Penetrative Convection Due to a Field of Thermals

M. J. MANTON[1]

Department of Mathematics, Monash University, Melbourne, Australia

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ABSTRACT

A model of a field of thermals is presented to describe the observed behavior of high-Rayleigh-number convection. It is assumed that the response time of thermals within the convection layer is much less than the time scale over which the bulk properties of the layer vary. Hence the depth of the layer does not affect the mechanics of the thermals. The interaction between the thermals is taken into account implicitly by imposing similarity conditions, and it is found that the thermals grow by entrainment at a rate much less than that of an isolated thermal. Thermals reaching the top of the convection layer tend to penetrate into the stable fluid above, causing the layer depth to increase and producing a net downward heat flux into the layer. This process is modeled so that the overall behavior of a convection layer is predicted in terms of parameters associated with the generation of thermals at the base of the layer.

1. Introduction

Laboratory experiments on high-Rayleigh-number convection between two horizontal surfaces indicate that the relationship between the heat flux and the temperature difference across the convection layer is independent of the layer depth; that is, the Nusselt number is proportional to the Rayleigh number to the one-third power (Spiegel 1971). Moreover, the temperature gradient of the body of the convection layer is negligible. These results are found at Rayleigh numbers $\gtrsim 10^6$. Convection in the atmosphere occurs typically over a depth of 1 km and is supported by a potential temperature difference of about 1 K. This corresponds to a Rayleigh number of 10^{17}. Observations in the lower atmosphere (e.g., Grant, 1965; Warner and Telford, 1967) show that above the superadiabatic layer near the ground, where the motion is dominated by mechanical turbulence, there is a region which is statically neutrally stable (or very slightly stable) and in which heat is transferred upward by the bulk motion of either continuous plumes or discrete thermals. The fraction f of the area at any height occupied by convecting elements is approximately constant, and Warner and Telford find that any increase with height in the horizontal dimension of the elements is small. All these observations suggest perhaps that the layer depth is not an important parameter in the determination of the local behavior of the convection process. This assumption is used in the present work to develop a model of high-Rayleigh-number convection.

It would seem that the observed convection can be modeled by a field of continuous plumes each of which extends over the whole layer and whose lateral spread is constrained by the presence of neighboring plumes. Indeed, such models yield some features which are consistent with the observations (Telford, 1970; Manton, 1974). On the other hand, laboratory observations of convection from a heated plate show that the breakdown of the thermal diffusion layer adjacent to the plate leads to the generation of individual thermals (Sparrow *et al.*, 1970). Glider pilots believe that atmospheric convection also is sustained by discrete elements (Yates, 1953). Although the lateral diffusion of a plume could be suppressed by anisotropic turbulence, a thermal (buoyant vortex) always entrains fluid in the neighborhood of its rear stagnation point which is present in a frame moving with the thermal (Woodward, 1959). An isolated thermal grows approximately linearly with height (Turner, 1969). It would therefore appear that a field of thermals would be marked by a significant increase with height in f and in the scale of each thermal, and so it would be incompatible with the observations of Warner and Telford. However, each element in a field of thermals, in which the thermals occupy a significant fraction of the volume, is not expected to act as an isolated thermal. The interaction between thermals is accounted for implicitly in the present model by imposing the observed condition that f is constant and by assuming the similarity condition that the mean potential density gradient at a given height is related to the density deficit of the thermals at that height.

The convection layer in the atmosphere is invariably capped by an inversion which rises slowly as the warmer air above is entrained. The development of the inversion due to the penetration of thermals from the convection

[1] Present affiliation: CSIRO, Division of Cloud Physics, Sydney, Australia.

layer into the stable region is modeled also. Thus a composite model of high-Rayleigh-number penetrative convection is presented.

2. Model of the convection layer

Sparrow *et al.* (1970) find that the uniform heating of a flat plate leads to the generation of sequences of thermals from more or less regularly spaced sources at the edge of the thermal diffusion layer; thermals arise periodically from each source. In the atmosphere, the diffusion layer corresponds to the mechanically well-mixed layer whose depth is of the order of the Obukhov length. We parameterize the generation process by considering the field produced by sources uniformly distributed with density N per unit area over the horizontal surface $z=0$ at the top of the diffusion layer. Each source generates thermals at a rate of $1/T$ per unit time. The mean heat flux at $z=0$ into the convection layer of depth H is $C_p\bar{\theta}_0 Q$, where C_p is the specific heat at constant pressure of the fluid and $\bar{\theta}_0$ the mean potential temperature. Thus $-Q$ is the mean potential mass flux into the convection layer from the diffusion layer near the ground.

