# Calculations of Electric Field Growth within a Cloud of Finite Dimensions

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ABSTRACT

A one-dimensional precipitative model of cloud electrification is outlined in which field growth results from the operation of either an inductive or non-inductive mechanism. The cloud is cylindrical, of finite dimensions, and charging is confined to a supercooled zone within which precipitation growth occurs. Account is taken of loss of negative charge arriving at the ground on precipitation and the storage of positive charge carried by the updraft to a level above the charging zone.

The most important conclusion is that previous models of cloud electrification, which have often been extremely elaborate, provide gross overestimates of the rate of field growth because they have assumed a cloud of infinite width. They also predict a "top-hat" distribution of field, which is shown to be quite unrealistic. The present calculations cast serious doubt on the capability of an inductive mechanism, *by itself*, to produce breakdown fields in the available time.

These calculations also indicate that 1) the retardation of field growth due to the effect of electrical forces on particle velocities is negligible; and that 2) the "lower positive charge" can be produced in the bases of clouds, in some circumstances, without having to invoke an additional charging mechanism.

## 1. Introduction

In recent years numerous papers have been published which present calculations of electric field development in thunderclouds, under the action of inductive mechanisms of cloud electrification. Examples are Sartor (1967), Kamra (1970, 1971), Mason (1972), Paluch and Sartor (1973) and Ziv and Levin (1974).

In one sense these treatments are extremely comprehensive, taking into consideration a large number of effects which—if properly incorporated into the calculations—should add significantly to their realism and quantitative accuracy. For instance, one or more of these papers have taken account of: the electrical relaxation time for charge transfer; distributions of sizes of hydrometeors; the stochastic nature of the interaction process; variations with particle size of separation probabilities; the influence of electric forces on the velocities of the hydrometeors; time-variations of precipitation rate; and dissipative effects due to ionic conductivity and point discharge. (We believe that the effect of point discharge has been greatly exaggerated because, in fact, a long time lag will exist between its creation at the ground and its entry into the cloud.) However, all of these calculations have been placed within the framework of an assumed parallel-plate capacitor model of cloud electrification, in which no account is taken of the finite dimensions of the cloud. We believe, for reasons to be developed in the following section, that the unrealistic nature of this assumption results in a gross overestimation of the rate of field growth; and that until such errors are rectified it is impossible to assess properly the efficacy of inductive mechanisms of cloud electrification. It follows that the introduction into field-growth calculations of refinements of the type mentioned above is not worthwhile until an acceptable model of cloud electrification has been constructed. As a first attempt to redress this deficiency we outline a somewhat rudimentary one-dimensional model of field growth, and some of the predictions that emanate from it.

## 2. Field growth calculations

The salient features of the parallel-plate capacitor model of field growth, used by the workers mentioned previously, can be summarized as follows. A "charging zone," of infinite horizontal extent, exists, within which interactions between cloud particles occur. These interactions transfer charge in a systematic fashion such that the larger particles acquire charge of one sign (usually negative) and the smaller particles charge of the opposite sign; the specific charging mechanism under consideration governs the *amount* of charge transfer per interaction, but does not affect the basic model of field growth. Gravitational separation of large and small particles (and the assumed presence of an updraft) causes a negative current to flow to the infinite horizontal electrode which is assumed to lie below the interaction zone and an equal positive current to flow to the corresponding electrode above this zone. The rate of growth of the field $E$ is governed by

$$\frac{\partial E}{\partial t}=(J_p-J_d)/\epsilon_0, \quad (1)$$

where $J_p$ is the charging current, $J_d$ the ionic dissipation current, and $\epsilon_0$ the permittivity of free space. The field is uniform between the electrodes and zero outside of them. Since the plates are infinite the contribution to the field at any point due to a surface density of charge $\sigma$ on one of the electrodes is given by

$$E_\sigma=\sigma/(2\epsilon_0). \quad (2)$$

In reality, of course, the dimensions of a cloud are finite, and the field produced at a point by a charge depends sensitively upon its distance from that point. We consider an electrified cloud in the form of a cylinder of diameter $D$ with its axis vertical. The field $E_\sigma$ produced at an axial point $P$, distant $H$ from a cylindrical disc, by a surface density of charge $\sigma$ on the disc, is

$$E_\sigma=\frac{\sigma}{2\epsilon_0}(1-H/L), \quad (3)$$

where $L=(H^2+D^2/4)^{\frac{1}{2}}$. This expression reduces to that for an infinite disc as $D$ tends to infinity. However, for example, if we take as typical dimensions for a cloud $H=D$, we note that $E_\sigma$ is reduced to $(\sigma/2\epsilon_0)[1-(4/5)^{\frac{1}{2}}]\approx 0.11\sigma/(2\epsilon_0)$. We see that the infinite-disc model, in addition to providing no information on the *structure* of the field, gives rise to a gross overestimate of the field strength and the rate of growth of the field; this error is particularly serious when a feedback-inductive process is being considered.

