Collision Efficiency of Water Drops in the Atmosphere

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(Manuscript received 18 October 1973, in revised form 17 March 1975)

ABSTRACT

A study has been made of the growth through collision of water drops in the atmosphere. The method of superposition of flow fields obtained from the numerical solution of the Navier-Stokes equations was used for the calculations, and only inertia, gravity, and drag force were considered.

The calculated linear collision efficiency is significantly less than the geometric collision efficiency between a large collector drop and a small collecting drop because of the effect of hydrodynamic forces. The linear collision efficiency is substantially higher than the geometric collision efficiency between similarly sized drops because of the wake effect.

To verify the validity of the calculations, the analytical results were compared with available experimental data. Satisfactory agreement was obtained for most drop sizes. It is concluded that in the absence of electricity and turbulence the dominant factor in the formation of precipitation is the collisional growth of similarly sized drops.

1. Introduction

The method of growth of water drops in the atmosphere can be classed as two types: condensational and collisional. Condensational growth, in which water vapor condenses on atmospheric aerosols, is the dominant mechanism for the growth of water drops with radii \( \leq 20 \mu m \). The Wegener-Bergeron-Findeisen process, in which the ice crystal is growing at the expense of supercooled water droplets in clouds as discussed in standard textbooks such as Fleagle and Businger (1970), is also considered as condensational growth of ice crystals. Collisional growth, in which accretion processes operate, is responsible for the growth of water drops with radii \( > 20 \mu m \). This study is concerned with the collisional growth of water drops, because it is directly related to the understanding of the precipitation phenomenon in clouds as well as to the scavenging process of pollutants.

Analytical studies of collisional efficiency have been made by using modified Stokes flow solutions, modified Oseen flow solutions, or available numerical solutions of the Navier-Stokes equations. Owing to the nonlinear nature of the flow field surrounding a relatively large drop, which is responsible for the collisional growth, considerable discrepancy occurs in the literature for the calculated collision efficiency. Extensive effort has been made to improve the calculation procedures for collision efficiency by improving the empirical relations for drag coefficients. Based on an improved method developed by Lin and Lee (1973) for flow field calculations, this study shows that the unsatisfactory flow field surrounding a moving water drop is responsible for the high order of magnitude of the discrepancy in collision efficiency, while the improvement of drag formulas can only account for a very small percentage of the discrepancy existing in the literature.

2. Historical background

Collision takes place when one moving drop (or collector drop), whose movement is caused by gravitational or other forces, collides with other drops (or collecting drops) which happen to be in its path. To examine the probability of collision, the following equation for linear collision efficiency is used:

\[
Y_r = y_r / R.
\]

(1)

Here \( R \) is the radius of the collector drop, and \( y_r \) is the radius of the collisional cross-sectional area within which any collecting drop of radius \( r \) will collide with the collector drop. Fig. 1 shows schematically the nomenclature used in this study. Ideally, the collisional cross-sectional radius should be equal to the sum of the radii of the collector and the collecting drops if the drops are not surrounded by air. The nondimensionalized form of this ideal collision cross-sectional radius is known as the geometric collision efficiency. However, as a result of the forces generated by the relative motion between the water drop and the air, laboratory observations of linear collision efficiency often differ substantially with the geometric collision efficiency. These phenomena were reported by Telford et al. (1955), Schotland and Kaplin (1956), Telford and Thorndike

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(1961), Woods (1965), and Beard and Pruppachar (1968). To understand the physical processes of the collisional growth of water drops, many investigators have conducted theoretical studies along one or the other of the following two approaches.

