The Evolution of Drop Spectra Due to Condensation, Coalescence and Breakup

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ABSTRACT

A numerical model of warm rain processes incorporating activation of cloud condensation nuclei, drop growth by condensation and stochastic coalescence, and drop breakup is described. A collisional breakup model is compared to a spontaneous disintegration model and found not only to dominate over the spontaneous disintegration model but to produce an exponential spectrum in fair agreement with average observed drop spectra. The steady-state spectrum was found to be quite insensitive to the number of satellite drops formed by collision or their size distribution.

The effect of the finite-difference solution to the collection and breakup equations is analyzed. A properly stochastic formulation for finite time steps is presented but found to differ only slightly from the simpler, "discrete" formulation. Time steps of 5 s and 45 size categories are found adequate to describe the essential quantitative features of more intensive treatments.

The model results were found to be somewhat sensitive to values used for the collection efficiency.

1. Introduction

Attempts to numerically simulate the conversion from the typically narrow, condensation-produced spectra (e.g., Neiburger and Chien, 1960, or Warner, 1969) to the drop spectra observed in natural rainfall (e.g., Marshall and Palmer, 1948) have been limited by their exclusion of various microphysical processes known to be important. Telford (1955) demonstrated the necessity of considering a stochastic collection model as opposed to the more simplistic continuous collection model. The importance of the condensation process on coalescence growth was demonstrated by Kovetz and Olund (1969) and more recently by Leighton and Rogers (1974). Srivastava (1971) combined the stochastic coalescence model with a drop breakup model based on Komabayashi *et al.* (1964) but notes that his computed drop distributions are "flatter" than observed distributions. Danielsen *et al.* (1972) included condensation, stochastic coalescence, breakup and sedimentation in a mixed phase cumulus model but parameterized important aspects of the condensation and breakup processes. Silverman and Glass (1973) include the activation of cloud condensation nuclei (CCN) and calculation of supersaturation in a warm rain model but obtain drop spectra similar to Srivastava's, using a similar drop breakup model.

This paper deals primarily with coalescence and breakup in a time-dependent, parcel framework. The model presented adopts the activation of cloud condensation nuclei and condensation growth and calculation of supersaturation described by Young (1973, 1974), and formulates several models of drop coalescence and breakup. Comparison of the predictions of this model utilizing "continuous," "discrete" and "Poisson" formulations of collection as well as "collision" and "spontaneous disintegration" breakup models are made. The sensitivity of the model predictions to the finite-difference solutions of the equation set is analyzed so as to reduce the computer time required while maintaining the essential features of the more intensive numerical solutions.

In applying these results to real clouds, one should bear in mind that this model neglects sedimentation and mixing. The intent of this paper is to examine the interaction between condensation, coalescence and breakup more closely than done previously, so as to assure a better physical understanding before placing this forformulation in a more comprehensive dynamical framework.

2. Formulation of the model

This model incorporates the activation of CCN, the diffusion growth or evaporation of drops, and their mutual coalescence and collisionally-induced breakup in the presence (or absence) of vertical motions. The calculation of supersaturation, the activation of CCN, the diffusion transfer of mass to or from the drops, and a discussion of the numerical techniques are given by Young (1974). It should be noted that the continuous bin technique employed eliminates numerical spreading while conserving number and mass. The calculation of supersaturation is based on an iterative procedure (predictor corrector) whereby that value of supersaturation is determined which balances the water

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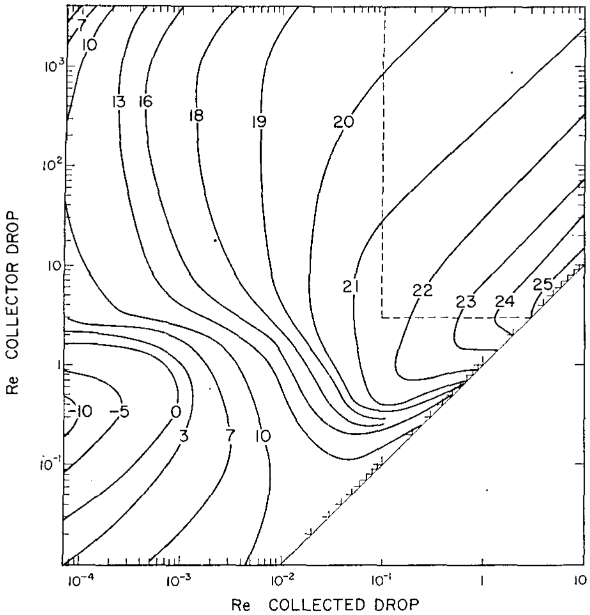

FIG. 1. Isopleths of 10 log E_{ij} (percent) as a function of collector and collected drop Reynolds number. Source of collection efficiency values is taken from a compilation by Young (1973).

mass condensed on the drops with the rate at which water vapor is being made available in the updraft and concomitant changes in the supersaturation.

