Field Generation and Dissipation Currents in Thunderclouds as a Result of the Movement of Charged Hydrometeors

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ABSTRACT

The calculations of Gay *et al.* of the terminal velocities of charged hydrometeors in the presence of electric fields have formed the basis of computations of the charging current density J flowing through a thundercloud as a result of the operation of a precipitative mechanism of cloud electrification. Values of J were calculated for a range of values of field strength E, precipitation rate p_0, precipitation content L, cloud water content C, charge distribution, total separated charge, and the fraction f of the small particles that have undergone a charging event.

It is found that the estimated field required for the initiation of a lightning stroke ($\sim$3.5 kV cm^{-1}) can be achieved only over a narrow range of conditions. The ease with which precipitative mechanisms can produce breakdown fields is considerably increased, however, if account is taken of spatial *inhomogeneities* in the field.

1. Introduction

Gay *et al.* (1974) presented a method for calculating the terminal velocities of charged water drops and droplets falling in electric fields. The calculations were confirmed by experiment over a range of values of drop radius r, charge q, and field strength E. Since changes in velocity for charges and fields of the magnitudes known to exist in the strongly electrified regions of thunderclouds are substantial, it is necessary to inquire whether they may significantly affect the growth of the electric field. Clearly, if a precipitation mechanism is responsible in whole or in part for cloud electrification, the electric forces, which tend to inhibit the gravitational separation of oppositely charged particles, will tend to retard the field growth, and eventually terminate it at $E=E_{max}$. This effect has been studied, using rather crude assumptions discussed by Gay *et al.*, by Kamra (1970, 1971), Kamra and Vonnegut (1971) and Mason (1972). For example, Kamra predicts values of E_{max} ranging from 1.7 kV cm^{-1} when the precipitation rate $p_0=10$ mm h^{-1} to 2.56 kV cm^{-1} when $p_0=50$ mm h^{-1}. If these values of E_{max} can be confirmed by more rigorous calculations, they place serious and perhaps overwhelming restrictions on the ability of precipitation mechanisms of any kind to generate fields sufficiently intense to initiate and propagate a lightning stroke.

This discussion is complicated by the fact that no general consensus exists as to the magnitude E_1 of the field required for the initiation of lightning. It seems probable that the value of E_1 will be at least as great as the field E_c required to initiate corona from large hydrometeors within the cloud. The studies of Richards and Dawson (1971) show that for individual large raindrops carrying high charges the value of E_c at a pressure of about 650 mm Hg is about 5.5 kV cm^{-1}. Even allowing, where appropriate, for the further small reduction in E_c which would occur at pressures existing in the central regions of a thundercloud, the value so obtained is still considerably larger than the largest fields that have been measured within thunderclouds, of about 4.0 kV cm^{-1} by Winn *et al.* (1974). Recent experiments by Griffiths and Latham (1974) have shown that under conditions likely to occur within thunderclouds corona may be initiated from ice hydrometeors in fields above about 4.0 kV cm^{-1}, provided that the temperature is warmer than -18°C. Parallel experiments by Crabb and Latham (1974) have demonstrated that collision of a pair of raindrops can produce temporarily a highly elongated liquid filament, leading to the initiation of corona in fields as low as 2.5 kV cm^{-1}. This lowest value of E_c, however, is below that field which is required to permit propagation of the positive streamers once initiated, for which Phelps (1971) has estimated a value of about 3.0–4.0 kV cm^{-1} within the thundercloud. In the light of this discussion it appears probable that a reasonable restriction to place on an acceptable mechanism of thundercloud electrification is that it should be capable of producing an electric field of at least 3.5 kV cm^{-1}. In view of Kamra's calculations, it is clear that E_1 and E_{max} may have very similar values, so that, in order properly to assess the efficiency of precipitation mechanisms of thundercloud electrification, it is necessary to take careful account of the influence of electric forces on particle velocities.

In the following sections we present calculations, based on the work of Gay *et al.*, and performed for a wide range of conditions of the current density J and maximum field $E_{\max}$, resulting from the operation within a thundercloud of a precipitative charging process. This permits us to assess the suggestion of Vonnegut (1963) that precipitative mechanisms may be incapable, *per se*, of explaining thunderstorm electrification.

