The Role of Electrical Forces in Charge Separation by Falling Precipitation in Thunderclouds

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(Manuscript received 19 November 1973, in revised form 23 July 1974)

ABSTRACT

Theoretical expressions for the maximum electric field, the total and leakage currents, and the recovery time of the electric field have been developed for the falling precipitation mechanism of charge separation in thunderclouds. The derivations take into consideration the electrical forces acting on the precipitation and smaller particles. Quantitative dependence of the derived electrical parameters on the precipitation intensity and on the charge density of the precipitation particles has been studied in the case of the ice crystal-hailstone collision mechanism and the drop-splintering mechanism. Certain conditions for optimum efficiency of this charge separation process have been established and the minimum rates of precipitation required to generate different values of electric field have been calculated. It is inferred that precipitation particles of larger sizes will contribute more efficiently than the smaller ones in generating high electric fields. The results suggest lower and upper limits for the average charge density on precipitation particles so that the theoretical results might match the experimental data. Furthermore, it is shown that when the precipitation intensity and the charge density on the precipitation particles are high, the currents due to electrical forces acting on precipitation particles, which hitherto have been ignored, become comparable or even larger than the sum of currents due to conductivity in the thundercloud and due to the point discharge from the ground. It is concluded from a comparison of the results to the available data that falling precipitation may not be the dominant cause in separating charges in thunderclouds with intense electrification.

1. Introduction

It is usually assumed that the separation of electric charges in thunderclouds is caused by falling charged precipitation particles. In all precipitation theories proposed to explain thundercloud electrification, larger precipitation particles acquire negative charge while the corresponding positive charge is given to smaller particles. Subsequently, larger particles, because of their greater terminal velocities, separate in space from the smaller particles, thus generating an electric dipole in the thundercloud with positive charge above and negative below. For most precipitation theories involving electric charge generation in thunderclouds, this mechanism of the separation of positive and negative charges has been claimed by its proponents to be sufficiently strong to build up the high electric fields required for the lightning flash.

It has never been demonstrated beyond doubt that precipitation particles in thunderclouds carry sufficient charge of proper polarity and that their separation from smaller particles is the cause of high electric fields. The adequacy of this charge separation process has generally been accepted, however. It is generally agreed that differential transport of charge by precipitation may cause some electrification. Frequently, however, this has been claimed as the only dominant mechanism separating electric charges in thunderclouds. Such claims have many times been doubted or denied on the basis of careful experimental observations and theoretical considerations of various meteorological and electrical conditions existing in and around the thunderclouds (Vonnegut and Botka, 1959; Vonnegut, 1965; Moore, 1965; Kamra, 1970; Kamra and Vonnegut, 1971; Colgate, 1972). As a result, serious disagreements have existed as to the dominant mechanism of charge separation.

Charged particles in a thundercloud should experience the following forces: (i) a gravitational force, acting downward, (ii) an aerodynamic force, acting opposite to the direction of movement of the particle relative to the surrounding air, and (iii) an electrical force, its direction depending upon the polarity of charge on the particle and the direction of the electric field. Most of the scientists in the past have either ignored the electrical force or considered it to be negligible compared to the other two. If we now consider the vertical component of the electric field, then in the negative electric field of thunderclouds, negatively charged precipitation particles will experience

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an electrical force upward and positively charged smaller particles a downward force. Thus, the effect of the electrical forces will be such as to reduce or, if the electrical forces are high enough, even to stop the further separation of oppositely charged particles. As a result, if we assume that the falling precipitation is the cause of electrification in a thundercloud, the electric field in the cloud should govern its own maximum value and its own rate of growth. This has earlier been pointed out by Gunn (1954) and Schonland (1964). Recently, Kamra (1970, 1971) has shown theoretically that for the induction charging mechanism of charge generation, the electrical forces acting on precipitation and smaller particles are important in determining the maximum electric field and the rate of charge separation in thunderclouds. Furthermore, Kamra and Vonnegut (1971) have shown experimentally that in the high electric fields of thunderclouds, the electrical forces acting on smaller particles can make them move downward with velocities comparable to those of falling precipitation particles.

The objective of this paper is to study the effect of electrical forces acting on both larger and smaller particles on the process of charge separation by the falling precipitation mechanism. Theoretical expressions for the maximum electric field, total and leakage currents inside a thundercloud, and the recovery time of the electric field will be obtained and the effect of various related parameters on these will be discussed. In this paper, in contrast to an earlier study (Kamra, 1970), I will take the electric charge on the larger and the smaller particles as independent of the existing electric field. A further objective of this paper is to discuss some correlations, among charge and size of the precipitation particles and the precipitation intensity, which must be satisfied for a precipitation mechanism to operate with optimum efficiency.

In accordance with the prevalent view among the proponents of precipitation theories of charge generation, that the point discharge and conduction currents of the thundercloud oppose the buildup of charges, it has been presumed that this assumption is true. However, it should be appropriate here to mention that there is another school of thought that subscribes to mechanisms in which these currents support electrification (Vonnegut, 1955). For a detailed discussion of the contribution of various currents to the thundercloud electrification, the reader is referred to a review article by Vonnegut (1963).

In this paper we will follow the convention that the electric field in a thundercloud, with positive charge above and negative below, is negative. Consequently, the charging current which brings positive charge up and negative down will be positive.

2. Calculations

We consider a mature thundercloud in which precipitation has developed and where precipitation particles coexist with smaller particles throughout the cloud with uniform density. Also, we suppose that by some unspecified process, precipitation particles have acquired negative charge and the smaller particles positive, but the cloud as a whole is neutral and there is no electric field. It is also assumed that there is no wind shear or turbulence. Then at time $t=0$, we postulate that precipitation particles start falling with respect to the smaller particles and hence start the separation of positive charge from the negative; the electric field starts growing. Now if we consider a vertical prism of unit cross section through the cloud, in which a net charge Q is being separated with velocity V, then the rate of growth of the electric field E is given by (Gunn, 1954; Sartor, 1961, 1967; Latham and Mason, 1962):

$$\frac{dE}{dt}=4\pi(QV-\lambda E-i), \tag{1}$$

where the first term on the right represents the convection current, the second term the conduction current (λ being the "effective" air conductivity inside the thundercloud), and the third term the leakage current due to point discharge currents from the ground. Following Latham and Mason (1962) we take $\lambda=2\times10^{-4}$ esu, which is about one-third the value of dry-air conductivity at about 3 km, and $i=1.8\times10^{-3}\,E$, so that

$$\frac{dE}{dt}+8\pi\times10^{-3}E=-4\pi\sum^{D}N_DQ_DV_D+4\pi\sum^{d}n_dq_dv_d, \tag{2}$$

where N_D, Q_D and V_D are the concentration, charge and velocity, respectively, of precipitation particles of diameter D and n_d, q_d and v_d are the concentration, charge and velocity of smaller particles of diameter d.

Measurements by Gunn (1949) and McCready and Proudfit (1965) of the electrical charges on precipitation particles in thunderclouds show that the particles are highly charged. The maximum charge on the particles is generally believed to be limited not because of the charging mechanism but because of the electric breakdown at the surface of the drops. Although there is no well-established correlation between the charge and size of the precipitation particles, available data of Gunn and McCready and Proudfit indicate that the average charge $\bar{Q}_D$ on a precipitation particle of weighted mean diameter $\bar{D}$ should be

$$\bar{Q}_D=K\bar{D}^2, \tag{3}$$

where K is a constant.

The maximum value of K should be determined from the critical breakdown potential at the surface of the drop and should thus be a complex function of the electric field, surface charge density of the particle, nature and distortion of the particle's surface, atmospheric pressure, etc. However, the maximum value of K can be roughly estimated. Available measurements of the size and charge of the particles, together with the considerations of corona limitations from the surface of the drops, suggest that $K<8.3$ (McCready and Proudfit, 1965) if $\bar{D}$ is in centimeters and $\bar{Q}_D$ in esu.