The expected number of drops collected by a single collector drop within a given time step (Δt) is given as

$$\lambda = \Delta n_j/n_i = \pi r_i^2 n_j E_{ij} |v_i - v_j| \Delta t, \qquad (1)$$

where n_i and n_j refer to the concentrations of collector and collected drops, v_i and v_j are their respective fall velocities, r_i the collector drop radius, and E_{ij} their mutual collection efficiency. In Fig. 1 E_{ij} is shown as a function of the Reynolds number of the collected and collector drops, based on the compilation made by Young (1973). This treatment allows E_{ij} to vary with height in the atmosphere in a realistic fashion.

All interactions between i and j drops, $j<i$, are considered; Δn_j drops are removed from the j category and their mass is added to drops in the i category. In the following section, three different methods of adding this mass are discussed. However, elsewhere in this paper, the "discrete" formulation is employed and mass is added in the following manner. If the expected number of collection events per collector drop (λ expressed in deciminal form as $k.d$) is <1, Δn_j of the collector drops receive a mass of one collected drop and the rest receive no mass. If the expected value is $\geqslant 1$, all collector drops receive a mass corresponding to k collected drops with a fraction ($0.d$) of the collector drops receiving an extra drop.

The breakup of drops is based on the theoretical and experimental work of Brazier-Smith *et al.* (1972, 1973) who showed that the collision and coalescence of drops >250 μm radius is frequently followed by their separation and the formation of satellite drops. The coalescence of two drops is considered to result in separation if the rotational energy (due to an off-center collision) exceeds the surface energy of the combined pair. They formulate this probability as

$$p = 1 - \frac{2.4\sigma(1+\gamma^3)^{11/3}[1+\gamma^2-(1+\gamma^3)^{\frac{2}{3}}]}{r_j \rho_w (v_i - v_j)^2 \gamma^6 (1+\gamma)^2},$$

where $\gamma = r_i/r_j$. They give the fraction of mass lost to satellite drop production as

$$\frac{m_s}{m_i + m_j} = 0.12\gamma^3/(1+\gamma^3)^2,$$

with an average of three satellite drops produced per breakup.

On the basis of their experiments, Brazier-Smith *et al.* assume that the number of satellite drops produced is independent of drop size. Although they assume three equal-sized satellites, the mass lost to satellite drop production is estimated to be partitioned among the three fragments as 1.6%, 11.4% and 87%.

Examination of Fig. 4 from Brazier-Smith *et al.* (1972) indicates the satellites are not of equal sizes. Knowing the sizes of collector and collected drop pairs used to generate the histogram of satellite drop sizes in Fig. 4 of Brazier-Smith *et al.* (1973), it is evident that the smallest fragment would need to be on the order of 1–2% of the mass loss and the largest would need to contain most of the mass in order to produce such a histogram. The sensitivity of the model results to this assumption and the assumed production rate of three satellites per separation is analyzed in a following section.

It is assumed that the collection efficiency (E_{ij}) includes those interactions which subsequently result in separation, and the percentage mass lost to satellite drop production is assumed the same for both collector and collected drops. In a subsequent section, this treatment of breakup is compared to that employed by Srivastava (1971).

3. Comparison of continuous, discrete and Poisson models of collection[2]

The essential differences between continuous and stochastic collection models are described by Telford (1955) and Berry (1967). The differential form of the collection equation as formulated properly describes a stochastic process as pointed out by Scott (1967) and Gillespie (1972), although this has been questioned in the past (e.g., Warshaw, 1967; Long, 1971). Recently, Gillespie (1975) has shown that the "stochastic" for-

[2] These models do not refer to different physical processes but rather to differences in our attempts to simulate the collection process in a realistic fashion.

mulation actually describes the evolution of the *expected* spectrum and shows how one may predict the magnitude of random deviations from the expected spectrum.

Here the pertinent question is whether the time steps commonly employed in the finite-difference solution to the collection equation result in significant differences between a formulation which retains the essential statistical nature of the process (Poisson model) and the one which is commonly employed (discrete model).

The use of a finite-difference solution can introduce errors in the stochastic model as Δt becomes large enough so that the expected number of collections defined by (1) is significantly different from the collection probability (Gillespie, 1972). These errors arise as the probability of multiple-collection events becomes significant.

This problem should be treated by considering the random spatial distribution of the drops and the probability distribution of all (multiple) collection events for each time step. Gillespie describes the random positioning of droplets using a binomial distribution; however, since the numbers involved are large and the probability of droplets overlapping is negligible, the Poisson distribution is more convenient (Brownlee, 1966). Thus, the probability distribution of collection events (x) for given collector drops may be described by

$$p(x)=\frac{\lambda^x}{x!}e^{-\lambda}. \qquad (2)$$

Gillespie points out that variations in the total (mean) population within a well-mixed volume of cloud due to the stochastic nature of the collection process are generally negligible in comparison to the variations due to the random positioning of the drops, provided the well-mixed volume is considerably larger than the swept volume for a given collector. Thus, this treatment properly describes the behavior of the mean population under the stochastic formulation.