The thermals at any given level in the convection layer are characterized by a potential density ρ, a volume v, and a vertical velocity w. These scales are assumed to be sufficient to describe the bulk properties of the thermals. Because the convection elements in the atmosphere are apparently quite elongated in the vertical direction, it might seem unreasonable to define a characteristic volume for developing thermals at a given level. However, we define the local volume in terms of the mean volume flux of thermals; that is

$$vN/T=fw, \tag{2.1}$$

where f is the fraction of the horizontal plane occupied by thermals. The environment through which the thermals rise is characterized by the velocity w_e and potential density ρ_e. Thus the mean potential mass flux balance at a height z above the diffusion layer is given by

$$f\rho w+(1-f)\rho_e w_e+\int_0^z \frac{\partial\bar{\rho}}{\partial t}dz+Q=0, \tag{2.2}$$

where $\bar{\rho}=f\rho+(1-f)\rho_e$ is the mean potential density at that level.

The equations for the conservation of momentum and mass for a thermal may be written as

$$\frac{d}{dt}(\rho v w)=(\rho_e-\rho)vg+\rho_e w_e\frac{dv}{dt}, \tag{2.3}$$

$$\frac{d}{dt}(\rho v)=\rho_e\frac{dv}{dt}, \tag{2.4}$$

where t is time and g the gravitational acceleration. All the dependent variables must be considered as potential quantities. This is necessary for the last terms on the right-hand sides of (2.3) and (2.4) to assert correctly that any increase in (potential) volume of a thermal corresponds to entrainment. At the base of the convection layer ($z=0$) we take

$$w=W \quad\text{and}\quad \rho=\bar{\rho}(1-\epsilon). \tag{2.5}$$

The initial temperature excess of thermals in the atmosphere is typically 1 K which corresponds to ϵ being of order 1/300. The initial updraft velocity W is found to be of order 1 m s^{-1} (Warner and Telford, 1967).

Warner and Telford plot the results of 136 observations of the area fraction f between the ground and a height of 2.5 km. The value of f lies between 0.45 and 0.5 for 127 of these observations, although extreme values of 0.33 and 0.6 are found. Less extensive measurements by Grant (1965) show f to be approximately constant with an average value of 0.45 over the first 600 m. It appears, therefore, that the thermals in high-Rayleigh-number convection interact with their environment in order to maintain a constant area fraction; thus, we impose the condition

$$f=\text{constant}. \tag{2.6}$$

(For numerical purposes we shall take f to be equal to $\frac{1}{2}$.) The physical process by which f remains constant in a field of thermals which must entrain as they rise is seen from (2.1). Each thermal accelerates as it grows in size so that the distance between successive thermals from a given source increases with height. Although the scale of each thermal encountered in a horizontal path through the field increases with height, the expected number of thermals encountered per unit area (which is proportional to $v^{\frac{1}{3}}N/wT$) decreases. We note that a decrease in the encounter rate of thermals does not necessarily imply that thermals are coalescing.

Eq. (2.6) is effectively an entrainment law for each thermal. It is seen from (2.1) that $v\propto w=dz/dt$. This may be compared with the behavior of an isolated thermal for which $v^{\frac{1}{3}}\propto z$ (Turner, 1969): the rate of increase in length scale is proportional to the velocity. In the former case, which implicitly accounts for the interaction between thermals, the rate of entrainment is proportional to the acceleration.

It is seen from (2.3), (2.5) and (2.6) that a characteristic length scale for the motion of a thermal is given by

$$D=(2-f)W^2/\epsilon g. \tag{2.7}$$

This is proportional to the distance through which a thermal with the initial buoyancy $\epsilon\bar{\rho}_0 g$ must accelerate without entraining any ambient fluid in order to obtain the initial velocity W. Thus we expect D to be a measure of the depth of the diffusion layer at the base of the convection layer, i.e., the Obukhov length in the atmosphere. For $W\approx 1$ m s^{-1} and $\epsilon\approx 1/300$, D is about 45 m.