The average velocities of precipitation and smaller particles, $\bar{V}_D$ and $\bar{v}_d$ respectively, moving in an electric field, can be deduced from their equations of motion, i.e., from

$$\bar{V}_D=\frac{\bar{m}_D g+\bar{Q}_D E}{3\pi\eta\bar{D}\left(\frac{C_D \mathrm{Re}}{24}\right)_{\bar{D}}}, \tag{4}$$

$$\bar{v}_d=\frac{\bar{m}_d g+\bar{q}_d E}{3\pi\eta\bar{d}\left(\frac{C_D \mathrm{Re}}{24}\right)_{\bar{d}}}, \tag{5}$$

where $\bar{m}_D$ and $\bar{m}_d$ are the respective masses of precipitation and smaller particles, C_D is the drag coefficient, Re the Reynold's number, η the viscosity of the air, and g the acceleration due to gravity. Since $(C_D\mathrm{Re}/24)$ is the ratio of the drag force to the viscous force acting on the particles, we can write for precipitation particles

$$\left(\frac{C_D \mathrm{Re}}{24}\right)_{\bar{D}}=\frac{\frac{1}{6}\pi\bar{D}^3\bar{\rho}g}{3\pi\eta\bar{D}\bar{V}_D}, \tag{6}$$

where $\bar{\rho}$ is the mean density of the particles.

If p is the precipitation intensity, then

$$\sum^{D} N_D=\frac{p}{\frac{1}{6}\pi\bar{D}^3\bar{\rho}\bar{V}_D}. \tag{7}$$

Now substituting (3)–(7) in (2), we have

$$\frac{dE}{dt}=-\alpha-\beta E, \tag{8}$$

where

$$\alpha=\left[\frac{24pK}{\bar{D}\bar{\rho}}-\frac{2\pi\bar{d}^2\bar{\rho}g\bar{n}_d\bar{q}_d}{9\eta\left(\frac{C_D \mathrm{Re}}{24}\right)_{\bar{d}}}\right], \tag{9}$$

$$\beta=\left[\frac{144pK^2}{\pi\bar{D}^2\bar{\rho}^2 g}-\frac{4\bar{n}_d\bar{q}_d^2}{3\bar{d}\eta\left(\frac{C_D \mathrm{Re}}{24}\right)_{\bar{d}}}+8\pi\times10^{-3}\right], \tag{10}$$

where $\bar{n}_d$ is the average concentration of smaller particles each having an average charge $\bar{q}_d$.

Now assuming that $E=0$ at $t=0$, (8) can be solved as

$$E=-\frac{\alpha}{\beta}(1-e^{-\beta t}). \tag{11}$$

Thus the maximum field that can be generated by any precipitation mechanism is

$$E_m=-\frac{\alpha}{\beta}. \tag{12}$$

The rate of growth of the electric field should depend upon the rate of precipitation growth and the rate of charging of precipitation particles. Because we have assumed these two factors to have already reached their maximum values at $t=0$, the growth of the electric field as calculated from (11) will not be realistic for the initial electrical state of thunderclouds. However, once precipitation has developed and the particles have acquired charges, as is generally the case after the first lightning discharge has occurred, (11) can reasonably well represent the growth of the electric field. Also, the maximum electric field as calculated from (12) will not be affected by these two assumptions.

To illustrate the phenomenon quantitatively, we shall consider, as examples, two precipitation mechanisms which are generally claimed to be dominant in generating high electric fields in thunderclouds. Other precipitation mechanisms may be treated similarly and the results are not expected to be much different.

a. *Ice crystal-hailstone collision mechanism*

From their laboratory experiments, Reynolds (1954) and Reynolds *et al.* (1957) provide a typical value of 5×10^{-4} esu of charge transfer in a single ice crystal-hailstone collision. The hailstone becomes negatively charged and the ice crystal positive. Latham and Mason (1961), however, give a much smaller value of 5×10^{-9} esu for the charge transfer in a single ice crystal-hailstone collision. Latham and Miller (1965) have tried to reconcile the large difference between these two values of charge transfer on the basis of the differences in the impact velocity and surface roughness of the hailstone. Here we shall take the value of Reynolds *et al.* (1957) for the charge transfer per collision because, as also suggested by Latham and Miller, it is closer to natural conditions inside a thundercloud. Thus, for the average charge on an ice crystal, we have $\bar{q}_d=5\times10^{-4}$ esu. Also, again following Latham and Mason (1961), we take the average values of the diameter and concentration of ice crystals as $\bar{d}=100$ μm and $\bar{n}_d=0.1$ cm^{-3}.

Ice crystals have very low terminal velocity but can attain a maximum velocity of 8 m s^{-1} (an average value for the terminal velocity of hailstones) under the action of electrical forces. Thus by taking 4 m s^{-1}

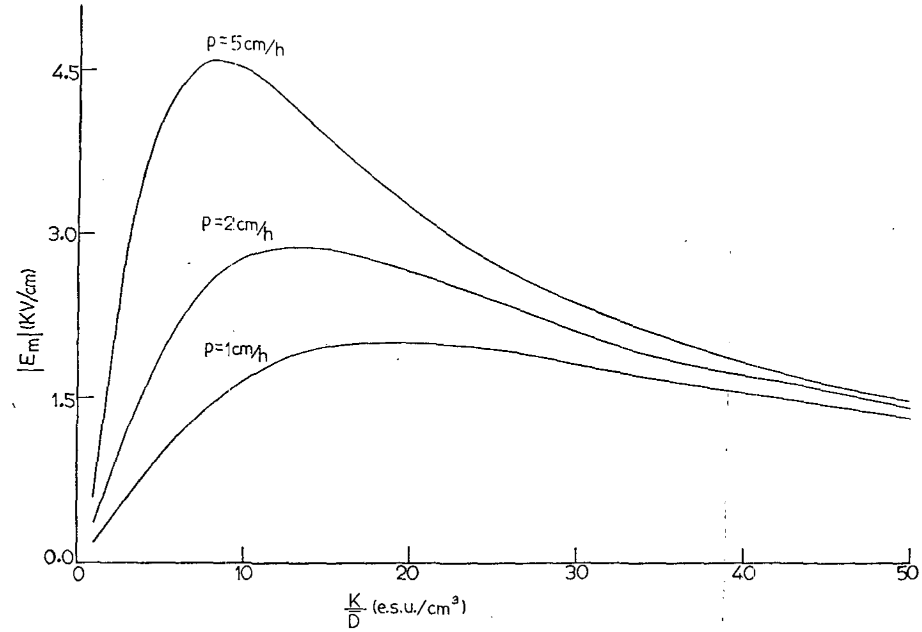

FIG. 1. Variation of the maximum electric field $|E_m|$ that can be generated as a function of $K/\bar{D}$ at fixed precipitation rates for the ice crystal-hailstone collision mechanism.

as the average fall velocity for ice crystals we can calculate the value of Re and then determine the corresponding value of C_D from a curve by Wieselsberger (Prandtl and Tietjens, 1957); thus we have $(C_D\mathrm{Re}/24)_{\bar{d}}=2.6$ for ice crystals. Since C_D and Re vary inversely, the value of $(C_D\mathrm{Re}/24)_{\bar{d}}$ will change little over quite a wide range of ice crystal terminal velocity; we shall see later that any such change does not significantly affect our results.