In an attempt to provide a more realistic description of cloud electrification the following model has been constructed. It is described here in terms of a mechanism involving collisions between small hail pellets and ice crystals, although it is equally applicable to any precipitative mechanism.

Ice particles are formed at a particular level within a rising airstream of velocity $U$ within the cloud. They grow as they ascend in the updraft and eventually, at a level $z=0$, they achieve a mass $m_0$ and radius $r_0$ at which their terminal velocity $V_0=U$. They then begin to fall toward the ground, in the direction of increasing $z$; typically, $r_0=500$ $\mu$m if $U=2$ m s$^{-1}$, and $r_0=1000$ $\mu$m if $U=4$ m s$^{-1}$. These particles are now growing principally by riming, and are termed ice pellets. It is assumed that at the level $z=0$ the pellets start to become charged by making rebounding collisions with ice crystals. As the pellets grow they fall faster and encounter ice crystals at an increasing rate. The charging mechanism can be either inductive or non-inductive. In the former case, of course, the charge transfer per collision, $q_c$, depends upon both the strength of the electric field at that point—which is variable in both time $t$ and $z$—and the charge $Q$ already residing on the pellet. Full account is taken of both these effects in calculating $q_c$, as outlined below.

We envisage, therefore, a precipitation shaft which starts at $z=0$ and grows, as time progresses, in the direction of increasing $z$ (downward), the largest pellets being always at the base of the shaft. Eventually, at some level $x_c$, the bottom of the shaft passes below the base of the charging zone (at 0°C or some chosen lower temperature) and pellets below this zone no longer engage in charging events, but simply fall toward the ground carrying the charges that they possessed at $z=z_c$. When pellets reach the ground their charges are assumed to be neutralized, and they therefore cease to contribute to the electric field. The flux of precipitation through any level $z$ is assumed to be independent of time—once the shaft has reached this level. The flux of charge carried on precipitation through any level $z$ is independent of time on a non-inductive model but changes with time on an inductive model, since $E$ and therefore $q_c$ is varying with time at all levels.

The ice crystals, which interact with the pellets, thereby separating charge, are assumed to have negligible fall velocities. Consequently they move upward through the cloud with a velocity $U$, interacting, as time progresses, with pellets at higher and higher levels. They engage in charging events at all levels between $z_c$ and $z=0$. Thereafter they rise to some level $z_a$ at a chosen distance above $z=0$ where they and their charges are stored; thereby representing, in some crude sense, a positively charged ice crystal "anvil." To date, the ice crystal concentration has been assumed to be independent of height within the charging zone, and the flux of crystals into this zone has been taken to be constant.

If we assume that the ice crystals have negligible sizes and fallspeeds then the rate at which pellets of radius $R$, velocity $V$ and concentration $N$ interact with them is $\pi R^2 nNV$ per unit volume, where $n$ is the concentration of crystals. If a fraction $f$ of these interactions result in charge transfer, of magnitude $q_c$, then the charge per unit volume residing on ice crystals varies as