The observation that mean properties do not vary greatly in the convection layer suggests that the behavior of the convection elements is determined by local conditions which respond very quickly to any overall change in the system. The local response time of a thermal within the convection layer is D/W, while the overall response time of the layer is H/W. Therefore, provided that the layer depth is large compared with the depth of the diffusion layer (i.e., $H/D\gg 1$), the behavior of a thermal is determined primarily by local conditions and so the total derivatives (d/dt) in (2.3) and (2.4) may be approximated by local advection terms $(w\partial/\partial z)$. By using (2.1) and (2.6), these equations now become

$$\left.\begin{aligned}[(\rho+\rho_e)w-\rho_e w_e]\frac{\partial w}{\partial z}&=(\rho_e-\rho)g\\ w\frac{\partial\rho}{\partial z}&=(\rho_e-\rho)\frac{\partial w}{\partial z}\end{aligned}\right\}. \tag{2.8}$$

We now introduce normalized variables such that

$$w=uW,\quad \rho=\bar{\rho}(1-\epsilon s),\quad z=xD, \tag{2.9}$$

where from (2.5)

$$u=1 \quad\text{and}\quad s=1 \quad\text{at}\quad x=0. \tag{2.10}$$

Since the normalized buoyancy excess ϵ is very small, it is used as an ordering parameter. It is clear from (2.2) and (2.5) that $Q\sim\epsilon\bar{\rho}W$ and $\partial\bar{\rho}/\partial t\sim -Q/H$. Thus by neglecting terms of order ϵ, (2.2) reduces to

$$w_e/W=-uf/(1-f); \tag{2.11}$$

that is, volume is conserved to the zeroth order in ϵ. The potential density of the ambient fluid is given by

$$\rho_e=\bar{\rho}[1+\epsilon sf/(1-f)]. \tag{2.12}$$

Putting (2.9)–(2.12) into (2.8), we find to the leading order in ϵ that the equations for the conservation of momentum and mass of a thermal reduce to

$$\left.\begin{aligned}u\frac{\partial u}{\partial x}&=s\\ \frac{\partial s}{\partial x}&=-(1-f)^{-1}(s/u)\frac{\partial u}{\partial x}+(\epsilon\bar{\rho})^{-1}\frac{\partial\bar{\rho}}{\partial x}\end{aligned}\right\}. \tag{2.13}$$

In the absence of any mean potential density gradient, the system (2.13) yields the similarity solution

$$u=[1+(3-2f)x/(1-f)]^{(1-f)/(3-2f)},$$
$$s=u^{-1/(1-f)},$$
$$\bar{\rho}=\bar{\rho}_0(t),$$

where $\bar{\rho}_0$ is the potential density at the base of the convection layer. If the thermals occupy a negligible fraction of the region (i.e., $f=0$), we obtain the well-known one-third power law for u and s in "free convection" (Turner, 1973). But when $f=\frac{1}{2}$, as is observed in the convection layer (Warner and Telford, 1967), a thermal accelerates such that its velocity is proportional to the one-fourth power of the height. Although such an increase is small, it is not consistent with the observations of Farmer (1975) who reports that the thermals produced in an ice-covered lake propagate at a constant speed over most of a deep convection layer. This implies that the layer must be stably stratified. To be consistent with the neglect of $\partial\bar{\rho}/\partial t$ in (2.8), $\partial\bar{\rho}/\partial z$ must be independent of H: it must depend upon the local parameters. If we assume further that $\partial\bar{\rho}/\partial z$ does not depend upon the local dynamics (characterized by w and g) but only upon the magnitude of the local density fluctuations $(\bar{\rho}-\rho)$, then it follows that

$$\frac{\partial\bar{\rho}}{\partial x}=-\epsilon as\bar{\rho}, \tag{2.14}$$

where a is a constant of order unity.

