| = |\tfrac{1}{6}\gamma_v h - \Delta_0| e^{-(6/\gamma_v h)(\Delta-\Delta_0)} e^{-t/\tau},$$

where

$$\tau = \gamma_v h^2 [7.2(\overline{\theta_v'w'})_0]^{-1}.$$

If τ is short compared to a time during which everything else is changing, then we may use the equilibrium solution (7).

3. An undisturbed BOMEX Phase III period

We consider an undisturbed period of BOMEX, 22–26 June 1969, for which analyzed results are available in the literature (Holland, 1972; Holland and Rasmussen 1973; Nitta and Esbensen, 1974). The results we need are that the averaged wind stress on the ocean surface $\tau_s = -0.65$ dyn cm^{-2}, the latent heat flux from the surface $Q_e = 350$ cal cm^{-2} day^{-1} corresponding to 6 mm day^{-1} of evaporation, and the sensible heat fluxes from the surface $Q_s = 30$ cal cm^{-2} day^{-1} [Nitta and Esbensen (1974) give slightly different values for Q_e and Q_s during the same period, about 10% higher].

The value of the Monin-Obukhov length can be computed from these values as follows. The buoyancy flux is given by[1]

$$(\overline{\theta_v'w'})_0 = Q_e(\rho c_p)^{-1}[b+0.08],$$

where $b = Q_s Q_e^{-1}$ is the Bowen ratio. The 0.08 arises from the buoyancy produced by evaporation, i.e., water vapor is lighter than air. With the above values for Q_e, $(\overline{\theta_v'w'})_0 = 13.6(b+0.08)$ [K cm sec^{-1}] and with the Bowen ratio of 0.086, $(\overline{\theta_v'w'})_0 = 2.26$K cm sec^{-1}. The friction velocity $u_* = (\tau_s\rho^{-1})^{\frac{1}{2}}$ is 23 cm sec^{-1}. The Monin-Obukhov length is therefore $-L = 7.0$ m$/(b+0.08)$. Even with no sensible heat transfer whatsoever, $-L = 88$ m, which is much smaller than any observed mixed layer height. We conclude that the mixed layer over this undisturbed BOMEX region during 22–26 June was convectively driven by buoyancy fluxes from the surface, equally due to sensible heat and evaporation from the ocean surface.

Before using these data to test the predictions of Eq. (7), one major comment must be made. Because of the large amount of water vapor in the mixed layer, there will be a large amount of radiative cooling. Holland and Rasmussen (1973) give this as -5×10^{-6} cal gm^{-1} sec^{-1}, corresponding to a cooling rate of about 2K day^{-1}, almost independent of height up to 1500 m. If we anticipate that the height of the layer is 500 m, the total radiative cooling from a column in the mixed layer is 28 cal cm^{-2} day^{-1}. The only source of heat to make up this radiative deficit (the air temperature stayed roughly constant during the 5-day period) is sensible heat transport from the surface. (Above the mixed layer radiative cooling is balanced by the mean subsidence, and in disturbed areas, even if the large-scale vertical velocity is upward, the cooling is balanced by the between-the-cloud motion which is usually downward.) Although it may seem that the sensible heat flux is used up in balancing the radiative cooling and is therefore not available for producing boundary layer turbulence, in fact it accomplishes this task by creating turbulent plumes whose downward between-the-plume motion, acting on a very slightly stable lapse

[1] Throughout this section, air density has been assumed constant throughout the mixed layer, the latent heat of evaporation has been assumed not to depend on temperature, and virtual temperatures have been replaced by temperatures when not entering as a difference. All these approximations should be good to 5%.

rate, heats the boundary layer by adiabatic compression. Therefore, the flux to be used in comparing Eq. (7) to the observations is the full flux $(\overline{\theta_v'w'})_0=2.26$K cm sec^{-1}. The effects of radiation only enter in the expression for the temperature discontinuity across the mixed layer inversion [Eq. (6b)] and there as the *difference* of radiative cooling rates above and below the inversion. The radiation data cited by Holland and Rasmussen in their Fig. 12 indicates no perceptible difference in radiative cooling rates across the inversion so that radiation has been neglected in Eq. (6b).