2. The interactions of raindrops or hailstones with cloud droplets

In order to calculate the charging currents we divide the hydrometeors into two categories: negatively charged raindrops, of radius R greater than some critical value R_c but not exceeding a maximum value of 0.3 cm, and cloud droplets, having radius r less than value r_c $(=R_c)$ carrying positive charge. The size distributions of these two classes are taken to be those given by Best (1951) for cloud droplets and Marshall and Palmer (1948) for raindrops. The Best equation can be written in the following form to give the concentration of droplets lying within a narrow radius band of width Δr centered around a radius r (cm):

$$n(r)\Delta r=\frac{3nAC}{4\pi\rho_w}r^{(n-4)}\exp(-Ar^n)\Delta r, \qquad (1)$$

where ρ_w (g cm^{-3}) is the density of water; the factor $A=(2\times10^4/a)^n$, where the values of the constants a and n are those listed for particular cloud types by Best; and C is the cloud water concentration (g m^{-3}). Similarly, the Marshall-Palmer expression for the concentration of raindrops in a band of width ΔR centered around a radius R (cm) can be written

$$N(R)\Delta R=0.16\exp(-82Rp_0^{-0.21})\Delta R, \qquad (2)$$

where p_0 is the field-free rainfall rate (mm h^{-1}). The corresponding rainwater content L (g m^{-3}) is given by

$$L=10^6\times\frac{4\pi\rho_w}{3}\sum_{r_c=R_c}^{R_{\max}}R^3N(R)\Delta R. \qquad (3)$$

It was considered desirable to retain flexibility in the choice of expressions for the charge carried on the particles, since this is known to be a very variable factor. Accordingly, the equation adopted for the charge Q (esu) carried on raindrops of radius R (cm) in a field of strength E was

$$Q=-kER^x. \qquad (4)$$

Experimental evidence (Gunn, 1950; Latham and Stow, 1969) suggests that in strongly electrified clouds the constant k may take values between 0.5 and 3, the most probable value being close to 2, and the constant x may lie between 1 and 3, with a value of 2 being typical of many charging processes. In the majority of calculations the charge q on a small cloud particle of radius r was assumed to be related to E and r by an equation of the same form as (4) with the same value of x as for the precipitation, but with the value of the constant k given a new value k' so as to make the net charge in the cloud equal to zero; i.e., at any value of E,

$$\sum_{r_{\min}}^{r_c}n(r)q(r)\Delta r=-\sum_{r_c=R_c}^{R_{\max}}N(R)Q(R)\Delta R. \qquad (5)$$

For some of the calculations the assumption was made that *all* of the cloud droplets were charged and the value of k' computed accordingly. In order to model the natural situation more realistically, calculations were also performed in which the positive charge was carried by a fraction f of the cloud droplets, the rest remaining uncharged, f being constant over the droplet size range. The value of k' was then computed for various values of f, while retaining the condition that the net charge on the aggregate of drops and droplets was zero. An additional restriction was that q was not allowed to exceed the Rayleigh limit.

If we define $v(r)$ and $V(R)$ as the velocities of the small and large hydrometeors, respectively, in the presence of electrical forces, then the charging current density J, which is responsible under our assumptions for the growth of the electric field, is given by

$$J=-\sum_{r_c=R_c}^{R_{\max}}N(R)Q(R)V(R)\Delta R-\sum_{r_{\min}}^{r_c}n(r)q(r)v(r)\Delta r, \qquad (6)$$

where the first and second terms on the right-hand side of (6) are the current densities carried by precipitation, J_p, and cloud droplets, J_d, respectively. In this notation J is directed upward, and $V(R)$ and $v(r)$ are directed downward. The second term J_d always makes a negative contribution to J, since $q(r)$ is always positive, while, in weak fields at least, the first term J_p makes an overall positive contribution, since $Q(R)$ is negative. However, as soon as any of the negatively charged precipitation becomes levitated or acquires an upward velocity due to the electrical force on it in the high field exceeding the gravitational force, there will be a negative component to J_p. In general, this is not sufficiently large to render the term negative overall, except in fields which are extremely high. Eqs. (1), (2), (4) and (5), with chosen values of C, a, n, p_0, k, x, $r_{\min}$, r_c, $R_{\max}$ and E, and the corresponding values of A, L and k' can then be substituted into Eq. (6) to find J for a given set of conditions.