Solving (2.13)–(2.14) subject to (2.10), we find that

$$u=\{1+[(3-2f)/(1-f)a](1-e^{-ax})\}^{(1-f)/(3-2f)}, \tag{2.15}$$

$$s=e^{-ax}u^{-1/(1-f)}, \tag{2.16}$$

$$\bar{\rho}=\bar{\rho}_0(t)\exp[-\tfrac{1}{2}\epsilon a(u^2-1)]. \tag{2.17}$$

Eq. (2.17) shows that $\partial\bar{\rho}/\partial t$ is constant (to the first order in ϵ) over the whole layer. Thus the potential mass flux $\overline{\rho w}$ is seen from (2.2) to vary linearly with height. (Clearly, the potential heat flux also varies linearly.)

Because $\partial\bar{\rho}/\partial t$ has been neglected in (2.8), Eq. (2.14) may be used only when $D/H\ll a$. Thus, away from the base of the convection layer, the thermals approach an asymptotic state. In particular, (2.15)–(2.17) imply that for $x\gg 1/a$

$$\left.\begin{aligned}u&\approx u_\infty=[1+(3-2f)/(1-f)a]^{(1-f)/(3-2f)}\\ s&\approx e^{-ax}u_\infty^{-1/(1-f)}\\ \bar{\rho}&\approx\rho_\infty=\bar{\rho}_0\exp[-\tfrac{1}{2}\epsilon a(u_\infty^2-1)]\end{aligned}\right\}; \tag{2.18}$$

that is, the buoyancy excess of a thermal decreases exponentially with height, while the velocity of a thermal and the mean potential density are constant. We see from (2.11) and (2.9) that thermals moving at a constant velocity are not growing by entrainment. Thus a thermal in a convection field behaves in a manner contrary to that of an isolated thermal. The latter is somewhat ellipsoidal in shape and entrains through the rear stagnation region which constitutes a significant fraction of its total surface area (Woodward, 1959). On the other hand, thermals in a convection layer apparently are elongated in the vertical direction. This implies that the entrainment region at the base of a thermal is only a small fraction of the total surface area, and so entrainment is suppressed. A thermal behaves locally like a plume which, in a field of plumes, has its lateral diffusion suppressed by the anisotropic nature of the turbulence (Manton, 1974). The elongation of a thermal could occur as it emerges from the diffusion layer at the base of the convection layer where its acceleration is large ($u\partial u/\partial x=1$ at $x=0$). In other words, the head of a thermal initially accelerates away from its tail.

Farmer (1975) finds that the behavior of the propagation velocity of thermals is essentially unchanged as the layer depth increases by a factor of 4. The constant asymptotic velocity is reached at a depth of about 10 m which suggests an e-folding depth of about 5 m, i.e., $D/a\approx 5$ m. Now since the diffusion layer beneath the ice "varied from a few centimeters to a little over 1 m", by taking $D\approx 50$ cm we find that

$$a\approx 0.1. \tag{2.19}$$

For $f=\frac{1}{2}$, Eqs. (2.18) and (2.19) imply that

$$u_\infty\approx 2.5; \tag{2.20}$$

that is, the asymptotic velocity of a thermal is $2\frac{1}{2}$ times greater than its value at the base of the convection layer. Farmer's results yield a constant velocity (Wu_∞) of 0.23 cm s^{-1}. Hence, taking a coefficient of thermal expansion of 2.5×10^{-5} K^{-1} and a water temperature of 275.5 K, we see from (2.7) that the present model predicts an initial temperature excess of order 0.01 K. The observed initial temperature excess was generally less than 0.02 K, although extreme values of 0.04 K were found. The estimate (2.19) of a therefore appears to be internally consistent for Farmer's data.

Although measurements in convection layers show the potential temperature to be essentially independent of height, Warner and Telford (1967) find that, averaged over 13 observations in a stable or neutrally stable atmosphere, the potential temperature increases by 0.21 K in a kilometer. This may be compared with (2.18) which predicts a temperature increase of 0.26 K over the depth of the layer, when $\epsilon\approx 1/300$ and a and u_∞ are given from (2.19) and (2.10) respectively.

Since the length scale of a thermal is proportional to $v^{\frac{1}{3}}$, Eqs. (2.1), (2.6), (2.9), (2.10), (2.18) and (2.20) imply that the asymptotic length scale is 1.4 times greater than its initial value. This is not inconsistent with the observations of Warner and Telford which show a small increase in length scale over the first 2 km of the atmosphere.