By way of illustration (see Fig. 2), consider that each collector drop, on the average, collects 0.5 drops during a given time step. The continuous model says that each collector drop collects a mass equal to 0.5 that of a collected drop. This implies that the mass of collected drops is uniformly distributed on a molecular scale. The discrete model says that half of the collector drops receive a mass equal to that of a single collected drop and the remainder collect none. This implies the drops are collected in discrete units but their distribution in space is uniform (since their arrival rate is uniform). The Poisson model says that the arrival of drops at the collector drop is Poisson distributed [Eq. (2)] with an expected value (λ) of 0.5 and $p(x)$ is the fraction of collector drops which receive x collected drops. Under the Poisson model then, 60.7% collect no drops, 30.3% collect 1 drop, 7.6% collect 2 drops, 1.3% collect 3 drops, etc.[3] This implies that the collected drops are distributed randomly in space. It may be seen that the discrete and Poisson formulations are equivalent as $\Delta t\rightarrow 0$.

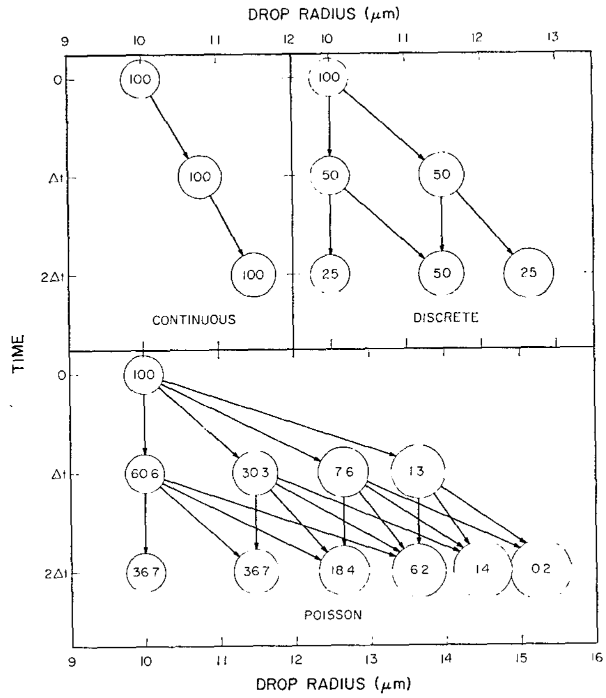

FIG. 2. The growth of 10 μm radius drops collecting 8 μm radius drops for the continuous, discrete and Poisson collection models. The expected number of collection events per collector (λ) within a given time step (Δt) is taken as 0.5. Numbers within circles refer to percent of drops of that size; arrows show growth paths. The Poisson model considers all event occurrences, $p(x)>10^{-4}$, although the diagram only shows a fraction of all interactions actually considered.

A comparison between the continuous, discrete and Poisson models using a time step of 5 s is shown in Fig. 3. The initial conditions assume a constant updraft velocity of 3 m s^{-1}, a cloud base temperature of +15°C, and a maritime CCN spectrum based on Fitzgerald (1972). For the discrete and Poisson models, the condensation process dominates for the first 15 min, after which a rapid conversion to precipitation sizes through

[3] In order to conserve mass, since the Poisson distribution is open-ended, once the event probability is $<10^{-4}$, the remaining mass of collected drops is partitioned among the remaining fraction of collector drops. This remaining fraction of collector drops for all $x\geqslant x^*$, where $p(x^*)<10^{-4}$, is

$$\delta n_i=1-\sum_{x=0}^{x^*-1}\frac{\lambda^x}{x!}e^{-\lambda},$$

and the remaining mass of collected drops per collector drop is

$$\delta m_j=m_j\left[\lambda-\sum_{x=1}^{x^*-1}\frac{\lambda^x}{(x-1)!}e^{-\lambda}\right].$$

Thus, the remaining fraction of collector drops each receive a mass of $\delta m_j/\delta n_i$.

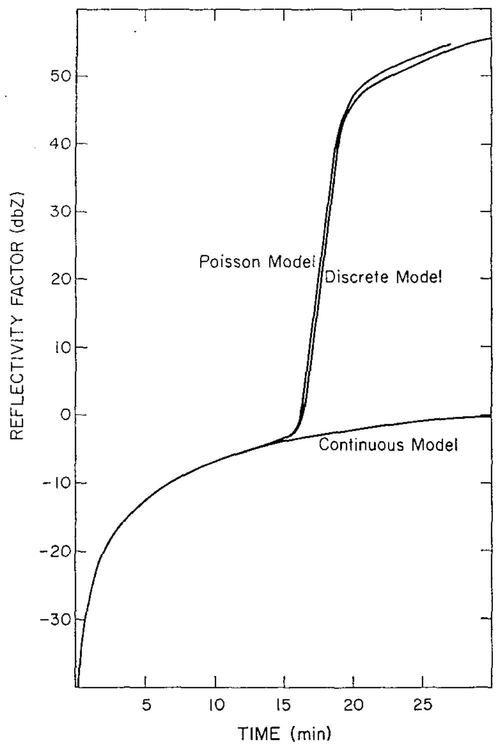

FIG. 3. Comparison of radar reflectivity factor as a function of time for continuous, discrete and Poisson models of collection. Initial conditions assume a cloud base temperature of +15°C, an updraft of 3 m s^{-1} and a maritime CCN spectrum. A time step of 5 s was used.

coalescence occurs. This is associated with a rapid rise in the radar reflectivity factor. The Poisson formulation predicts this conversion to occur ~7 s earlier than does the discrete formulation. This difference is felt to be insignificant, especially since the required computer time is increased twofold for the Poisson formulation. Differences in the drop spectra produced by these two models are extremely slight although, as expected, the Poisson model produces slightly more of the largest drops earlier than does the discrete model.