$$\frac{d\rho_c}{dt}=\pi R^2 nNVfq_c, \quad (4)$$

and the charge on a pellet may be expressed by

$$\frac{dQ}{dt}=-\pi R^2 nVfq_c. \quad (5)$$

For a non-inductive process $q_c$ is constant, but for an

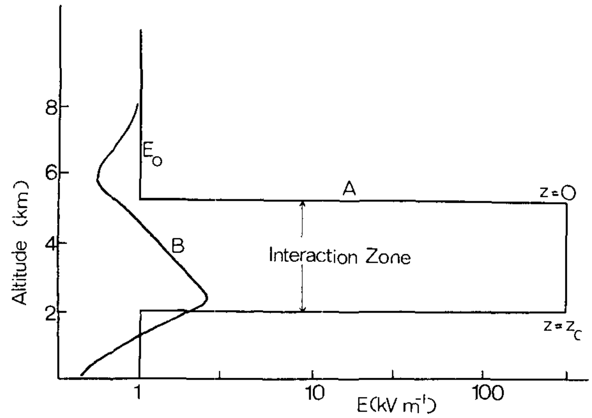


FIG. 1. Field profiles obtained from an inductive mechanism after 20 min of growth from an initial field $E_0=1$ kV m$^{-1}$ for the infinite disc, "top-hat" model used by other workers (A) and for the present model with $D=800$ m (B). Other parameters are the same for both models, the ice crystal sizes and concentration are those used by Mason (1972), and the hail pellet flux is equivalent to a precipitation rate of 36 mm h$^{-1}$. The pellet radius grows from 0.5 to 3 mm while falling through the interaction zone, the first pellets leaving the zone after 15 min. $U=2$ m s$^{-1}$.

inductive process it is of the form

$$q_c=\left(4\pi\epsilon_0\gamma_1 E\cos\theta+\frac{\gamma_2 Q}{R^2}\right)r^2, \tag{6}$$

where $r$ is the radius of the ice crystal, assumed spherical, $\theta$ the co-latitude of the position of interaction of the ice crystal on the pellet, and $\gamma_1$ and $\gamma_2$ are geometric factors tabulated by Latham and Mason (1962). The probability of ice crystals rebounding from the underside surface of a pellet is assumed to be independent of $\theta$.

The evolution and distribution of field has been calculated using a small computer. Both the rising air mass, containing ice crystals, and the falling precipitation are split up into appropriately sized discs. The charges separated in a small time interval $\delta t$ at all levels due to interactions between pellets and crystals are calculated. This gives rise to a new charge distribution from which the new field distribution at all levels between the ground and the anvil is calculated using Eq. (3). The charged particles have now moved to a different position; the crystals having moved upward through $U\delta t$ and the pellets downward through $(V-U)\delta t$, where $V$ is the terminal velocity appropriate to their radius. This computational cycle is then repeated throughout the period of interest. After each time step the total net charge over all the discs is calculated, to check that it is not significantly non-zero. The image charges in the earth are neglected, as their contribution to the field within clouds of finite dimensions is extremely small.

The calculations show that the field distribution is far from the "top-hat" function implicitly assumed by workers who have based their field-growth calculations on the infinite-disc model. A typical example of a field distribution produced by an inductive mechanism of electrification is shown in Fig. 1; it relates to a situation in which the maximum field (on the "infinite disc" model) is approaching breakdown values. We see that the field varies appreciably both within and outside of the charging zone, the largest value ($E_{max}$) occurring near the bottom of this zone, as a consequence of the large interaction rates exhibited by the pellets at this level. The profile varies with time also, of course, and in some circumstances, with an inductive mechanism, a field reversal can occur within the interaction zone, thus causing a reversal in the sign of $q_c$ in this region. Perhaps the most striking fact revealed by Fig. 1, however, is that the maximum field predicted ($E_{max}$) is very much less than that ($E_\infty$) which—on an infinite disc model after the same time and with similar charging parameters—is predicted to exist *throughout* the charging zone. This reduction occurs because the finite diameter results in a lower field than with the infinite disc, and this lower field results in less charge being separated. Thus the feedback process becomes less efficient. When a non-inductive process is considered the field profiles and growth rates are somewhat different, of course, than for the inductive case. In this case, with no feedback, the reduction is the simple geometrical factor. However, the salient features remain: the field has a maximum value $E_{max}$ at just above $z_c$ which is much less than that predicted on the infinite-disc model.

There appears to be no doubt that the introduction of the finite geometry of a cloud into calculations of its electrical development reduces very severely the rate of field growth. A further conclusion when an inductive mechanism is operating is that a lower positive charge, reported to exist in the bases of some thunderstorms, may be produced. This will occur when the negative charge center, which forms the lower part of the main dipole within the cloud, lies within the interaction zone. In this case the field is reversed in the lowest parts of the interaction zone, and interactions between hydrometeors will tend to confer positive charge on the falling pellets. Finally, we mention that in a cloud of finite dimensions the influence of the electric field on the motion of the charged particles will constitute a much less serious obstacle to the final stage of field development than has been predicted by several workers, using the infinite disc model. The reason is that the regions of highest field do not coincide with those of the largest hydrometeor charges or charge-to-mass ratios. It seems much more likely that the onset of lightning is the ultimate field-limiting mechanism, rather than leakage currents or the levitation of hydrometeors.

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