3. Penetration of thermals into stable layer

The convection layer in the atmosphere is invariably capped by a layer of stably stratified air. The interface between the layers is well defined by an inversion across which the potential density drops markedly. We consider now the behavior of thermals which rise through the convection layer and into the stable layer.

The bulk properties of a thermal at the top of the convection layer are given by the asymptotic result (2.18); that is, the thermal has negligible excess buoyancy and is travelling at the constant velocity $u_\infty W$. Thus the head of a thermal entering the stable layer is strongly negatively buoyant relative to the ambient air. As discussed in Section 2, entrainment is suppressed in the thermal because it is elongated in the vertical direction, and so buoyancy is the dominant force acting on a thermal in the stable layer. We therefore assume that the momentum equation for fluid entering the stable layer can be approximated by

$$\frac{dw}{dt}=(\rho_s/\rho_\infty-1)g, \tag{3.1}$$

where $\rho_s(z)$ is the mean potential density in the stable layer and ρ_∞ is the mean potential density in the convection layer. A similar model of penetration is given by Stull (1973). However, he assumes that entrainment retards a thermal and he accounts for this effect by introducing an arbitrary constant of proportionality.

By solving (3.1), it is seen that a thermal penetrates a distance L into the stable layer, where

$$\tfrac{1}{2}W_\infty{}^2=\int_H^{H+L}(1-\rho_s/\rho_\infty)g\,dz, \tag{3.2}$$

with $W_\infty=u_\infty W$; that is, the fluid simply gives up its kinetic energy to potential energy. The distance L is generally very small and it is reached in a short time. For example, if $1-\rho_s/\rho_\infty$ is 1/300 (corresponding to a 1 K inversion) and W_∞ is 1 m s^{-1}, then L is 15 m and that distance is covered in about 30 s. At the penetration height ($z=H+L$), a thermal has no momentum but it is negatively buoyant relative to the ambient fluid. It therefore disintegrates and mixes with the ambient air below $z=H+L$ until the next thermal reaches the top of the convection layer a time T after it. During this period, the region below $z=H+L$ ought to become well mixed with fluid from the thermal, provided that the thermal reaches its penetration height in a time small compared with the period T between thermals. Glider pilots believe that T is of order 5–15 min (Yates 1953); thus this condition is satisfied in general.

The thermals at the top of the convection layer give rise to a volume flux into the stable layer of fW_∞. Thus the expected flushing time for the region of penetration by the thermals is L/fW_∞. If

$$L/(fW_\infty T)<1, \tag{3.3}$$

then, during one period T, all the stable air in the region $H<z<H+L$ is flushed into the convection layer and is replaced by air from the thermals. Hence the penetrating thermals tend to form a sharp inversion because buoyancy forces suppress any mixing with fluid above the penetration height while the fluid below is well mixed. Although the inversion is locally well-defined

in the neighborhood of a penetrating thermal, the mean inversion (obtained by taking a horizontal average of the potential temperature) is diffused over the penetration depth L. This is because the arrival times of thermals at the top of the convection layer are spatially uncorrelated so that the local inversion level varies between $z=H$ and $z=H+L$.

If condition (3.3) is not satisfied then the penetration distance is large and thermals arrive at the top of the convection layer before the mixing of previous thermals is completed. They pass through the partially mixed region and then penetrate further into the stable layer. Because the penetration region is not well-mixed, the inversion becomes very diffuse and a sharp inversion does not form until a region of air stable enough to satisfy (3.3) is reached.

During one period T, we see that the inversion in the neighborhood of the thermal rises from $z=H$ to $z=H+L$. Thus, although the deepening of the convection layer occurs intermittently as a thermal penetrates the inversion at any given location, the rate of rise averaged over a large area is given by

$$\frac{dH}{dt}=\frac{L}{T}. \tag{3.4}$$

[Condition (3.3) ensures that dH/dt is small compared with the local velocities in the convection layer.] As a thermal enters the stable layer it displaces an equal volume of ambient fluid into the convection layer. Thus, at the top of the convection layer, there is a net potential mass flux

$$Q_H=[\rho_\infty-\rho_s(H)]dH/dt, \tag{3.5}$$

which corresponds to a net downward heat flux.