4. Comparison of breakup by collision and spontaneous disintegration

Previous treatments of drop breakup have considered only the fragmentation of large drops ($r>1.5$ mm), prescribing the spontaneous disintegration of all drops in excess of a given size (e.g., Danielsen *et al.*, 1972) or a disintegration rate strongly dependent on drop size alone (e.g., Srivastava, 1971). Since these treatments prescribe a single-body breakup, this is termed "spontaneous disintegration" as contrasted to the collisionally-induced breakup model as presented by Brazier-Smith *et al.* (1972, 1973), here termed "collision." The treatment of spontaneous disintegration follows Srivastava's treatment which is based on the observational data of Komabayashi *et al.* (1964); that of collision breakup is described in Section 2.

These two models of drop breakup were treated separately and then combined to determine whether either of these models would dominate and whether or not either (or both combined) would produce a physically realistic, i.e., exponential, spectrum. As in the previous section, the model's initial conditions were an updraft velocity of 3 m s^{-1}, a cloud-base temperature of +15°C and a maritime CCN spectrum.

The drop spectra resulting after 30 min for each of the three models (two separately and one combined) of drop breakup are shown in Fig. 4. It is evident that the collision breakup model dominates over the spontaneous disintegration model since differences between the combined and collision-only models are negligible. The drop spectrum resulting from the spontaneous disintegration-only model is markedly dissimilar from observed drop spectra whereas the models with collisional breakup produce realistic, exponential spectra.

This spectrum may be compared to the $n=0.08\ e^{\Lambda D}$ relationship given by Marshall and Palmer (1948). The rainfall rate (R) corresponding to the collision-only model at 30 min is 225.55 mm h^{-1}, which gives $\Lambda=13.1$ cm^{-1} $[=41\ R^{-0.21}]$. A linear approximation to the portion of the curve shown in Fig. 4 for $r_w>500$ μm, gives a slope of 9.4 and an intercept of 0.076. Since sedimentation would be expected to remove more large drops from the parcel than small drops, the slope of the distribution would be expected to increase under more realistic conditions. This would improve the agreement

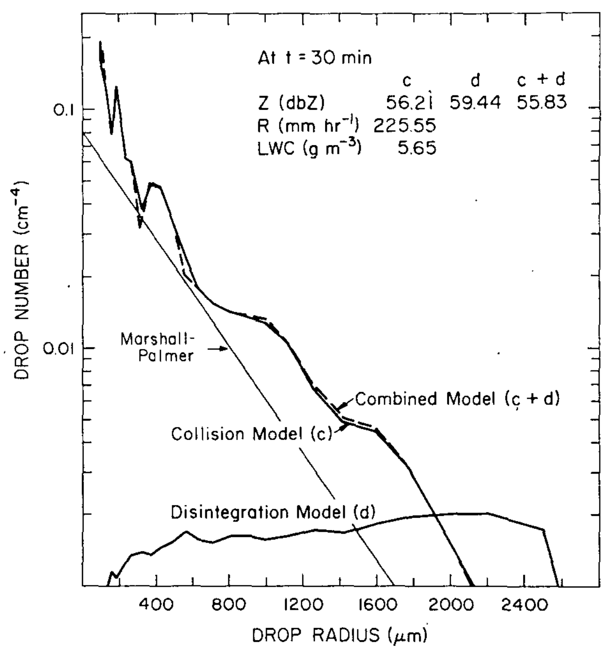

FIG. 4. Drop spectra at 30 min resulting from collision and disintegration models separately and combined. Initial conditions assume a cloud base temperature of +15°C, an updraft of 3 m s^{-1} and a maritime CCN spectrum.

with the Marshall-Palmer relationship. This agreement is considered significant, in particular with regard to agreement in the intercept which is not sensitive to sedimentation.

List (1974) doubts that the spontaneous disintegration mode is significant, citing his observation that drops as large as 5 mm radius can be floated stably in a vertical air stream. He cites three collisional breakup modes, namely necks (as treated here), sheets and discs. The two latter modes are noted to occur only for large drops ($r_w > 3$ mm). Since the inclusion of the spontaneous disintegration model, which is similarly restricted to drops >3 mm radius, has little influence on the raindrop spectrum, it is anticipated that the sheet and disc breakup modes are similarly unimportant and that the essential features of raindrop spectra may be simulated using only the collision-breakup model as outlined. Brazier-Smith *et al.* (1973) come to similar conclusions regarding the influence of the breakup of very large drops on rainfall rates.