TABLE 1. Values of k for various values of x and rainfall rate p_0: $Q=-2ER^2$, $r_c=50$ μm.

p_0 (mm h^{-1})	k ($Q=-kER^x$) $x=1$	$x=2$	$x=3$
10	0.0811	2.0	33.699
20	0.0932	2.0	29.161
30	0.1012	2.0	26.809
40	0.1073	2.0	25.268
50	0.1122	2.0	24.144
60	0.1164	2.0	23.271
70	0.1201	2.0	22.564
80	0.1234	2.0	21.975
90	0.1263	2.0	21.472
100	0.1290	2.0	21.036
110	0.1315	2.0	20.652
120	0.1338	2.0	20.311
130	0.1359	2.0	20.005
140	0.1380	2.0	19.728
150	0.1398	2.0	19.475
160	0.1416	2.0	19.244
170	0.1433	2.0	19.030
180	0.1449	2.0	18.833
190	0.1465	2.0	18.649
200	0.1479	2.0	18.478

This model of the precipitative development of the electric field clearly has many features which are unlike those existing in a real cloud. First, the model is one-dimensional, and assumes a uniform electric field distribution within any volume of cloud to which the calculations may apply; and the net charge on the cloud is zero. Such simplifications, with their attendant disadvantages, have been made by the great majority of workers in calculations of field growth. Second, the value of electric field selected for any of our calculations is not *derived* from the actual distribution of charge within the cloud. Finally, the model is time-independent. Nevertheless, despite these limitations, the method provides a means of assessing the possible range of conditions under which a precipitation mechanism becomes ineffectual, or continues to augment the existing electric field.

The velocities of the charged drops and droplets were calculated using the method described by Gay *et al.* (1974). The constants a and n in the Best equation were assigned the values 27 and 3.29, respectively, these being characteristic values for a cumulus congestus cloud. The minimum droplet radius $r_{\min}$ was taken as 5 μm, and the maximum drop radius $R_{\max}$, 0.3 cm. The value of $r_c=R_c$, above which drops carry negative charge, was generally taken to be 50 μm, and a further set of calculations was performed with $r_c=100$ μm. The precipitation rate p_0 was varied from 10 to 200 mm h^{-1}. The values of k employed when $x=2$ were 0.5 and 2.0. Corresponding values of k for $x=1$ and $x=3$ are presented in Tables 1 and 2; these values were calculated subject to the condition that the total charge distributed over the precipitation spectrum was the same for all values of x.

Values of J were calculated for various altitudes Z, using Eqs. (1), (2), (4) and (6), the velocity expression of Gay *et al.*, and a small computer. The droplet spectrum was divided into 44 intervals, and the drop spectrum into 29 intervals. The effect of changing Z was to vary the velocities of the hydrometeors owing to changes in air density and viscosity. For all calculations it was assumed that the cloud had its 0°C isotherm at 2.5 km, and a lapse rate of 6°C km^{-1}. Values of air density and viscosity were taken from Mason (1971).

Similar calculations of J were made for a cloud in which the larger class of hydrometeors were spherical hailstones instead of raindrops. In this case the velocities of the hailstones were calculated using the method of Gay *et al.*, and the Reynold's number-drag coefficient relationship given for spheres by McDonald (1960). A mean density ρ of 0.7 g cm^{-3} was assumed for the hailstones, and their size distribution was assumed to be the same as that used for the raindrops, with the number of hailstones in a given radius range increased by the factor $1/\rho$ so as to conserve the mass of water for a given value of L. Calculations were made for the same range of values of precipitation and cloud water content as was used for raindrops. The charge-size relation for hailstones was taken to be the same as that for drops, and therefore, since the number of hailstones in a given radius range is greater than the corresponding number of liquid water drops by a factor $1/\rho$, the total separated charge was also greater by the same factor.

3. Results of the computations

Fig. 1 shows graphically the computed values of J as a function of E for liquid hydrometeors, with $p_0=50$ mm h^{-1} at $Z=5.5$ km, where the pressure $P=500$ mb and the temperature $T=-18$°C. In this

TABLE 2. Values of k for various values of x and rainfall rate p_0: $Q=-0.5ER^2$, $r_c=50$ μm.