Now taking $\bar{\rho}=0.5$ g cm^{-3}, $\eta=1.8\times10^{-4}$ poise, $g=980$ cm s^{-2}, and other values from (9) and (10), and expressing p in centimeters per hour, we have

$$\alpha=13.3\times10^{-3}p\frac{K}{\bar{D}}-3.6\times10^{-3}, \tag{13}$$

$$\beta=5.2\times10^{-5}p\frac{K^2}{\bar{D}^2}+18\times10^{-3}. \tag{14}$$

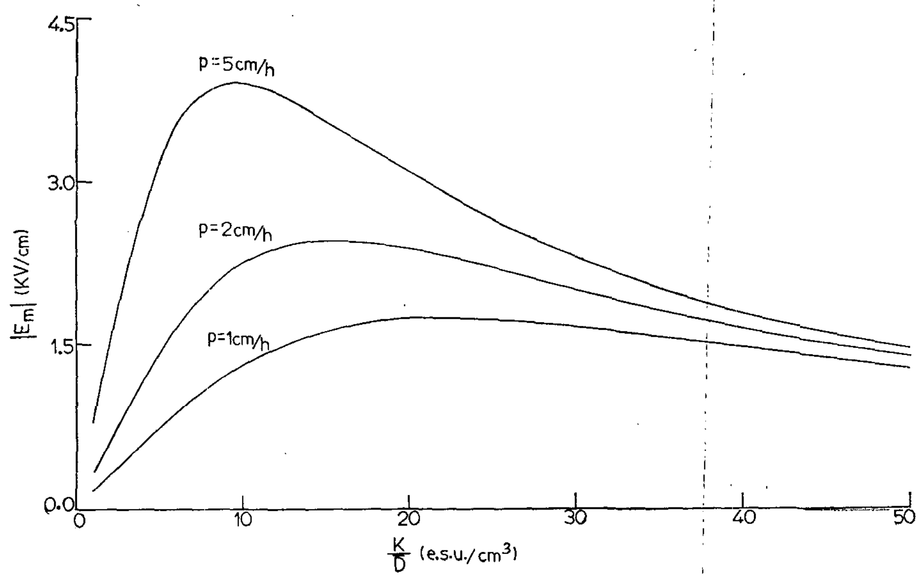

FIG. 2. As for Fig. 1 except for the drop-splintering mechanism.

Thus

$$E_m = -\frac{13.3\times10^{-3}p\dfrac{K}{\bar{D}} - 3.6\times10^{-3}}{5.2\times10^{-5}p\dfrac{K^2}{\bar{D}^2} + 18\times10^{-3}}. \quad (15)$$

b. The drop-splintering mechanism

Latham and Mason (1961) observed experimentally that a charge of 4×10^{-6} esu separated each time a supercooled droplet in the size range of 30–80 μm collided with a hailstone. While the negative charge is given to the hailstone, positive charge resides on the ice splinters which are ejected from the hailstone during riming. They estimated that on the average 12 splinters each of mean diameter, $\bar{d}=20$ μm, were ejected in a single droplet-hailstone collision. If we assume that charge is distributed equally on the ice splinters, then the average charge on an ice splinter will be $\bar{q}_d=3.3\times10^{-7}$ esu. Browning and Mason (1963) have shown that the concentration of ice splinters produced by this action should be one-tenth of the concentration of supercooled droplets of diameter 25 μm between -20 and -30°C. Thus, if we take the concentration of supercooled droplets as 10 cm^{-3}, the average concentration of splinters should be

$$\bar{n}_d = 1 \text{ cm}^{-3}.$$

As in the previous section the value of $(C_D\text{Re}/24)_{\bar{d}}$ for ice splinters is calculated as $(C_D\text{Re}/24)_{\bar{d}}=1.6$.

Putting these values along with others as in the previous section, and expressing p in the same units, we find that

$$\alpha = 13.3\times10^{-3}p\frac{K}{\bar{D}} - 1.50\times10^{-6}, \quad (16)$$

$$\beta = 5.2\times10^{-5}p\frac{K^2}{\bar{D}^2} + 25.1\times10^{-3}. \quad (17)$$

Thus

$$E_m = -\frac{13.3\times10^{-3}p\dfrac{K}{\bar{D}} - 1.50\times10^{-6}}{5.2\times10^{-5}p\dfrac{K^2}{\bar{D}^2} + 25.1\times10^{-3}}. \quad (18)$$

To understand the significance of the following results and the discussion it should be pointed out that K is a measure of the surface charge density of the precipitation particles, and $K/\bar{D}$ a measure of their specific charge or charge per unit mass. Quantitatively we can express these relations as

$K=\pi\times\{$surface charge density of the precipitation particles$\}$,

$\dfrac{K}{\bar{D}}=\frac{1}{6}\pi\rho\times\{$specific charge of the precipitation particles$\}$.

3. Results

a. Maximum electric field

Figs. 1 and 2 show the variation with $K/\bar{D}$ of the maximum electric field that can be generated [as calculated from (15) and (18)] for different rates of precipitation, by the ice crystal-hailstone collision mechanism and by the drop-splintering mechanism. The initial increase of $|E_m|$ with $K/\bar{D}$ is obviously due to increased transport of charge. After reaching a maximum however, $|E_m|$ starts decreasing as $K/\bar{D}$ increases because electrical forces acting on the charged particles become so strong that they start opposing further separation of the particles. Some of the highly charged particles in such electric fields may start moving in an opposite direction so as to decrease the existing electric field. It is evident from Figs. 1 and 2 that even with moderate precipitation intensities, such as 2 cm h^{-1}, electric fields $\geqslant 3$ kV cm^{-1}, whatever the value of $K/\bar{D}$, cannot be generated by these two charge generation mechanisms. This contradicts Latham's conclusion (Latham, 1971) that a precipitation rate of only about 1.5 cm h^{-1} is sufficient to meet the essential requirements of a tenable theory of charge generation in thunderclouds.

From Eqs. (15) and (18) we can calculate E_m for different precipitation rates taking $K/\bar{D}$ as constant.

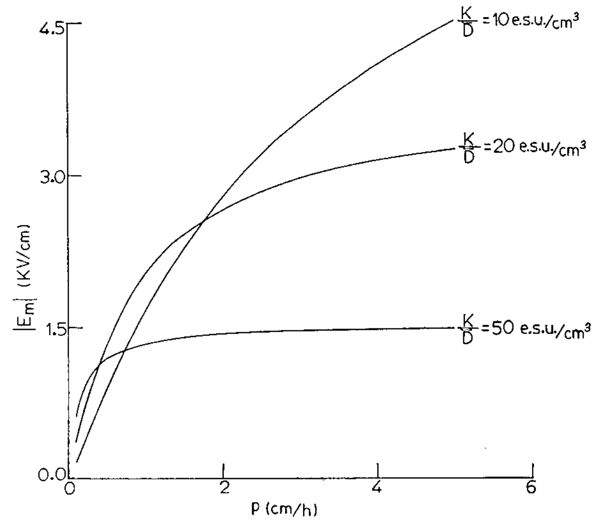

FIG. 3. Variation of the maximum electric field $|E_m|$ that can be generated as a function of precipitation rates at fixed values of $K/\bar{D}$ for the ice crystal-hailstone collision mechanism.

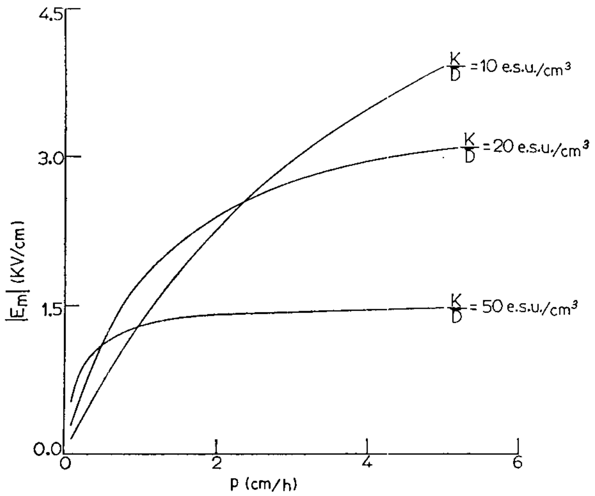

FIG. 4. As in Fig. 3 except for the drop-splintering mechanism.