The drop spectra resulting from collisional breakup were found to be highly insensitive to the number of fragments produced or to their size distribution. Simulations were run using from 2 to 5 satellite drops per breakup and varying their distribution from equal-sized satellites to one in which the smallest fragment contained only 0.001 of the total satellite mass. Differences in rainfall rate at 30 min were <2% and differences in drop concentrations in the large drop size generally <5%.

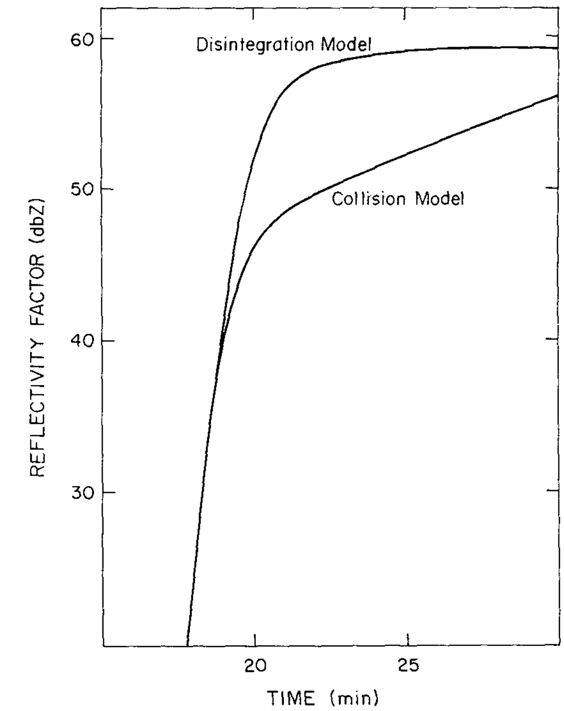

FIG. 6. Comparison of the reflectivity factor as a function of time for collision and disintegration breakup models. Initial conditions assume a cloud base temperature of +15°C, an updraft of 3 m s⁻¹ and a maritime CCN spectrum.

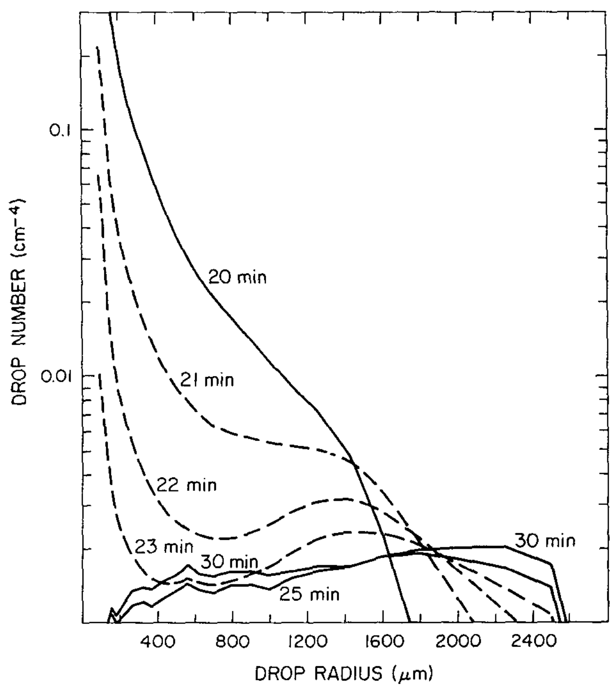

FIG. 5. Drop spectra for the disintegration breakup model at different times. Solid curves give spectra at 5 min intervals; dashed curves at intermediate times. Initial conditions assume a cloud base temperature of +15°C, an updraft of 3 m s^{-1} and a maritime CCN spectrum.

Since the size spectrum resulting from the disintegration model shown in Fig. 4 is considerably flatter than those obtained by Srivastava (1971) or those observed, it is of interest to consider the time evolution of the drop population under this model. Fig. 5 shows that the relatively exponential spectrum at 20 min rapidly collapses in the radius range 400–1000 μm. Actually, the percentage removal of drops increases with decreasing size <1200 μm. This is expected as the swept volume for these drop sizes increases with decreasing collected drop size. The production of drops via this breakup model adds an increasing number of drops at smaller sizes and nearly offsets the effect of increasing sweep-out, resulting in a nearly uniform size distribution. The spectrum at 21 min is quite similar to those presented by Srivastava, although in this model the spectrum continues to evolve, reaching a significantly different steady-state solution. The reasons for these differences are not obvious although these models are not strictly comparable. Inclusion of sedimentation would be expected to remove large drops preferentially, i.e., such spectra produced under the spontaneous disintegration model would look more realistic with sedimentation included. However, such spectra characteristically lack drops in the size range 100 to 400 μm radius (e.g., Tag, 1974).