The observed large-scale vertical velocity profile is computed from the synoptic-scale convergence measurements and turns out to be approximately linear, $-\bar{w}=4.3\times10^{-6}z-0.07$ (cgs), in the region 200 to 1500 m (see Holland and Rasmussen 1973, Fig. 4). The ambient potential temperature stratification is about $\gamma_\theta=3.5\times10^{-5}$ K cm^{-1} near 600 m height and the specific humidity stratification is $\gamma_q=-3.5\times10^{-8}$ cm^{-1} giving a virtual potential temperature stratification of about $\gamma_v=2.9\times10^{-5}$K cm^{-1}. Inserting these values into Eq. (7) yields $h=1500$ m and $\Delta\theta_v=0.72$K. The observed height of the mixed layer, however, was about 500 m, and the observed cloud base was about 600 m.

To see that this discrepancy between observation and prediction is real, let us invert the question and ask what subsiding velocity it would take to keep the mixed layer at the observed height of 500 m. Eq. (7) now yields $-\bar{w}=1.86$ cm sec^{-1} and $\Delta\theta_v=0.24$K. Thus in order to agree with the observed mixed layer height, the mixed layer grows upward with respect to the ambient air at a rate of 1.86 cm sec^{-1} which itself must be descending with velocity -1.86 cm sec^{-1} in order that the mixed layer be stationary with respect to the ground.

Another anomaly may be seen by examining the moisture budget. The moisture flux through the top of the mixed layer due to turbulence is about 90% of the moisture flux evaporated at the surface (Holland and Rasmussen, 1973, Fig. 6). We can write (2) separately for the temperature and moisture:

$$\frac{dh}{dt}-\bar{w}=-\frac{(\overline{\theta_v'w'})_i}{\Delta\theta_v}=-\frac{(\overline{\theta'w'})_i}{\Delta\theta}=-\frac{(\overline{q'w'})_i}{\Delta q},$$

so that

$$(\overline{q'w'})_i=\frac{\Delta q}{\Delta\theta_v}(\overline{\theta_v'w'})_i=-0.2\frac{\Delta q}{\Delta\theta_v}(\overline{\theta_v'w'})_0,$$

since the virtual flux at the mixed layer inversion is related to the surface flux by the factor -0.2. Now since the moisture flux through the top is 90% of the evaporation at the surface, $0.2\delta\theta\Delta q=-0.9\ \Delta\theta_v$ and using the equilibrium value $\Delta\theta_v=0.24$K, we find $\delta\theta\Delta q=-1.08$K or $\Delta q=-5.8$ gm kg^{-1}. Since $\Delta\theta_v=\Delta\theta+\delta\theta\Delta q$, we find $\Delta\theta=1.32$K. The value for Δq is far larger than that observed through the mixed layer inversion.

The conclusion we must come to is that there is a mechanism which provides an additional source of subsiding air at the top of the mixed layer, and that also carries a large flux of water vapor upward through the inversion. Both of these tasks are of course accomplished by the trade cumulus clouds themselves. We will see in the next section that this has interesting implications for some recent cumulus parameterization schemes.

4. Implication for cumulus parameterization schemes

Yanai *et al.* (1973) were the first to use large-scale data to derive the properties of a cloud ensemble. However, they averaged over both disturbed and nondisturbed events and derived a mean cloud ensemble, essentially that ensemble of clouds that maintains the mean state of the tropical atmosphere. This approach is useful for determining long-term budgets but very little information can be extracted about the mixed layer which responds differently to different synoptic situations. Nitta (1974) has recently performed a similar analysis on BOMEX data and has separately considered undisturbed, transition and disturbed regimes.

Nitta's undisturbed regime was 22–26 June 1969, the same period of time we have considered in the previous section. During this time the trade inversion was prominent at about 2000 m and the mixed layer inversion was taken to be fixed at 600 m. Nitta's derived cloud mass distribution is shown in his Fig. 7, and, as we expect, shallow trade cumulus dominate, with hardly any clouds breaking through the trade inversion. It is indeed just these non-precipitating and shallow trade cumulus that perform the crucial role of moisturizing the lower tropical troposphere and maintaining the mean conditional instability.