Figs. 3 and 4 show that $|E_m|$ initially increases with precipitation quite rapidly but then more slowly. Thus, for a specific value of $K/\bar{D}$, $|E_m|$ has a certain maximum value when it almost stops increasing with precipitation intensity. These corresponding maximum values of $K/\bar{D}$ and E_m for some finite precipitation intensity can be calculated by writing (15) and (18) in the form

$$p=\frac{3.6\times10^{-3}-18\times10^{-3}E_m}{5.2\times10^{-5}\dfrac{K^2}{\bar{D}^2}E_m+13.3\times10^{-3}\dfrac{K}{\bar{D}}}, \qquad (19)$$

$$p=\frac{1.50\times10^{-6}-25.1\times10^{-3}E_m}{5.2\times10^{-5}\dfrac{K^2}{\bar{D}^2}E_m+13.3\times10^{-3}\dfrac{K}{\bar{D}}}. \qquad (20)$$

Now since E_m is negative and $K/\bar{D}$ positive, the numerators of (19) and (20) will always be positive. For the denominator to be positive and thus for p to be finite, it is essential that

$$\frac{K}{\bar{D}}E_m<-256.$$

This relation gives us the corresponding maximum values of $K/\bar{D}$ and $|E_m|$ for any finite value of p. For example, if an electric field of 3 kV cm^{-1} is to be generated, then, whatever the precipitation intensity, $K/\bar{D}<25.6$, i.e., the specific charge on a hailstone should be less than 99 esu gm^{-1}.

The minimum rate of precipitation required to generate $|E_m|\geqslant3$ kV cm^{-1} (generally believed to be the breakdown value of the electric field in thunderstorms) for any specific value of $K/\bar{D}$ can be calculated by putting $E_m=-10$ esu in (19) and (20). Thus,

$$\left.\begin{aligned} p&=\frac{183.6}{13.3\dfrac{K}{\bar{D}}-0.52\dfrac{K^2}{\bar{D}^2}}\\ p&=\frac{251}{13.3\dfrac{K}{\bar{D}}-0.52\dfrac{K^2}{\bar{D}^2}}\end{aligned}\right\}.$$

Fig. 5 illustrates the variation of precipitation intensity with $K/\bar{D}$ required to generate $|E_m|=3$ kV cm^{-1} by the two precipitation charge generation mechanisms. Once again it is seen that with precipitation rates $\lesssim2$ cm h^{-1}, whatever the value of $K/\bar{D}$ it is not possible to generate electric fields $\geqslant3$ kV cm^{-1}. Furthermore, if $K/\bar{D}$ lies between 5 and 21 esu cm^{-3}, then the precipitation intensity required to generate $|E_m|=3$ kV cm^{-1} is more or less constant. However, if $K/\bar{D}$ is <5 or >21 esu cm^{-3}, then the required precipitation intensity increases very rapidly. These lower and upper limits of $K/\bar{D}$ will be discussed further in later sections.

b. Total electric current inside thunderclouds

From (8) we can calculate the total vertical current flowing inside a thundercloud. If I_T is the vertical

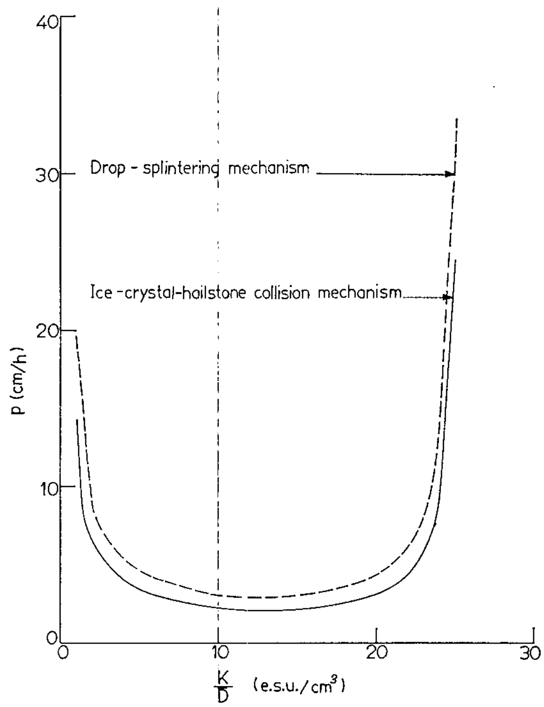

FIG. 5. Minimum precipitation rates p_m as a function of $K/\bar{D}$ required to generate an electric field of 3 kV cm^{-1}.

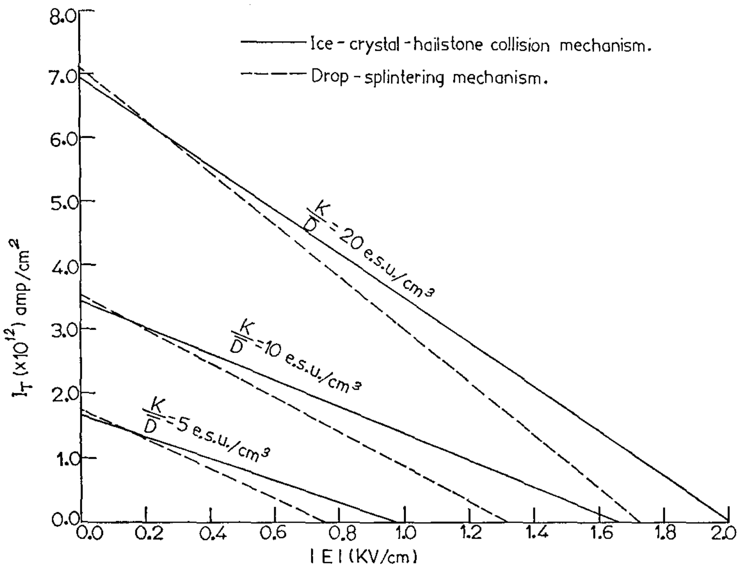

FIG. 6. Variation of the total vertical current density I_T with electric field for various values of $K/\bar{D}$ and for $p=1$ cm h^{-1} for the two charge separation mechanisms.

current density, then

$$I_T=\frac{1}{4\pi}(\alpha+\beta E), \qquad (21)$$

where the first term on the right is the total charging current and the second term the total discharging current which is proportional to the electric field. The variation of the total electric current with electric field for precipitation rates of 1 and 5 cm h^{-1} is shown in Figs. 6 and 7, respectively. It is evident that as the electric field increases, the total current density decreases. At some high value of electric field which is the maximum that can be generated with these particular values of $K/\bar{D}$ and precipitation intensity, the current density becomes zero. At this point the discharging current exactly balances out the charging current. Thus, if vertical electric fields higher than this critical value exist in a thundercloud, then they cannot be due to the falling of charged precipitation particles. It should be noted from Figs. 6 and 7 that the total current density decreases with electric field

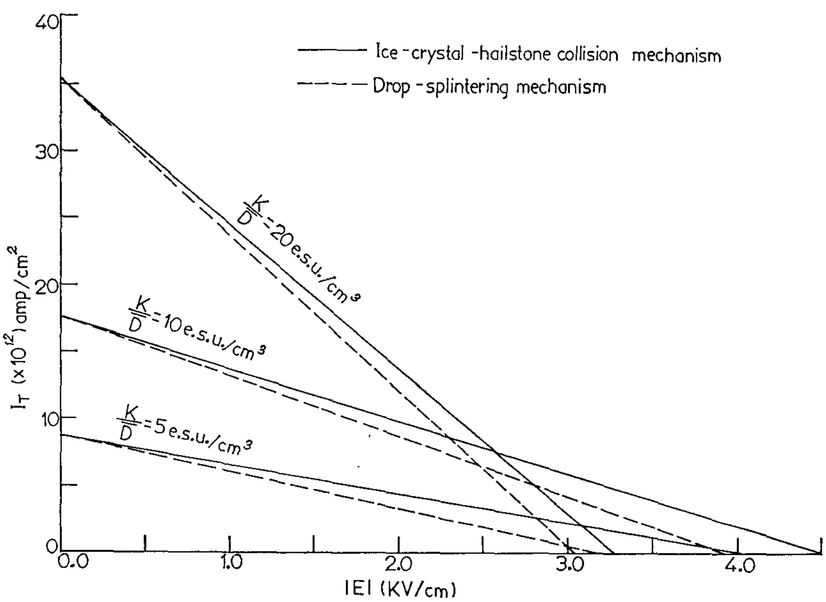

FIG. 7. As in Fig. 6 except for $p=5$ cm h^{-1}.