p_0 (mm h^{-1})	k ($Q=-kER^x$) $x=1$	$x=2$	$x=3$
10	0.0203	0.5	8.425
20	0.0233	0.5	7.290
30	0.0253	0.5	6.702
40	0.0268	0.5	6.317
50	0.0281	0.5	6.036
60	0.0291	0.5	5.818
70	0.0300	0.5	5.641
80	0.0309	0.5	5.494
90	0.0316	0.5	5.368
100	0.0323	0.5	5.259
110	0.0329	0.5	5.163
120	0.0335	0.5	5.078
130	0.0340	0.5	5.001
140	0.0345	0.5	4.932
150	0.0350	0.5	4.869
160	0.0354	0.5	4.811
170	0.0358	0.5	4.758
180	0.0362	0.5	4.708
190	0.0366	0.5	4.662
200	0.0370	0.5	4.619

TABLE 3. Values of J_p and J_d for various values of E and f, for raindrops: $p_0=50$ mm h^{-1}, $Z=5.5$ km, $L=C=2.4$ g m^{-3}, $Q=-2ER^2$, $r_c=50$ μm.

E (kV cm^{-1})	J_p (μA m^{-2})	J_d (μA m^{-2}) $f=1$	$f=0.2$	$f=0.1$	$f=0.05$
0.6	6.2×10^{-2}	-1.7×10^{-4}	-1.3×10^{-3}	-5.4×10^{-3}	-3.2×10^{-2}
1.2	1.2×10^{-1}	-3.7×10^{-4}	-5.4×10^{-3}	-3.2×10^{-2}	-2.3×10^{-1}
1.8	1.6×10^{-1}	-6.0×10^{-4}	-1.5×10^{-2}	-1.0×10^{-1}	-7.1×10^{-1}
2.4	1.8×10^{-1}	-9.1×10^{-4}	-3.2×10^{-2}	-2.3×10^{-1}	-1.6
3.0	1.6×10^{-1}	-1.3×10^{-3}	-6.0×10^{-2}	-4.3×10^{-1}	-2.9
3.6	7.6×10^{-2}	-1.8×10^{-3}	-1.0×10^{-1}	-7.1×10^{-1}	-4.7
4.2	-6.9×10^{-2}	-2.4×10^{-3}	-1.6×10^{-1}	-1.1	-6.9
4.8	-2.7×10^{-1}	-3.2×10^{-3}	-2.3×10^{-1}	-1.6	-9.8
5.4	-5.0×10^{-1}	-4.2×10^{-3}	-3.2×10^{-1}	-2.1	-13.3
6.0	-7.4×10^{-1}	-5.3×10^{-3}	-4.3×10^{-1}	-2.8	-17.4

case $r_c=50$ μm and $L=C=2.38$ g m^{-3} with $Q=-2ER^2$. For the case where all the droplets are charged, i.e., $f=1$, it is seen that J rises to a peak value of 0.18 μA m^{-2} at a field $E=2.4$ kV cm^{-1}, and becomes negative at $E_{max}\approx4.0$ kV cm^{-1}. The value of J plotted is the combined density due to both drops and droplets. However, as can be seen from Table 3 the droplet contribution for $f=1$ is minor except over a small range of E close to E_{max} when it is comparable to the precipitation current density; at this point the total current density J is changing from a positive to a negative value. If the values of f are less than 1, then the smaller numbers of droplets that are more highly charged carry a greater current because of their greatly increased terminal velocities, and their contribution to J is such as to reduce E_{max} to 3.6, 2.2 and 0.9 kV cm^{-1} for $f=0.2$, 0.1 and 0.05, respectively, as shown in Table 4, with correspondingly smaller values of J_{max}.