a. Evaluation of forces between two drops

Hocking (1959) analyzed the Stokes equations and calculated the force exerted between two drops to determine collision efficiency. Davis and Sartor (1967) and Hocking and Jonas (1970) extended Hocking's approach to include rotation of the water drops. However, because they assumed that the inertial force is negligibly small in Stokes equations, their approach can only apply to cases of practically zero Reynolds number (Re~0). For freely falling water drops in a standard atmosphere, the drop sizes and terminal falling velocities vs Reynolds numbers are shown in Fig. 2. It can be seen that a 30 µm drop has a free fall velocity of 10 cm s⁻¹ that corresponds to a Reynolds number of 0.4. The effect of inertial force on collision efficiency was considered by Klett and Davis (1973) using a modified Oseen's equation made by Carrier (1953). Their results indicated that the collision efficiency is considerably larger for approximately equal size drops. However, due to the inherent limitation of the Oseen's equation, this method is only valid for relatively small drops.

b. Superposition of flow field

Langmuir (1948) suggested that the flow field around a single drop could be superposed on the flow field of another drop to determine the relative motion between the two drops. Pearcey and Hill (1956) developed this approach by using Goldstein's (1929) solution of Oseen's (1910) linearized equations for flow around a spherical drop at a very low Reynolds number (Re<1). Shafrir and Neiburger (1963), Shafrir (1965) and Neiburger (1967) improved the superposition approach by using Jenson's (1959) method to obtain solutions of the Navier-Stokes equations for an intermediate Reynolds number range (Re<20). Shafrir and Tzvi (1971) extended the Reynolds number range (Re<104) by using Rimon and Cheng's (1969) method with a modified boundary condition. Beard and Grover (1974) utilized the flow field calculated by LeClair et al. (1970), and obtained the collision efficiency for raindrops colliding with micron size particles. Since the particle is assumed much smaller than the raindrops, the existence of the particle does not alter the flow field around the raindrop (collector). Consequently, no superposition is necessary if the radius ratio between the particle and the raindrop is less than 10%.

A brief summary of the literature is shown in Fig. 3. It is noted that the collision efficiency determined from existing theories does not seem to be consistent. For example, the calculated collision efficiency between a 20 µm collector drop and a 19 µm collecting drop varies from 0 to 2. To resolve some of these discrepancies, the available literature was re-examined. Hocking's approach, which appears to be limited to very small drops, is not practical when applied to pollutant scavenging and weather modification. Langmuir's approach does not appear to have any size restriction for most atmospheric applications provided that the solution of the Navier-Stokes equations can be accurately determined. The availability of high-speed digital computers makes it possible both scientifically and economically to meet this requirement. Le Clair et al. (1970) and Lin and Lee (1973) obtained numerical solutions for flows around spherical drops at steady and transient states, respectively. The accuracy of their solutions has been verified by experimental data on drag coefficients, separation angles, and wake lengths. This study uses Langmuir's approach by superimposing the flow fields obtained from Lin and Lee's (1973) numerical solution of the Navier-Stokes equations.

3. Analysis

Water drops in the atmosphere may be influenced by many parameters, such as gravity, inertia, turbulence, updraft and electrical charge. To understand the significance of each parameter on droplet growth, laboratory experiments have to be conducted in a
controlled environment. Analytical studies are being established so that the effect on droplet growth by each parameter can be verified with experimental data. With a well-established knowledge concerning each individual parameter, the overall effect on droplet growth by the combination of the many physical parameters actually existing in the atmosphere can then be investigated.

If one considers an environment where only gravity, inertia and drag forces are significant, the equations of motion of a freely falling drop can be written as

\[
\frac{dV_L}{dt} = g \left( \frac{C_D \text{ Re}}{24} \right)_L \frac{6\pi\mu R (V_L - U_d)}{M_L},
\]

(2)

\[
\frac{dV_s}{dt} = g \left( \frac{C_D \text{ Re}}{24} \right)_s \frac{6\pi\mu r (V_s - U_L)}{M_s},
\]

(3)

in which the subscripts \(L\) and \(s\) designate the larger collector drop and the smaller collecting drop, respectively. The drop velocity is \(V\) and the flow field velocity \(U\). The mass of the water drop is \(M\), the gravitational acceleration is \(g\), and \(\mu\) is the dynamic viscosity. The drag coefficient \(C_D\) is given by Oseen (1910) for the Reynolds number region of \(0 < \text{Re} \leq 1.6\) as

\[
C_D = \frac{24}{\text{Re}} \left( 1 + \frac{1}{6} \text{ Re} \right),
\]