But Nitta finds that the cloud mass flux at the top of the mixed layer corresponds to a velocity of 3.3 cm sec^{-1}. The compensating subsidence at this height is therefore 3.3 cm sec^{-1} plus the mean subsidence to give a total between-the-cloud subsidence of 3.5 cm sec^{-1}. It is a basic assumption of the theory that the clouds cover a small fractional area, so that on the average, the mixed layer would see a subsiding velocity of 3.5 cm sec^{-1}. As seen in the previous section, the location of the mixed layer is approximately consistent with the mixed layer seeing a downward velocity of 1.86 cm sec^{-1}. *There is no way a mixed layer could exist near 500 m in the presence of a downward subsiding velocity of 3.5 cm sec^{-1}.*

The situation may be summed up as follows. Either Nitta's analysis is in error by a factor of 2 or the subsidence compensating the cloud mass flux is not fully felt at the top of the mixed layer. In the first case the boundary layer considerations presented here can be used as a valuable constraint on cloud inversions of the Yanai and Nitta type. If the Nitta results are valid, statements can be made about the magnitude of the compensating subsidence below cloud base as will be done in the next paragraph. It should be emphasized that the resolution of the inconsistency presented here is important and should be examined observationally, especially in GATE.

A picture of the undisturbed BOMEX mixed layer consistent with both the considerations of Section 3 and Nitta's results can be formulated as follows. The mixed layer is close enough to the zero buoyancy level that random fluctuations (rising plumes, local temperature anomalies, waves on the mixed layer inversion, etc.) overcome the stabilizing effects of the mixed layer inversion and push up to the level of zero buoyancy, whereupon they rise as clouds, having since become visible at the lifting condensation level. Depending on their lateral dimensions, which in general are small, the clouds rise under their own power and detrain somewhere between the level of zero buoyancy and the trade inversion. The clouds set up a local circulation around themselves in such a way that although the upward mass flux into the clouds is 3.5 cm $\sec^{-1}$ averaged over the BOMEX region at 600 m, only 1.71 cm $\sec^{-1}$ subsides below cloud base to push down the top of the mixed layer at 500 m. The mixed layer sees the sum of this local cloud circulation and the mean subsidence of 0.15 cm $\sec^{-1}$ and is balanced by a total subsiding flow of 1.86 cm $\sec^{-1}$.

Thus, the answer to the old question of whether or not the trade cumulus have their roots[2] in the mixed layer seems to be the discouraging one that trade cumulus *partially* have their roots in the mixed layer—about 50% in the BOMEX region. The trade cumulus themselves play an important role in the dynamics of the mixed layer, and the simple mixed layer theory of Deardorff, Betts, and Tennekes is not by itself adequate.

The situation becomes more interesting in Nitta's analysis of the disturbed BOMEX regime which arrived two days later, 28–29 June. Nitta takes the mixed layer height to be 400 m during this period. The mean vertical velocity at mixed layer height has turned around to upward at about 0.1 cm $\sec^{-1}$ (Nitta and Esbensen, 1974, Fig. 11). The derived vertical cloud mass flux corresponds to a velocity of 3.9 cm $\sec^{-1}$ and this flux seems to be into deep non-entraining cumulonimbus. Deep cumulus *are* commonly believed to have their roots in the mixed layer (Arakawa and Schubert, 1974), so that the total between-the-cloud mass flux at the top of the mixed layer would be -3.8 cm $\sec^{-1}$. Unless the surface flux of virtual temperature increases by a factor of 2, this between-the-cloud subsiding velocity is *not* consistent with Nitta's assumption of a mixed layer stationary at 400 m. Possible alternatives are that the deep clouds, too, do not have their roots completely in the mixed layer, that the fluxes did increase by a factor of 2, and that the mixed layer was *not* present at 400 m having been pushed much lower.

[2] Clouds will be said to have roots where they have mass flux and therefore compensating subsidence.

5. Discussion and conclusions

There are several sources of error in the above analysis which prevent us from making more than qualitative statements. First, the data itself, while suited to large-scale budget analysis, are not particularly appropriate for locating the mixed layer height as a function of time. Radiosonde data are averaged over 10-mb layers so that the statement that the mixed layer height was 500 m may itself be in error by 20% (to this accuracy, we have used 1 mb = 10 m throughout). Data on fluxes, vertical velocities, etc., are probably no better than 20–30%.