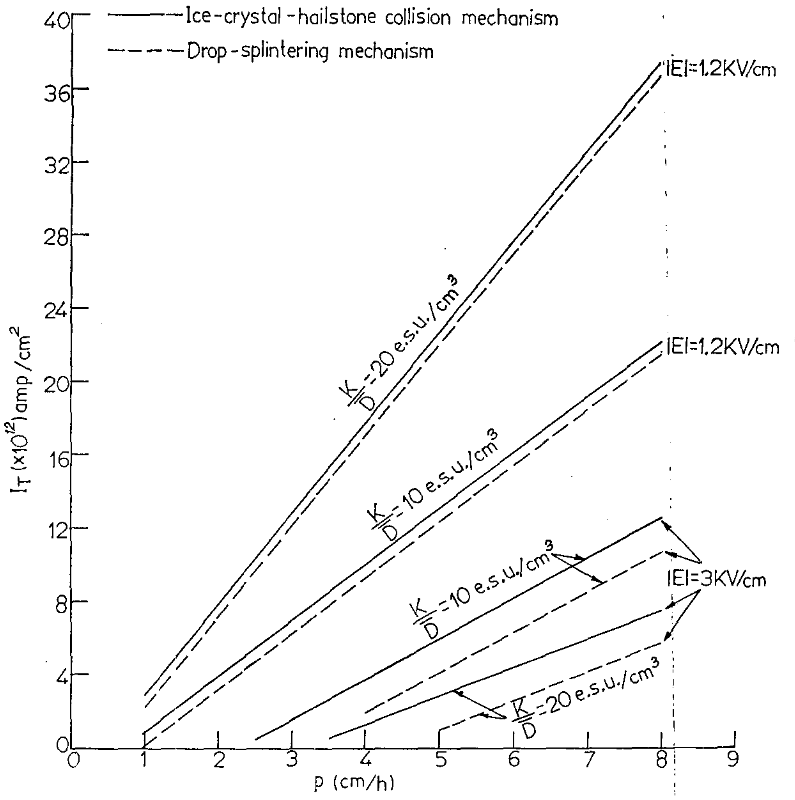

FIG. 8. Variation of the total vertical current density I_T with precipitation intensity for various values of electric field and $K/\bar{D}$, for the two charge separation mechanisms.

more rapidly for higher values of $K/\bar{D}$ and precipitation intensity than for lower ones.

Fig. 8 shows the variation of the total vertical current density with precipitation intensity. The current density increases as precipitation intensity increases. As $K/\bar{D}$ increases, however, the current density first increases and then after reaching a maximum, starts decreasing (Fig. 9). The value of $K/\bar{D}$ at which the maximum current flows is the same for all precipitation intensities if the electric field remains constant. This value of $K/\bar{D}$ can be obtained if we differentiate I_T in (21) with respect to $K/\bar{D}$ (taking E to be constant) and equate it to zero. Hence, for I_T to be maximum,

$$E(K/\bar{D})\approx 128. \tag{22}$$

Thus, if the current, i.e., the rate of growth of the electric field, is to remain maximum, $K/\bar{D}$ should follow the changes in the electric field according to (22). Since such changes in $K/\bar{D}$ are not likely to occur, the total current should change with the electric field.

Ignoring the effect of electrical forces on cloud particles, Latham (1971) concludes that a precipitation intensity of about 1.5 cm h^{-1} is sufficient to achieve a current of 10^{-11} A cm^{-2}. According to our calculations, the electrical forces acting on the particles have such a significant effect on the current that much higher precipitation rates are required to achieve such currents. For example, when $|E|=3$ kV cm^{-1} and $K/\bar{D}=10$ esu cm^{-3}, a precipitation intensity of 6.8 cm h^{-1} (for the ice crystal-hailstone collision mechanism) is required to obtain a current of 10^{-11} A cm^{-2}. However, in lower electric fields, lower precipitation rates can provide such currents (Fig. 8).

A vertical current density of the order of 10^{-12} A cm^{-2} is generally believed to flow in a thundercloud. This is based on the experiments conducted in clear air above thunderclouds by Gish and Wait (1950) who suggested a net current of 0.5–1 A per thundercloud. If we consider a thundercloud with a circular horizontal base area of 2 km radius then according to our calculations the maximum current that can flow in it when $K/\bar{D}=10$ esu cm^{-3} and $p=5$ cm h^{-1} is 2.2 A. However, as the electric field increases to 3 kV cm^{-1}, the total current decreases to only 0.7 A. The total current will become zero when the electric field increases to 4.5 kV cm^{-1}. Anytime during the growth of the electric field, however, a lightning discharge can occur and reduce the electric field to some lower value. The total vertical current will consequently increase to some higher value.

It should be pointed out here that because of our assumption that the precipitation intensity and the charge on the precipitation particles have already

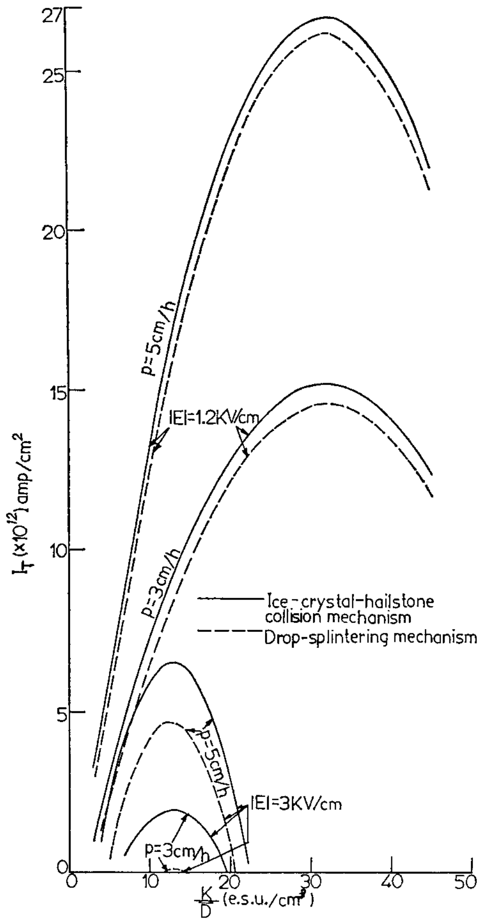

FIG. 9. Variation of the total vertical current density I_T with $K/\bar{D}$ for various values of electric field and precipitation intensity, for the two charge separation mechanisms.

reached their maximum values, Eq. (21) may not exactly represent the flow of currents in the early stages of the thundercloud. However, in the mature thundercloud stage when precipitation has developed and the particles become charged, (21) should reasonably well predict the variations in the total vertical current. For example, since the recovery currents after the lightning discharge should follow (21), the recovery time of the electric field should be determined from this relation.

c. Leakage currents

In the past the leakage currents considered in calculating the growth of electric field in thunderclouds were only those due to conductivity of the thundercloud and to point discharge currents from the ground. In these calculations, we have also included those leakage currents which result from the electrical forces acting on precipitation and smaller particles. In (21), the second term on the right-hand side represents the total leakage current density, say I_L, so that

$$I_L=\frac{E}{4\pi}\left[\frac{144pK^2}{\pi\bar{D}^2\bar{\rho}^2g}-\frac{4\bar{n}_d\bar{q}_d^2}{3\bar{d}\eta(C_D\,\mathrm{Re}/24)_{\bar{d}}}+8\pi\times10^{-3}\right], \quad (23)$$

where the first term in the brackets is the leakage current due to electrical forces acting on precipitation particles, the second term the leakage current due to electrical forces acting on smaller particles, and the last term the sum of the conduction and point discharge currents. In our calculations we have prescribed a definite value for the second term for each charge generating mechanism. The first term varies with p and $K/\bar{D}$. From the ratio X of the first and the third terms we have an idea of the relative magnitudes of the leakage currents due to precipitation particles and those due to conduction and point discharge; thus

$$X=2.07\times10^{-3}p\frac{K^2}{\bar{D}^2}. \quad (24)$$

Fig. 10 shows the variation of X with $K/\bar{D}$. It is evident that at higher values of p and $K/\bar{D}$, when $X>1$, the leakage current due to electrical forces acting on precipitation particles is many times the leakage currents due to conduction and point discharge and thus they should not be neglected.

