On the Heat Flux and Friction Velocity in Free Convection Near the Ground

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5 December 1973

ABSTRACT

Measurements on wind and temperature fields near the ground as reported by Swinbank and Dyer are re-examined. Through the use of the free convection wind and temperature profiles, H and u_* as derived by the present authors have accuracies that are comparable to or better than those obtained earlier through the use of the exponential wind profile and the integrated KEYPS profile.

1. Introduction

The careful measurements on the wind and temperature fields near the ground (Swinbank, 1964; Swinbank and Dyer, 1968) have been examined by several authors. Two of these authors have derived the friction velocity u_* and vertical heat flux H, by applying different models of the wind profile to these experimental data. Swinbank assumed the exponential wind profile that he

TABLE 1. Comparison between measured and calculated values of H and u_*.

Test run	Heat flux ($mW\ cm^{-2}$)				Friction velocity ($cm\ s^{-1}$)			
	Measured*	Calculated from model (Swinbank)	(Klug)	(Ting)**	Measured	Calculated from model (Swinbank)	(Klug)	(Ting)**
1	33.4	34.3	39.1	29.6	28.5	27.8	31.0	28.6
2	29.3	24.9	27.1	31.3	26.3	25.8	28.5	28.2
3	22.9	24.7	29.4	24.9	24.8	24.9	27.2	25.4
4	22.2	21.0	19.2	24.6	26.3	25.9	28.4	28.1
5	15.8	14.1	13.7	17.8	22.5	22.8	24.3	24.2
6	10.6	12.2	10.2	12.2	22.4	25.1	24.5	23.2
7	13.5	9.2	7.4	10.8	22.3	22.2	23.8	23.4
8	18.0	21.3	30.0	25.8	22.1	22.1	24.5	22.8
9	33.2	23.9	35.0	25.7	22.5	20.8	24.9	22.7
10	31.6	26.7	37.9	26.2	23.8	24.3	26.5	23.7
11	32.2	32.7	46.5	27.3	24.9	22.9	27.6	24.2
12	23.8	27.6	31.3	29.5	26.8	27.2	29.2	27.8
13	29.2	25.4	36.6	23.7	24.1	23.1	26.6	24.0
14	23.8	25.7	28.3	23.1	26.0	25.2	28.2	26.3
15	37.8	26.4	30.9	27.8	25.2	24.7	27.6	26.1
16	28.6	24.1	30.8	22.4	24.3	23.8	26.7	24.4
17	24.0	22.2	25.6	20.0	23.8	23.3	26.1	24.1
18	16.0	20.2	23.7	14.2	23.2	24.4	25.5	22.6
19	13.6	8.4	6.7	13.0	23.9	25.6	25.7	25.8
20	1.2	2.4	2.0	2.9	23.5	23.7	24.9	21.1
21	33.2	27.6	30.5	31.4	27.5	28.3	29.9	28.8
22	30.8	27.6	31.5	31.7	26.2	27.1	28.6	27.5
23	8.4	8.8	8.8	8.1	19.3	19.5	20.9	19.7
24	36.0	27.6	23.8	27.6	34.3	35.7	36.7	34.5
25	27.2	37.5	31.3	30.3	37.5	38.9	40.1	36.7
26	38.0	44.9	39.4	32.9	37.9	35.3	40.1	36.7
27	(35.2)	46.1	39.8	40.6	38.6	26.8	40.9	38.5
28	34.2	45.9	40.5	42.6	37.1	37.8	39.8	37.7
29	32.0	48.6	48.0	41.2	35.3	35.4	38.0	35.3
30	(35.4)	36.2	31.4	31.3	35.5	37.8	38.4	35.6
31	29.2	36.7	30.7	27.4	36.9	38.5	39.5	35.5
32	26.1	35.1	28.7	26.4	37.7	40.3	40.4	36.2
33	21.4	32.4	26.6	25.2	41.1	39.0	43.4	38.4
34	19.0	22.6	20.9	22.8	41.4	40.9	43.7	38.7
Standard deviation†		6.6	6.3	4.8		1.3	2.3	1.2

* The measured heat fluxes reported in 1968 differ from those reported in 1964. The former have been used in this table. Measured fluxes in runs 27 and 30 were not included in the former, and these are taken from the 1964 report.