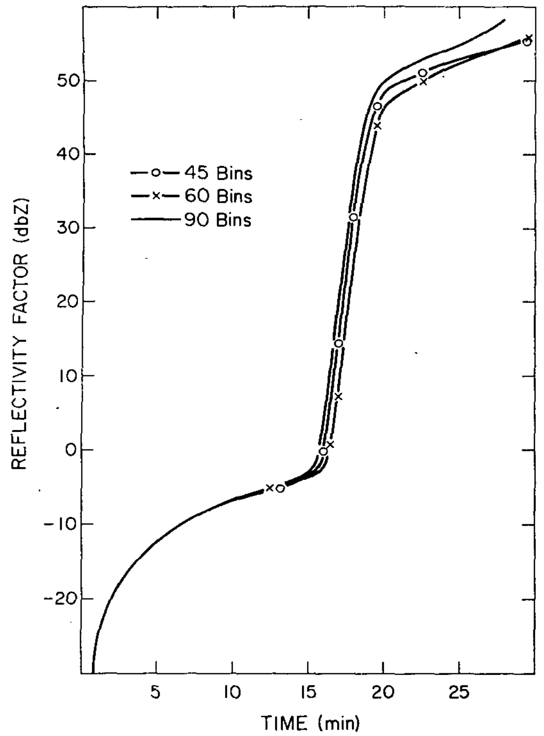

FIG. 7. Comparison of reflectivity factor with time for bin arrays containing 45, 60 and 90 categories. Initial conditions assume a cloud base temperature of +15°C, an updraft of 3 m s^{-1} and a maritime CCN spectrum.

Fig. 6 compares the reflectivity factor for the collision and disintegration breakup models. Since the collision breakup model produces fragments earlier in the evolution of the drop spectrum, it might be anticipated that this model would predict the conversion to a raindrop spectrum (and high reflectivity) to occur sooner. This is not the case as differences are evident only after much of this conversion has taken place and then the disintegration model yields higher values of reflectivity since it concentrates more mass in the largest drop sizes.

5. Analysis of model sensitivity to the finite-difference solution

Errors in the collection treatment in this model arise from two aspects of the finite-difference solution, namely those errors resulting from the finite time step and those errors resulting from the finite bin width. In addition, the numerical formulation which treats coalescence and breakup simultaneously does not provide for the possibility that a single drop will experience both a breakup and a collection event within a given time step.

a. Time step sensitivity.

Although the model does consider multiple-collection events within a given time step, including multiple collection involving collected drops of different sizes, the progressive removal of collected drops alters the concentration of such drops as seen by successive collector drops. This model assumes that the removal of collected drops does not significantly alter their concentration within a given time step. In addition, the assumption that a given drop does not undergo both breakup and collection events within a given time step may be tested by determining the time step sensitivity.

The time step sensitivity was evaluated on the basis of the time required to produce the rapid rise in reflectivity factor for the warm, maritime updraft modeled in previous sections. Time steps of 1, 2, 5 and 10 s were compared. Taking the 1 s time step as standard, the remaining steps resulted in delays of 4, 20 and 46 s in producing this rise. The 20 s delay for the 5 s time step is considered acceptable and this time step was employed for all other sections of this paper.

b. Bin size sensitivity.

Although the continuous bin method employed in this model assumes a linear distribution of drops within a bin for the purposes of drop growth (by condensation or collection), values used for collection efficiency, fall velocity, collected drop mass and radius of the collector drop are single-valued within a given bin, based on mean *or* midpoint values. Errors may be expected to arise from this simplification. However, a potentially more serious effect is that this treatment does not permit collection between drops within the same bin although the actual or physical range of drop fall velocity for these drops would normally permit such interactions. While such errors may be reduced by employing more bins, the computer time required increases roughly as the square of the number of bins.

Errors may also be expected to result when the width of maxima (or minima) in the drop spectra are on the same order as the bin width. This is expected to be important at small drop sizes where the condensation-produced drop spectra are typically narrow and peaks are normally expected. Thus, the various arrays of bin sizes employed in this analysis have a relatively higher density of bins (on a logarithmic scale) at smaller radii than at larger.

Three arrays of bin sizes were employed for this analysis, containing 45, 60 and 90 bins. The 60-bin array has more bins at larger sizes and fewer bins at small sizes than the 45-bin array. The 45-bin array has greatly abbreviated detail at larger sizes with 7 bins between 1 and 5 mm radius whereas the 90-bin array contains 29 such bins.

The warm, maritime updraft situation previously modeled was also used to test the bin size sensitivity. The results in Figs. 7 and 8 show the change in the reflectivity factor with time and the resultant drop spectra at 28 min. From Fig. 7, it is evident that differences are small during the condensation and conversion phases of the drop spectrum evolution. However, the