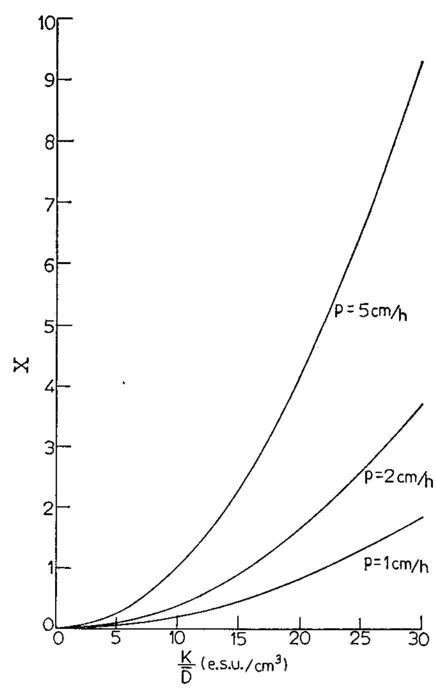

FIG. 10. The ratio X of the leakage currents due to electrical forces acting on the precipitation particles to the sum of the leakage currents due to conductivity of the thundercloud and to point discharge from the ground as a function of $K/\bar{D}$ for various precipitation rates.

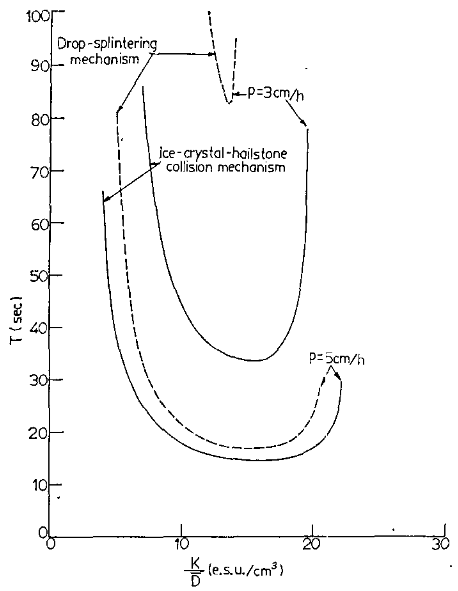

FIG. 11. Recovery time T of the electric field as a function of $K/\bar{D}$ for various precipitation rates.

d. Recovery time of the electric field

As the electric field increases and reaches the electric breakdown value of cloudy air, a lightning discharge occurs. Consequently the electric field drops to a lower value. However, as the charge separation process continues, the electric field starts growing again to its original value. If the electric field just before the discharge is E_1 and after the discharge E_2, then the time the electric field takes to grow again from E_2 to E_1 is called the recovery time of the electric field. Now if we assume that the precipitation intensity and the charges on the cloud particles remain constant before and after the discharge and through the growth period of the electric field, then the recovery time T from (11) is given by $(E_m > E_1 > E_2)$

$$T = \frac{1}{\beta} \ln\left(\frac{E_2 - E_m}{E_1 - E_m}\right). \tag{25}$$

If we assume that the lightning discharge occurs at 3 kV cm^{-1} and after the discharge the electric field drops to 1.2 kV cm^{-1}, i.e., if $E_1 = -10$ esu and $E_2 = -4$ esu, then

$$T = \frac{1}{\beta} \ln\left(\frac{4 + E_m}{10 + E_m}\right). \tag{26}$$

Fig. 11 shows the variation of the recovery time with $K/\bar{D}$ for precipitation intensities of 3 and 5 cm h^{-1}. A prominant feature of Fig. 10 is that as $K/\bar{D}$ increases, the recovery time first decreases and then after reaching a minimum again starts increasing very rapidly. This is apparently because of the fact that when $K/\bar{D}$ is very large the electrical forces acting on the precipitation particles retard the rate of separation of larger and the smaller particles. Another important characteristic of the rate of recovery of the electric field is that the smaller the difference between E_1 and E_m, the larger will be the recovery time. That is the reason why in Fig. 11 the recovery times for the drop-splintering mechanism are significantly higher than those for the ice crystal-hailstone collision mechanism when the precipitation intensity is 3 cm h^{-1}; the difference is not so pronounced when the precipitation intensity is 5 cm h^{-1}.

The variation in recovery time with precipitation intensity for the two charge generating mechanisms is shown in Fig. 12. It is obvious that as the precipitation intensity increases from its minimum value required to generate the breakdown-electric field, the recovery time decreases sharply. At very high values of precipitation intensity, the difference in recovery times for different values of $K/\bar{D}$ is comparatively much smaller.

From the preceding calculations it can be concluded that the charge density on the cloud particles and the precipitation intensity both determine the frequency of the lightning flashes in thunderclouds. Lightning flashes at the rate of 10–20 s^{-1} have been observed in giant electrical storms by Vonnegut and Moore (1959). Such high frequencies of flashes are hard to explain with recovery times such as shown in Figs. 11 and 12 unless we consider that the storm consists of very large number of different electrically active centers, each having high precipitation intensities.

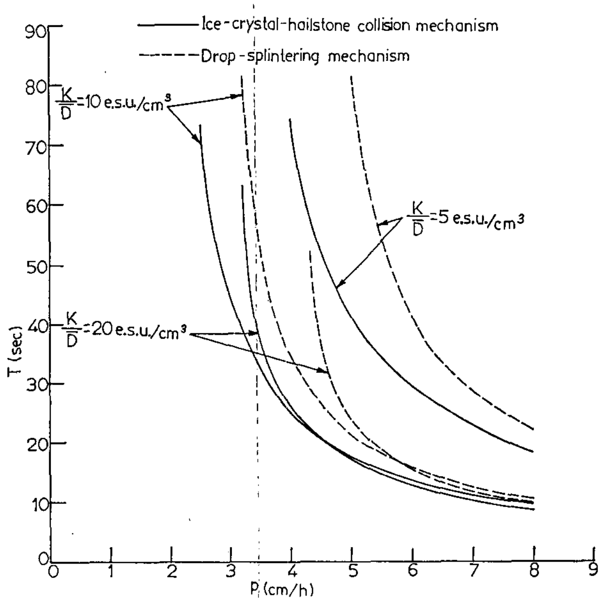

FIG. 12. Recovery time T of the electric field as a function of precipitation intensity for various values of $K/\bar{D}$.