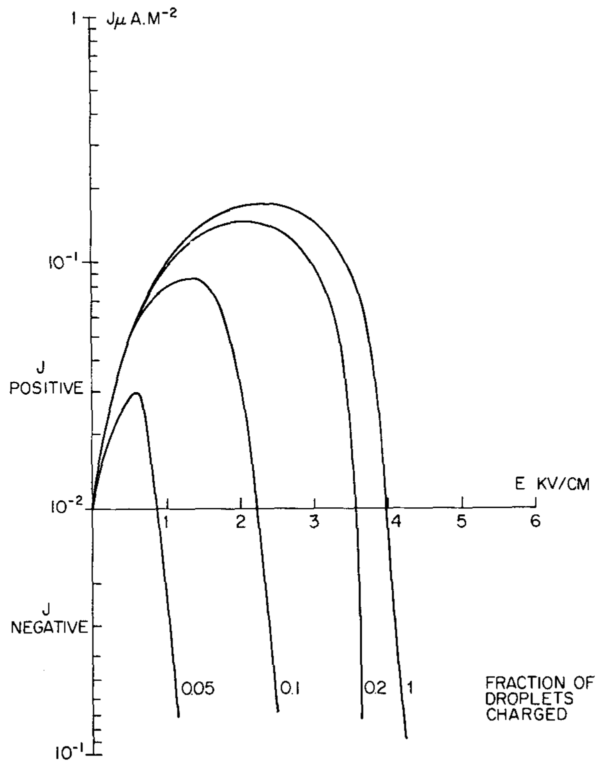

FIG. 1. The calculated variation of total current density J with field strength E, for liquid hydrometeors, for various values of f. E_{max} is the value of field strength at which the J-curve crosses the abscissa. $Q=-2ER^2$, $Z=5.5$ km, $L=C$, $r_c=50$ μm, $p_0=50$ mm hr^{-1}.

Table 5 presents values of J_{max}, E_{max} and the field E at J_{max} for $f=1$, $r_c=50$ μm, $Z=5.5$ km, various values of p_0 and the three charge distributions. It is seen that whereas J_{max} is approximately proportional to p_0, E_{max} increases only slightly as the initial precipitation rate is raised from 30 to 100 mm h^{-1}. This is, of course, because J depends on the total flux of charged particles, which rapidly increases with p_0, while E_{max} is sensitive to p_0 only because the size spectrum of the raindrops shifts to the larger particles as p_0 increases. The time taken to achieve E_{max} would, of course, decrease rapidly with increasing p_0. Another interesting observation is that J_{max} increases markedly as x increases from 1 to 3, while E_{max} is essentially the same for $x=2$ and $x=3$, although substantially lower for $x=1$. The reason for this is that as x increases a greater proportion of the charge is localized on hydrometeors whose velocities are relatively unaffected by the electric field. The opposition to gravitational separation of charge is therefore reduced and higher current densities exist. Also the most mobile particles carry higher charges when $x=1$ and consequently they become levitated in lower fields, thus reducing E_{max}.

TABLE 4. Values of J_{max}, E_{max} and E at J_{max} for various values of f, for raindrops: $p_0=50$ mm h^{-1}, $Z=5.5$ km, $L=C=2.4$ g m^{-3}, $Q=-2ER^2$, $r_c=50$ μm.

f	J_{max} (μA m^{-2})	E at J_{max} (kV cm^{-1})	E_{max} (kV cm^{-1})
1	0.18	2.4	4.0
0.2	0.15	2.3	3.6
0.1	0.09	1.2	2.2
0.05	0.03	0.6	0.9

TABLE 5. Values of J_{max}, E_{max}, E at J_{max} and k' for various values of p_0, L, k and x, for raindrops: $Z=5.5$ km, $L=C$, $f=1$, $r_c=50$ μm.

p_0 (mm h^{-1})	L (g m^{-3})	x	k	k'	J_{max} (μA m^{-2})	E at J_{max} (kV cm^{-1})	E_{max} (kV cm^{-1})
30	1.56	1	0.1012	3.13×10^{-5}	0.05	1.8	2.7
		2	2	3.12×10^{-2}	0.11	2.4	3.9
		3	26.809	2.78×10	0.16	2.4	3.7
50	2.38	1	0.1122	2.82×10^{-5}	0.08	1.8	2.7
		2	2	2.82×10^{-2}	0.18	2.4	4.0
		3	24.144	2.51×10	0.24	2.4	4.0
100	4.21	1	0.1290	2.45×10^{-5}	0.15	1.8	3.0
		2	2	2.45×10^{-2}	0.32	2.4	4.2
		3	21.036	2.19×10	0.44	3.0	4.2

The form of the charge distribution is therefore seen to have a significant effect on the limiting electrical conditions.

Table 6, derived for $r_c=100$ μm, illustrates the reassuring finding that the predicted values of E_{max} and J_{max} are extremely insensitive to the chosen dividing line between the droplets and the precipitation. The slight increase in E_{max} over that displayed for the same p_0 in Table 5 is attributable to the shift of the charge distributions toward the larger, more immobile, end of the raindrop spectrum.

Table 7 shows that if the mean charge carried by the raindrops is reduced from the $-2ER^2$ (or its equivalent for $x=1$ or 3) used in the former calculations to $-0.5ER^2$ (or its equivalent), the values of J_{max} are reduced and those of E_{max} increased. This is because the velocity changes in a given field are less if the particles carry smaller charges while a higher field is required in order to levitate drops carrying less charge. In these calculations J was still positive at $E=6$ kV cm^{-1} and the calculations were terminated at this point since we do not believe that higher fields can exist within a thundercloud in the presence of precipitation.