Second, it is not clear how well the constant 0.2 in Eq. (5) is known. Deardorff (1974) found the value 0.18. Lenschow (1973), however, on the basis of boundary layer measurements over the Great Lakes, concluded that this number must be less than 0.08. The relation between height and vertical velocity given by the equilibrium condition (7) can be rewritten, for an arbitrary value α of this numerical constant, as $L=(1+\alpha)(\overline{\theta_v'w'})_0[\gamma_v(-\bar{w})]^{-1}$ which is not very sensitive to the precise value of α as long as it is small compared to unity. The discontinuity at the mixed layer inversion in equilibrium, however, is given by $\Delta\theta_v=\alpha(1+\alpha)^{-1}\gamma_v h$ and *is* sensitive to the precise value of α.

Third, no account has been taken of radiative cooling due to clouds and, as mentioned, due to the temperature and moisture discontinuities at the mixed layer inversion. Since such cooling affects $\Delta\theta_v$ directly, its importance cannot be discounted.

Finally, although significant diurnal variations in both temperature and mean vertical velocities are indicated by the data (Nitta and Esbensen, 1974), we have neglected them and instead considered the mean undisturbed mixed layer in equilibrium with the mean vertical subsidence. For the fluxes used, the adjustment time was on the order of a few hours so this should be a reasonable assumption. In the future, however, it would be most valuable to consider these diurnal variations. In particular, the amount of trade cumulus should be greater the closer the top of the mixed layer is to the level of zero buoyancy. Nitta and Esbensen's data seem to show counteracting effects in that morning mean vertical velocities seem to be downward implying a depressed mixed layer, while the temperature of the air is cooler implying increased sensible heat fluxes and therefore a raised mixed layer. The effects are subtle and the full equations (6) should be used to study the diurnal variations. Detailed height measurements, say by acoustic radar, would also be most valuable.

The balance between mixed layer growth and downward subsidence can no longer obtain when the lowlevel vertical velocity turns upward, as it did during the period 28–29 June. In this case the mixed layer grows faster than the mean upward velocity, eventually reaching the level of zero buoyancy, and active convection must commence. In the absence of compensating trade cumulus subsidence, the mixed layer would grow freely at a rate (Tennekes, 1973)

$$\frac{dh}{dt}=\frac{14}{5\gamma_v}(\overline{\theta_v'w'})_0\approx 1 \text{ cm sec}^{-1},$$

so that it would go from 500 to 800 m, say, in about 12 hr. In the presence of compensating trade cumulus subsidence, it would, of course, take longer. When deep convection sets on, as it did on 28 June, the compensating between-the-cloud subsidence is enough to push the mixed layer downward. Nitta and Esbensen report a disturbed mixed layer height of 400 m. Other observers have, in regions of intense deep convection, noted that a well-defined mixed layer does not exist at all. This would be consistent with the compensating between-the-cloud subsidence being so strong as to push the mixed layer down below the Monin-Obukhov height. In either case, a depressed mixed layer would make it harder for parcels to reach the level of zero buoyancy and this would suppress convection. Clearly a model of a tropical wave interacting with a mixed layer would be of enormous value in understanding tropical convective patterns. Progress is currently being made on such a model and will be reported in due course.

To summarize we have applied turbulent boundary layer theory to the tropical undisturbed mixed layer. We have found that buoyancy-driven boundary layer turbulence does not by itself explain the location of and transports through the mixed layer inversion and that effects of the trade cumulus must be considered as part of the mixed layer theory even though it has been commonly believed that trade cumulus roots are not to be found in the mixed layer. The prediction of the height and transports of the mixed layer depend not only on the intensity of the trade cumulus mass flux, but also on the details of the trade cumulus cloud dynamics and moisture transports. While such considerations do not invalidate diagnostic studies such as those of Yanai *et al.* (1973) and Nitta (1974), they are crucially important for predictive schemes (e.g., Arakawa and Schubert, 1974).

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