** For the present computation, $g=9.81\ m\ s^{-2}$, $c_p=1000\ J\ kg^{-1}\ K^{-1}$, $T_0=$ air temperature at higher 1 m, and $\rho T_0=346.4\ kg\ m^{-3}\ K$ corresponding to ambient pressure 1000 mb. Wind data for heights below 1 m are excluded because of uncertainty in the anemometer position.

† The corresponding standard deviations of calculated heat flux relative to the measured fluxes reported in 1964 are 4.9, 5.5 and 3.6, respectively.

proposed earlier (1960), in conjunction with the definition of drag coefficient and the Monin-Obukhov length. Alternatively, Klug (1967) used the integrated form of the KEYPS profile that relates wind speed to the Monin-Obukhov length and the friction velocity. The present note describes a third approach that is based upon the free convection wind and temperature profiles, and that leads to improved prediction of u_* and H.

2. Temperature and wind profiles

We begin with the profiles of temperature and wind gradients in a free convection regime (see, e.g., Zilitinkevich, 1973). These relate the potential temperature $\bar{\theta}$, wind speed $\bar{u}$ and height z to the heat flux H and the friction velocity u_* through the equations:

$$\partial\bar{\theta}/\partial z = -C_1(H/\rho c_p)^{\frac{2}{3}}(g/T_0)^{-\frac{1}{3}}z^{-\frac{4}{3}}, \quad (1)$$

$$\partial\bar{u}/\partial z = C_2 u_*^2(Hg/\rho c_p T_0)^{-\frac{1}{3}}z^{-\frac{4}{3}}, \quad (2)$$

where ρ, c_p, g and T_0 are density, specific heat, gravitational acceleration and reference temperature, respectively, and C_1 and C_2 are numerical constants. Integration of (1) and (2) yields

$$\bar{\theta} = 3C_1(H/\rho c_p)^{\frac{2}{3}}(g/T_0)^{-\frac{1}{3}}z^{-\frac{1}{3}} + (\text{constant}), \quad (3)$$

$$\bar{u} = -3C_2 u_*^2(Hgz/\rho c_p T_0)^{-\frac{1}{3}} + (\text{constant}). \quad (4)$$

The parameters H and u_* are evaluated by fitting (3) and (4) to Swinbank's observational data using the method of least squares. In this computation, the values of C_1 and C_2 are adjusted to yield the least overall deviations between calculated and measured H and u_*. This leads to $C_1=0.81$ and $C_2=1.77$. These may be compared with the range of values $C_1=0.77$–1.20, and $C_2=1.02$–2.03, as reported for many observations in free convection elsewhere (Monin and Yaglom, 1971; Wyngaard and Coté, 1972).

3. Calculated H and u_*

Table 1 presents the results of the calculations of heat flux and friction velocity. Comparisons are made for each of 34 sets of experimental observations reported by Swinbank. The second and sixth columns give the measured H and u_*, respectively; the third, fourth, seventh and eighth columns present the corresponding derived values as reported by Swinbank and Klug. Derived H and u_* as based upon the present approach are given in the fifth and ninth columns, respectively. The standard deviations between the sets of measured and derived H and u_* are listed in the last row of Table 1. Inspection shows that the present approach, using the $-\frac{1}{3}$ law of free convection in temperature and wind profiles with $C_1=0.81$ and $C_2=1.77$, leads to accuracy in derived H and u_* that is comparable to or better than the approaches using the exponential wind profile or the integrated KEYPS profile.

Acknowledgments. The authors wish to thank Dr. H. C. Martin of the Atmospheric Environment Service (Canada) for his reading of the orginal draft.

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Comments on "Dynamic Stability of an Isentropic Shear Layer in a Statically Stable Medium"[1]

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2 August 1974 and 21 October 1974

In a recent paper Gossard (1974) examined the possibility of shearing instability in isentropic shear layers embedded in an otherwise statically stable atmosphere. He illustrated the paper with data showing radar returns from billows in the clear atmosphere but temperature soundings were not available close enough