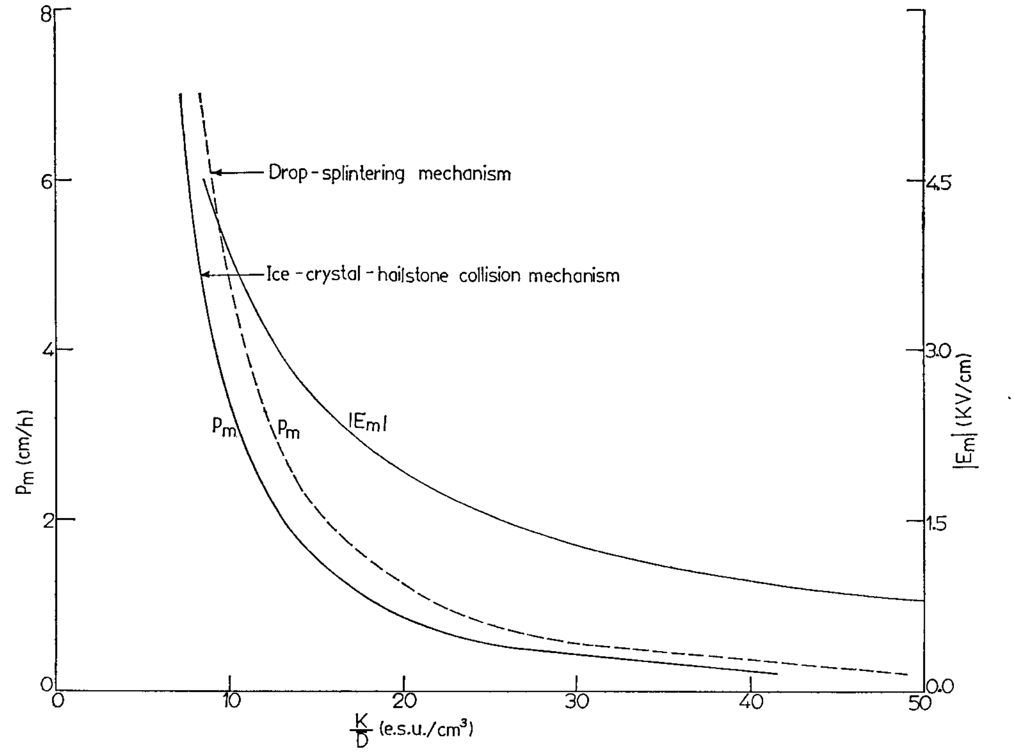

FIG. 13. Minimum precipitation intensity p_m and maximum electric field $|E_m|$ as functions of $K/\bar{D}$ for the two charge generating mechanisms to operate with optimum efficiency.

It is being increasingly recognized in recent years that the formation of screening layers (Grenet, 1947; Gunn, 1955; Vonnegut *et al.*, 1962; Eden and Vonnegut, 1965; Brown *et al.*, 1971; Hoppel and Phillips, 1971; Klett, 1972) on the surfaces of the cloud obscures the effects of electrical processes occurring inside the cloud from being observed outside it. Illingworth (1971) has been able to show that the recovery curve is not primarily the result of electrical processes going on within the cloud, but is in fact the response of the electrical environment of the cloud. Therefore, the comparison of recovery times as calculated for the charge generating processes occurring within the clouds, to those inferred from the recovery curves of the electric field, will be of little relevance. However, the primary objective of presenting the above calculations is only to stress the necessity of including the effect of electrical forces acting on cloud particles in calculating the recovery times for various charge generating mechanisms operating within the clouds.

4. Conditions for optimum efficiency of a charge separation process

A charge generating mechanism will be considered to operate with optimum efficiency when the maximum value of E_m (say E_M) will be generated with the minimum precipitation intensity p_m. For a fixed precipitation intensity this can be achieved (as can be seen from Fig. 1 and 2) only with a certain specific value of $K/\bar{D}$. The conditions for optimum efficiency can be obtained by writing (12) in the form

$$E_m=\frac{A(K/\bar{D})-B}{C(K/\bar{D})^2+D}, \tag{27}$$

where

$$\left.\begin{aligned} A&=24p/\bar{\rho}, & B&=\frac{2\pi\bar{d}^2\bar{\rho}g\bar{n}_d\bar{q}\bar{V}_d}{9\eta(C_D\,\mathrm{Re}/24)_{\bar{d}}} \\ C&=144p/(\pi\bar{\rho}^2g), & D&=\frac{4\bar{n}_d\bar{q}_d^2}{3\bar{d}\eta(C_D\,\mathrm{Re}/24)_{\bar{d}}}-8\pi\times10^{-3} \end{aligned}\right\}.$$

Now differentiating (27) with respect to $K/\bar{D}$ and equating the differential to zero, we obtain

$$(K/\bar{D})^2-2\frac{B}{A}(K/\bar{D})-\frac{D}{C}=0,$$

so that

$$\frac{K}{\bar{D}}=\frac{B}{A}\pm\left(\frac{B^2}{A^2}+\frac{D}{C}\right)^{\frac{1}{2}}.$$

Now from (13), (14), (16) and (17) we see that

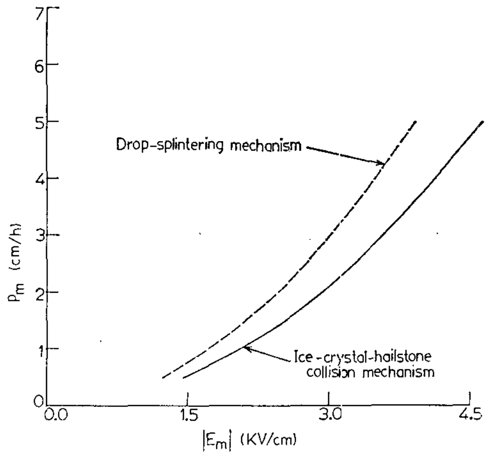

FIG. 14. Minimum precipitation intensities p_m required to generate the maximum values of the electric field if the charge generating mechanisms operate with optimum efficiency.

$B/A \ll D/C$, so that (taking positive sign only)

$$\frac{K}{\bar{D}} \approx \left(\frac{D}{C}\right)^{\frac{1}{2}}.$$

Now, when $E_m = E_M$, we can call the corresponding precipitation intensity as the minimum required to generate E_M, i.e., $p = p_m$. Thus, for the ice crystal-hailstone collision mechanism,

$$p_m \frac{K^2}{\bar{D}^2} \approx 346, \quad (28)$$

and for the drop-splintering mechanism,

$$p_m \frac{K^2}{\bar{D}^2} \approx 483. \quad (29)$$

Now using (28) and (29) to eliminate p_m from (27), we have for both charge generating mechanisms, the following relationship between E_M and $K/\bar{D}$:

$$E_M(K/\bar{D}) \approx 128. \quad (30)$$

After eliminating $K/\bar{D}$ from (28), (29) and (30) we have, for the ice crystal-hailstone collision mechanism,

$$E_M \approx 6.88\sqrt{p_m}, \quad (31)$$

and for drop-splintering mechanism,

$$E_M \approx 5.81\sqrt{p_m}. \quad (32)$$

Eqs. (28)–(32) represent the conditions for the charge generating mechanisms to operate with optimum efficiency. The variations of E_M and p_m with $K/\bar{D}$ [given in (28), (29) and (30)] for the two charge generating mechanisms are shown in Fig. 13. If $|E_M| = 3$ kV cm^{-1}, then from Fig. 13 the corresponding value of $K/\bar{D}$ is 12.8 (i.e., the specific charge on a hailstone is 49.5 esu g^{-1}) and for $K/\bar{D} = 12.8$, the minimum value of precipitation intensity is 2.11 cm h^{-1} for the ice crystal-hailstone collision mechanism and 2.95 cm h^{-1} for the drop-splintering mechanism. This can also be seen directly from Fig. 14 which illustrates (31) and (32).

Eqs. (30) and (22) are identical and provide the conditions for producing the maximum electric field and the maximum vertical current respectively. Thus, the magnitude of the electric field and its rate of growth will both be maximum for the same values of $K/\bar{D}$. In other words, if the values of p, $K/\bar{D}$ and E are such that the total vertical current is the maximum, then that value of E will be the maximum (E_M) that can be generated for the same values of p and $K/\bar{D}$.