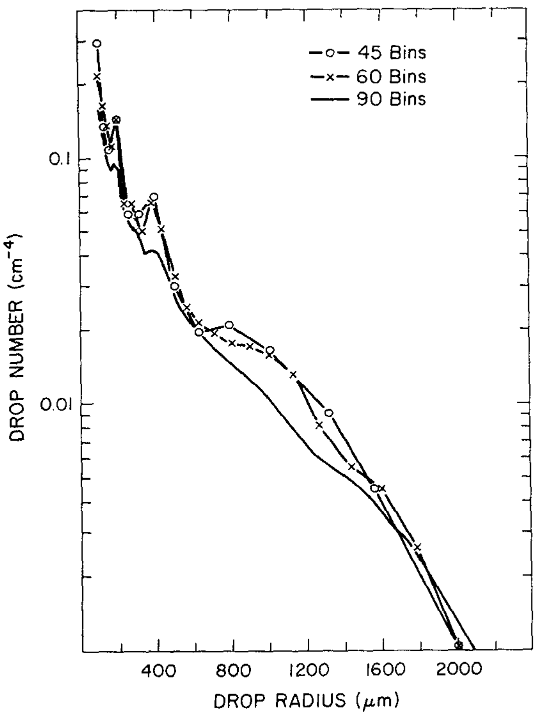

FIG. 8. Drop spectra at 28 min resulting from use of arrays containing 45, 60 and 90 bin sizes. Initial conditions assume a cloud base temperature of +15°C, an updraft of 3 m s^{-1} and a maritime CCN spectrum.

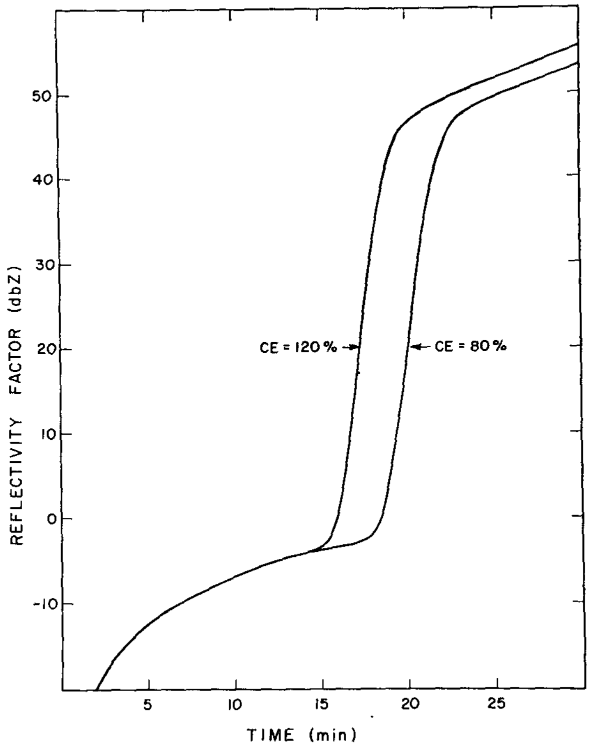

FIG. 9. Comparison of reflectivity factor as a function of time for model using values of collection efficiency of 80 and 120% of those given in Fig. 1.

90-bin array produces a somewhat higher reflectivity factor as the evolution becomes dominated by the coalescence and breakup interactions. At this time (28 min), the drop spectrum calculated using 90 bins has slightly more drops >1.8 mm radius and rather fewer drops between 0.7 and 1.6 mm radius than the spectra calculated using 45 and 60 bins. The 45- and 60-bin arrays do reproduce the essential features found using the 90-bin array with greatly reduced computational time; the 60-bin array is used in all other parts of this paper. Silverman and Glass (1973) find that 45 bins provide an optimal balance between computational time and reliability of results.

6. Analysis of model sensitivity to uncertainties in the collection efficiency

The sensitivity of the model results to uncertainties in values used for collection efficiency was examined by both adding and subtracting 20% to the values of collection efficiency[4] (in percent) obtained from Fig. 1. Higher values of collection efficiency would be expected to produce the coalescence conversion more rapidly, as demonstrated in Fig. 9. Here, a difference of 3 min is noted in reaching 20 dBZ over a range of collection efficiencies which probably represents rough 90% confidence limits. This reflects changes in collection efficiencies for drops on the order of 15 to 100 μm radius.

Changes in collection efficiencies for mutual large drop interactions would be expected to have relatively little influence on the drop spectra since the probability of breakup through a given interaction is constant. That is, drops may grow more slowly but they lose mass through satellite formation less frequently. Changes would be expected for small-large drop interactions, i.e., those interactions which produce growth without possibility of breakup. Thus, higher collection efficiencies should produce fewer drops total but more large drops.

The drop spectra after 30 min for the 0.8 and 1.2 times collection efficiency cases shown in Fig. 10 are similar in shape although the slope of the spectrum for the 0.8 case is noticeably steeper. The total drop concentration for the 0.8 case is more than twice that for the 1.2 case.

This analysis demonstrates that model predictions of warm rain processes and raindrop spectra are not overly sensitive to uncertainties in collection efficiencies.

7. Discussion and summary

The model formulated improves on the previous models of condensation and coalescence of Kovetz and Olund (1969) and Leighton and Rogers (1974) and the

[4] Collection efficiency values were not permitted to exceed geometric values.