Changes in altitude Z from 2.5 to 10 km were found to have a negligible influence on E_{max}, but J_{max} was found to increase somewhat with decreasing Z. This is clearly a consequence of the larger terminal velocities which occur at higher altitudes, principally as a result of the reduction in air density.

Table 8 shows calculated values of E_{max} and J_{max} for hailstones over ranges of other parameters identical to those utilized in constructing Table 5 for raindrops. No significant differences between hailstones and raindrops are observed in the values of J_{max}, but the values of E_{max} are about 20% lower in the case of hailstones. This is attributable to the fact that since the hail is less dense than the water its velocity will be more significantly affected by the electric field. The insensitivity of J_{max} to the type of precipitation particle is a result of the approximate cancellation of the two effects of the reduced density ρ in increasing the number of hailstones while reducing their velocity at least over the greater part of their size range.

4. Discussion

We are now in a position to estimate whether precipitation mechanisms of cloud electrification, insofar, at least, as they can be described in terms of our simple model, are capable of generating the fields and currents which must exist in clouds which produce lightning. We have already established that a satisfactory mechanism must be capable of producing at least the corona onset and propagation field $E_c\approx3.5$ kV cm^{-1}. The compilation by Latham (1971) of the results of other workers suggests that an acceptable mechanism must also be able to produce a current density of about 0.1 μA m^{-2} in order to withstand leakage effects. Griffiths *et al.* (1974) have shown that ionic leakage currents will be negligible over the range of E of interest. Fig. 1 and Table 5 combine to suggest that if $f=1$ and $x=2$ or 3, the required conditions can be met if $p_0\gtrsim30$ mm h^{-1}. The production of breakdown fields becomes more difficult as f is reduced below 1 and cannot occur (for $p_0=50$ mm h^{-1}) if f falls below 0.2. If a lower mean charge is assumed ($k=0.5$), E_{max} is higher, and considerably in excess of E_c, but higher values of p_0

TABLE 6. Values of J_{max}, E_{max}, E at J_{max} and k' for selected values of p_0, L, k and x, for raindrops: $Z=5.5$ km, $L=C$, $f=1$, $r_c=100$ μm.

p_0 (mm h^{-1})	L (g m^{-3})	x	k	k'	J_{max} (μA m^{-2})	E at J_{max} (kV cm^{-1})	E_{max} (kV cm^{-1})
50	2.38	1	0.1159	2.80×10^{-5}	0.08	1.8	3.0
		2	2	2.80×10^{-2}	0.18	2.4	4.0
		3	24.033	2.50×10	0.24	2.7	4.0

TABLE 7. Values of J_{max}, E_{max}, E at J_{max} and k' for various values of p_0, L, k and x, for raindrops: $Z=5.5$ km, $L=C$, $f=1$, $r_c=100$ μm.

p_0 (mm h^{-1})	L (g m^{-3})	x	k	k'	J_{max} (μA m^{-2})	E at J_{max} (kV cm^{-1})	E_{max} (kV cm^{-1})
30	1.56	2	0.5	7.76×10^{-3}	0.06	4.2	>6
		3	6.662	6.92	0.08	5.1	>6
50	2.38	2	0.5	6.99×10^{-3}	0.09	4.8	>6
		3	6.008	6.24	0.12	5.4	>6

are required in order to produce the required current densities; even with $f=1$, p_0 must exceed about 50 mm h^{-1}. In the case of hailstones the conditions are somewhat similar, although the predicted maximum fields for $k=2.0$ are always marginally below E_c, even in the most advantageous circumstances.

It is clear that consideration of the influence of electric forces on the velocities of hydrometers imposes some severe restrictions on the role of precipitation mechanisms in explaining thundercloud electrification; although, in contradiction to the calculations of Kamra, we do find ranges of conditions in which breakdown fields can be attained. However, the reports by Moore *et al.* (1962) of the occurrence of lightning in clouds where the precipitation rate does not exceed a few millimeters per hour are clearly inexplicable in terms of a precipitation mechanism. The fraction f of small particles which carry charges is clearly an important parameter. On a crude argument, bearing in mind that as a precipitation particle becomes increasingly charged, the charge transfer q_c per collision with small particles will decrease, we might expect that if the mean charge $\bar{Q}=-2E\bar{R}^2$ then $q_c\approx0.5\ E\bar{r}^2$, where $\bar{R}$ and $\bar{r}$ are mean radii of the large and small hydrometeors respectively. Conservation of charge therefore demands that