The ratio of the mass flux out of the convection layer at the top to that at the base (i.e., the ratio of the downward heat flux at the top to the upward heat flux at the base) is given by (3.4) and (3.5) as

$$Q_H/Q=[\rho_\infty-\rho_s(H)]L/QT. \tag{3.6}$$

In general, this ratio depends upon the detailed behavior of the potential density in the stable layer. This is contrary to the common assumption that Q_H/Q is a constant in the absence of a mean wind shear. On the other hand, when

$$-\rho_\infty^{-1}d\rho_s/dz\ll[1-\rho_s(H)/\rho_\infty]^2 g/W_\infty^2 \tag{3.7}$$

(i.e., the density jump at the inversion is large), then it is found from (3.2) and (3.6) that the flux ratio is independent of $\rho_s(z)$; in particular,

$$Q_H/Q=\rho_\infty W_\infty^2/(2gTQ). \tag{3.8}$$

4. Bulk properties of convection layer

A primary aim of any model of a convection layer is the prediction of the temporal development of the potential density and depth of the layer. We see from (2.2), (2.17) and (2.18) that the overall mass balance equation for the convection layer is

$$H\frac{d\rho_\infty}{dt}+Q_H+Q=0. \tag{4.1}$$

By specifying the nature of the potential density profile $\rho_s(z)$ of the stable layer, the development of the potential density ρ_∞ and depth H of the convection layer is given in terms of the source parameters Q, W and T through (3.2), (3.4) (3.5) and (4.1).

For a given profile $\rho_s(z)$, the rate of change of the jump in potential density at the inversion can be calculated from (3.2) and (3.4); that is,

$$\frac{d\Delta}{dt}=\frac{d\rho_\infty}{dt}+\rho_s\Gamma\frac{dH}{dt}, \tag{4.2}$$

where $\Delta=\rho_\infty-\rho_s$ and $\Gamma=-(1/\rho_s)d\rho_s/dz$ at $z=H$. The sign of $d\Delta/dt$ is arbitrary and it is negative if Γ is small. Thus, if the region above the convection layer is only weakly stable then Δ might decrease with time. But since the fluid above the convection layer must always be lighter than that in the layer, Δ cannot become negative in practice. If Δ is calculated from (3.4) and (4.1) to be negative then the appropriate boundary condition at the top of the convection layer becomes

$$\rho_s(H)=\rho_\infty; \tag{4.3}$$

that is $\Delta=0$ and there is no effective inversion. Since there is also no net mass flux due to entrainment, the overall mass balance equation (4.1) reduces to

$$\frac{d\rho_\infty}{dt}=-\frac{Q}{H}. \tag{4.4}$$

The behavior of the convection layer depth is therefore governed by the equation

$$\rho_s\Gamma H\frac{dH}{dt}=Q.$$

As discussed in Section 3, thermals rapidly penetrate the region of weak stability until a layer stable enough to support an inversion is reached. (As Γ decreases, dH/dt becomes limited by the speed of the thermals at the top of the convection layer, i.e., $dH/dt\lesssim W_\infty$.)

An inversion begins to form at the height where $d\Delta/dt$ first becomes positive. By neglecting the curvature of the potential density profile, the penetration depth for an individual thermal is found from (3.2) to be

$$L=W_\infty/(\Gamma g)^{\frac{1}{2}} \tag{4.5}$$

when (4.3) is satisfied. Hence, putting (3.4), (4.4) and (4.5) into (4.2), we see that

$$d\Delta/dt=-Q/+H(\rho_\infty W_\infty/T)(\Gamma/g)^{\frac{1}{2}},$$

which is positive when

$$\Gamma > (Q/\rho_\infty W_\infty)^2 (gT^2/H^2). \quad (4.6)$$

It is seen that a less stable layer is needed to cap a deep convection layer than a shallow one. For example, if $Q \approx \rho_\infty W_\infty/300$ and $T \approx 10^3$ s, then (4.6) implies that the lapse rate required to form an inversion in the atmosphere decreases from 21 K km^{-1} to 5.3 K km^{-1} as the layer depth increases from 500 m to 1 km.

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