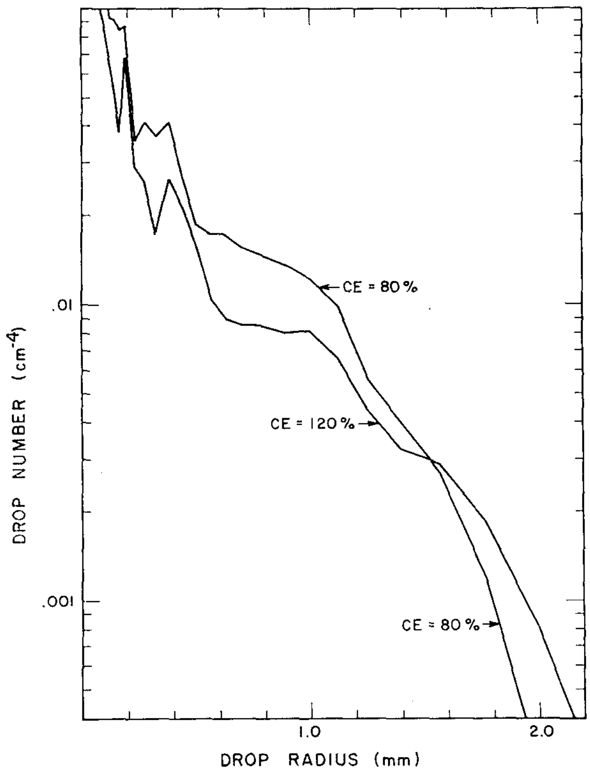

FIG. 10. Comparison of drop spectra of 30 min obtained using collection efficiencies of 80 and 120% of those given in Fig. 1.

model of coalescence and breakup of Srivastava (1971) by improving the condensation and breakup treatments and combining these with a "discrete" collection treatment and the activation of CCN. This enables modeling of warm cloud processes from the activation of CCN, through condensation and coalescence growth of droplets and their breakup.

The "stochastic" nature of the finite-difference form of the collection equations is examined in terms of "discrete" and "Poisson" formulations. Scott (1967) and Gillespie (1972, 1975) have demonstrated that the differential form of the collection equation is properly stochastic. However, when finite time steps are employed in the solution, the numerical formulation commonly employed tacitly assumes the drops to be uniformly distributed in space (a constant arrival rate at the collector drop) within the time step (discrete formulation). Since drops are expected to be randomly positioned within the cloud volumes being considered (Gillespie, 1972), their arrival rate at the collector drop is properly given by a Poisson distribution with an expected value as calculated for the discrete case (the average number of collected drops per collector).

Such a Poisson model may be expected to produce larger drops more rapidly than the discrete model, the discrepancy being dependent on the time step used and approaching zero as $\Delta t \rightarrow 0$. In the comparative case modeled, a 5 s time step was used and the stochastic model predicted the cloud-to-raindrop spectrum conversion to occur $\sim$7 s earlier than the discrete model. Since the computational time involved is roughly doubled for the Poisson model, the use of the discrete model is felt to be justified.

Two conceptually different models of drop breakup are considered, namely the collisional breakup described by Brazier-Smith *et al.* (1972, 1973) and the "spontaneous disintegration" described by Komabayashi *et al.* (1964) and employed by Srivastava (1971). The "steady-state" spectrum produced under the spontaneous disintegration model is even flatter than those obtained by Srivastava, casting severe doubts as to the sole applicability of this model of drop breakup in treating the drop spectra evolution. Multilevel models utilizing spontaneous disintegration for drop breakup (e.g., Tag, 1974) tend to exhibit exponential spectra for sizes >1 mm radius but exhibit unrealistically low concentrations of drops between 100 and 500 μm radius. In contrast, the collisional breakup model produces a "steady-state" spectrum which is exponential in form and is in fair agreement with spectra reported by Marshall and Palmer (1948), particularly at sizes <1 mm radius, although it is anticipated that the inclusion of sedimentation would have improved this agreement still further.

The collisional breakup model may be visualized as a random walk problem. The size history of a single, large drop may be described by discrete jumps in mass, coalescence resulting in a size increase dependent on the collected drop mass and collisional breakup resulting in a size increase dependent on the mass lost to satellite drops. Since the interaction between two large drops of similar size is relatively rare and frequently results in separation, step-wise changes in size are relatively small. Thus, a single large drop in a population of drops tends to wander up and down the size spectrum. This tends to reduce the effect of size sorting by sedimentation since those sizes removed would tend to be replaced by random changes in individual drop sizes with the exponential distribution representing the steady-state spectrum.

The finite-difference solution of the collection equation introduces potential sources of error. In this model, errors result from the successive removal of collected drops during a given step, the failure to consider interactions between drops in the same bin, and the failure to consider the possibility that a given drop will undergo both breakup and collection events within a given time step. An analysis of the sensitivity of the model results to time step and bin size indicated that 5 s time steps and 45 bins were adequate to preserve the essential quantitative features of the more intensive treatments.

The model results were found to be markedly insensitive to the numbers and size distribution of satellite drops formed by collisional breakup. Varying the collection efficiencies over ranges felt to be representative of the uncertainties in these values, produced a 3 min change in the time required to reach a 20 dBZ "first echo" and changed the slope of the resultant raindrop spectrum. These uncertainties are not felt to be of great concern regarding development of more complex models of warm rain processes.

Subsequent papers in this series will examine more closely some aspects of the microphysical interactions within the parcel framework and then extend this treatment to include sedimentation in a multi-level framework.

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