$$f\sim4\left(\frac{\bar{r}}{\bar{R}}\right)\frac{L}{C}.$$

If we take $\bar{r}=30$ μm, $\bar{R}=1$ mm and $L=C$, we find that $f\approx0.14$, which we see from Fig. 1 to be about the lowest permissible value. Some alleviation is afforded by the facts that (i) larger, less mobile droplets are more likely to separate from precipitation particles after colliding with them than are smaller droplets; consequently, the effect of E on velocity change will be somewhat reduced; (ii) the ratio L/C will generally be less than 1, thereby decreasing the minimum allowable value of f. It is worth noting that since all calculations presented have been for $L/C=1$ the parameter f can be replaced in all the tables and Fig. 1 by the more flexible parameter fC/L, i.e., the effect on J of reducing f by a given factor is exactly compensated by increasing C/L by the same factor.

Further amelioration of the severe limitation imposed by our calculations on precipitative mechanisms may be afforded by the following argument. If we consider that there exists a *variation* in E throughout the volume of the thundercloud, and not a uniform field as assumed in our model and by other workers, E may continue to increase in a region of strong field, even though the local current density is negative, because of the overriding effect of positive current densities in more widespread surrounding regions of somewhat lower field. Evidence for spatial inhomogeneities in electrified clouds of all types, including thunderclouds, has been provided by Reiter (1972) and Winn *et al.* (1974). A crude assessment of the possible importance of this effect may now be made.

We assume that the electrically active volume of the cloud is cylindrical, with its axis vertical. The field within this cylindrical region, of diameter D, is assumed to be constant, of strength E_D, except that within a central cylindrical region of diameter d a higher field E_d exists. Typical dimensions for D and d might be 1 km and 200 m, respectively. If we take $p=50$ mm h^{-1}, $Q=-2ER^2$ and $f=0.2$, we see from Fig. 1 that if the background field $E_D=0.5$ kV cm^{-1} the current density is about 4×10^{-2} μA m^{-2}.

TABLE 8. Values of J_{max}, E_{max}, E at J_{max} and k' for various values of p_0, L, k and x, for hailstones: $Z=5.5$ km, $L=C$, $f=1$, $r_c=50$ μm.

p_0 (mm h^{-1})	L (g m^{-3})	x	k	k'	J_{max} (μA m^{-2})	E at J_{max} (kV cm^{-1})	E_{max} (kV cm^{-1})
30	1.56	1	0.1012	4.47×10^{-5}	0.05	1.2	2.4
		2	2	4.46×10^{-2}	0.12	1.8	3.0
		3	26.809	3.98×10	0.15	1.8	3.0
50	2.38	1	0.1122	4.02×10^{-5}	0.08	1.2	2.5
		2	2	4.02×10^{-2}	0.18	1.8	3.3
		3	24.144	3.58×10	0.25	2.4	3.3
100	4.21	1	0.1290	3.50×10^{-5}	0.14	1.2	2.5
		2	2	3.50×10^{-2}	0.33	2.4	3.3
		3	21.036	3.12×10	0.45	2.4	3.3

Consequently, since the ratio of the cross-sectional areas in the high- and low-field regions is 1/25, the field E_d can increase to a value of 5.7 kV cm^{-1} when the charging current becomes about -1.0 μA m^{-2}. Winn *et al.* have shown that the background fields and scale of fluctuations within thunderclouds can be close to those assumed. We may therefore conclude that the existence of variations in electric field removes some of the restrictions, previously discussed, imposed upon precipitative mechanisms.

These calculations illuminate the point that the values of E_{max}, the predicted maximum field produced by a precipitative process, E_c, the minimum field in which corona can be produced from hydrometeors, and E_1, the field required to sustain a discharge, once initiated, may be very close to each other. Since the production of lightning by this process depends critically upon the relative values of these fields it is imperative that further research be conducted in order to establish them more accurately. It is also clear that we need much more information concerning the distributions of charge on hydrometeors, the electrical structure of thunderclouds, and the relation between precipitation development and the growth of field. Our tentative conclusion that precipitation processes are capable of explaining thundercloud electrification in the range of circumstances described above by no means proves that they are responsible for the production of lightning.

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