Fig. 15 shows the variation of $\bar{D}$ with K at different values of p_m and E_M for the two mechanisms to operate with optimum efficiency. It is evident that as the surface charge density of the larger particles increases, higher values of $\bar{D}$ will contribute more efficiently to generate high electric fields. The variation of $\bar{D}$ with K for different values of E_M also represents, as it follows from (22), the condition for the maximum current to flow. These results do not seem to be in agreement with Latham's conclusion (Latham, 1971) that particles 0.1–0.2 cm in diameter contribute most to thunderstorm electrification. The disagreement is obviously due to the fact, as also pointed out by Vonnegut (1972), that in his calculations Latham ignores the electrical forces acting on

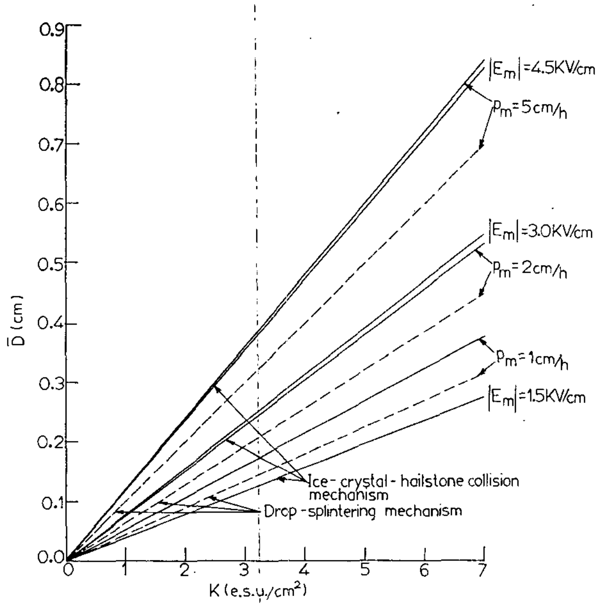

FIG. 15. Weighted mean diameter $\bar{D}$ of the precipitation particles as a function of K ($\pi\times$surface charge density of the precipitation particles) for various values of the maximum electric fields and the minimum precipitation rates if the charge generating mechanisms operate with optimum efficiency.

the particles. Our calculations show that to have a quantitative estimate of the most effective mean diameter, it is essential to know among other things, the specific charge on the precipitation particles. Unfortunately, at present our knowledge of this parameter is very scanty. The measurements of Gunn (1949) and McCready and Proudfit (1965) indicate very high surface densities of charge on some of the precipitation particles. It is pointed out by these observers that the charge on the larger particles are generally limited by the breakdown gradient on their surfaces rather than by the charging mechanism. Actually, McCready and Proudfit have suggested that as a rough estimate, the maximum charge (esu) on a liquid drop of diameter $\bar{D}$ can be represented by $Q=8.3\,\bar{D}^2$ and on a rimed particle by $Q=5.8\,\bar{D}^2$, where $\bar{D}$ is in centimeters. If such high electric charges reside on smaller precipitation particles then the electrical forces acting on them in relatively lower electric fields should be enough to leviate them in free space and thus stop their further separation from the smaller particles. Thus, with the same precipitation intensity and the same charge density on precipitation particles, larger values of $\bar{D}$ will build up higher electric fields than will do the smaller ones.

5. Discussion

We have applied our theory to two charge generating mechanisms which have been often claimed as the dominant ones in generating electric fields in thunderclouds. A similar treatment of the induction charging mechanism has been reported earlier (Kamra, 1970, 1971). These investigations have emphasized the important role played by electrical forces on differential transportation of positive and negative charges. The results can now be safely regarded as the general limitations to generating high electric fields in thunderclouds by any charge generation precipitation mechanism.

A satisfactory theory of charge separation should be capable of producing the high electric fields required to cause the lightning flash. Furthermore, it should ensure the flow of adequate charging currents even during high electric fields. To satisfy these requirements our calculations put some definite restraints on various parameters which are crucial to the separation of charges by falling precipitation. An essential ingredient for these requirements seems to be the presence of a precipitation intensity of at least several centimeters per hour. Other parameters upon which the transportation of charge depends are the specific charge (charge per unit mass) on the cloud particles and the separation velocity of the larger and the smaller particles. The latter, even in the case of large precipitation particles and atmospheric ions, cannot be more than a few meters per second. Since this separation velocity decreases with increasing values of specific charge on precipitation particles, the specific charge, too, cannot be increased beyond a certain limit to achieve high electric fields and high currents. Thus, in the charge separation process by falling precipitation we are essentially faced with upper limits of the electric field and charging current. These upper limits can be derived from our calculations if reliable measurements of various parameters are available.

The difficulty in deciding what mechanism dominates in building up high electric fields arises mainly because of our lack of satisfactory data of the electrical properties of thunderclouds. For example, there are conflicting views at present even about the order of magnitude of the electric field that is necessary to produce the electric breakdown in the thundercloud. Evans (1969) reports electric fields of up to about 500 V cm^{-1} only. On the other hand, Gunn (1948) measured on the belly of an aircraft an electric field of 3.4 kV cm^{-1} just before lightning struck the aircraft. Winn and Moore (1971), firing instrumented rockets in thunderclouds, observed electric fields of up to 3.0 kV cm^{-1}; and in one case (Winn and Moore, 1972) they report a single value in excess of 4.0 kV cm^{-1}. Clark (1971) sent balloonborne field mills enclosed in spherical housings into thunderclouds and repeatedly measured electric fields, at the surface of the sphere, of 13.3 kV cm^{-1} which he points out to be close to the dielectric breakdown value of cloudy air. After corrections for the concentration of the field by the sphere and for the field due to charge on the sphere, the maximum undisturbed ambient field was calculated to be about 2.5 kV cm^{-1}. Because aircraft, balloons and parachutes are liable to distort the electric field by acquiring charges on themselves or by producing point discharge in high electric fields (e.g., see Vonnegut, 1969), it is expected that much larger electric fields than the reported ones might exist in thunderclouds. Reynolds (1954) has suggested that an electric field of 15 kV cm^{-1} might be required to trigger a lightning discharge.

Charge density on precipitation particles is another parameter which must be known for the assessment of the relative efficiencies of various charge generating mechanisms. Available charge distribution data on particles of different sizes are not very reliable because of the unsatisfactory collection techniques used. Therefore, at present, we have only a rough estimate of the average charge density on precipitation particles. The maximum charge on precipitation particles, however, has been pointed out by some investigators to be limited because of the dielectric breakdown on their surfaces. From our calculations we can make some rough estimates of the lower and upper limits of the average charge densities on precipitation particles. If we assume that electric fields of the order of 3 kV cm^{-1} exist in clouds then from the nature of the variations of E_m, I_T and T with $K/\bar{D}$ (Figs. 1, 2, 9 and 11), it seems that $K/\bar{D}$ most likely lies between 5 and 20 esu cm^{-3}, i.e., the average specific charge on the precipitation particles lies between 19 and 77 esu g^{-1}. Gunn's measurements (Gunn, 1949) of drop charges give a rough estimate of 40 esu g^{-1} in a cold front and 85 esu g^{-1} in an active thunderstorm. These values of average specific charges are in reasonable agreement, considering the roughness of the experimental data, with our theoretically predicted values.

Our present ambiguous understanding of the electrical characteristics of thunderclouds, as discussed above, makes it impossible to completely discard or accept some particular theory of charge generation. Thus, a better understanding of such variables as the electric charge densities on the cloud particles and the electric fields and currents in the thunderclouds is needed for a better evaluation of the charge separation process by falling precipitation.

Our calculations specify some quantitative restrictions on various meteorological and electrical parameters of the thundercloud under which this process of charge separation must operate. Evaluating these restrictions on the basis of the available data it seems that although the contribution of the falling charged precipitation particles is an important factor in the electrical budget of the thundercloud, its role as the dominant cause of electrification is doubtful. The maximum value of the vertical electric field that occurs in thunderclouds seems to be a deciding factor whether or not falling charged precipitation is the dominant cause of thundercloud electrification. If the maximum electric field in thunderclouds is only a few hundred volts per centimeter, as reported by Evans (1969), then the electric field should have little or no effect on the differential transportation of positive and negative charges. On the other hand, if electric fields of order of kilovolts per centimeter exist in thunderclouds and the precipitation particles are highly charged, as observed by many investigators, then the separation of charged precipitation particles from smaller ones may be claimed, at most, only as a secondary process of charge separation.

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