(4)

and by Lin and Lee (1973) for the Reynolds number

\[
\text{Re} = \frac{\text{Re}_L}{\text{Re}_s}
\]
region of $1.6 < \text{Re} \leq 1000$ as

$$C_D = \frac{24}{\text{Re}} (1 + 0.2207 \text{Re}^b + 0.0125 \text{Re}).$$

(5)

The Reynolds number is evaluated by using the relative velocity between the drop and its surrounding flow field, i.e.,

$$\text{Re}_L = \frac{2\rho |V_L - U_s| R}{\mu},$$

(6)

$$\text{Re}_s = \frac{2\rho |V_s - U_L| r}{\mu},$$

(7)

in which $\rho$ is the density of the medium in which the water drop is traveling.

To calculate the position of each drop, Eqs. (2) and (3) are integrated numerically in both vertical and horizontal directions. The vertical direction is along the axis connecting the center of the collector drop and the center of gravity of the earth. The horizontal direction is normal to the vertical direction from the center of the collector drop toward the vertical axis of the collecting drop. The integration is performed in two steps as discussed by Conte (1965). The first step is an estimation which uses the Adams-Bashforth’s prediction formula

$$V_{n+1} = V_n + \frac{\Delta t}{24} \left[ 55 \left( \frac{dV}{dt} \right)^n - 59 \left( \frac{dV}{dt} \right)^{n-1} + 37 \left( \frac{dV}{dt} \right)^{n-2} - 9 \left( \frac{dV}{dt} \right)^{n-3} \right],$$

(8)

in which the subscripts $n, n+1, \ldots$, denote the time steps of $n, n+1, \ldots$, respectively. The velocity at the $(n+1)$st time step is refined by the Adams-Moulton correction formula

$$V_{n+1} = V_n + \frac{\Delta t}{24} \left[ 9 \left( \frac{dV}{dt} \right)^{n+1} + 19 \left( \frac{dV}{dt} \right)^n - 5 \left( \frac{dV}{dt} \right)^{n-1} + \left( \frac{dV}{dt} \right)^{n-2} \right].$$

(9)

By integrating $V$ with respect to time again, the position of each drop can be determined. These processes of integration are repeated by assuming the distance of the initial separation between the collector and the collecting drops of given sizes. Collision is considered to take place if the closest distance between the two drops is less than 1% of the smaller collecting drop radius. An initial vertical separation distance of 54 radii of the larger collector drop is used because the flow pattern calculations indicate that the flow fields are not noticeably disturbed by the presence of each other. The initial horizontal separation distance is the variable to be determined. The largest initial horizontal separation distance, which results in collision between the collector and the collecting drops, is known as the radius of the collisional cross-sectional area, $y_c$.

4. Results and discussion

The collision trajectory of a collector drop and a smaller collecting drop is shown in Fig. 4. For a collector drop of 30 $\mu$m to collide with a collecting drop of 9 $\mu$m, the collisional cross-sectional radius is about 25 $\mu$m. On the other hand, the 30 $\mu$m collector drop can only collide with a 3 $\mu$m collecting drop within a collisional cross-sectional radius of 1.5 $\mu$m. The hydrodynamic force in the vicinity of the frontal surface of the collector drop pushes the collecting drop away from its falling axis. The collecting drop, as a result of its own inertia, tends to maintain its own course. A larger collecting

![Fig. 4. Collisional trajectories.](image-url)
drop possesses a greater inertia and thereby has a larger collisional cross-sectional radius.

The relative position of two drops of approximately equal size is shown in Fig. 5. For a collector drop of 194 μm to collide with a collecting drop of 183 μm, the collisional cross-sectional radius is 425 μm. In other words, for a pair of water drops with a radius ratio of 0.94, the linear collision efficiency is 2.19, which is larger than the geometric collision efficiency of 1.94. The reason for the high collision efficiency is that the disturbed air in the back of the collecting drop drags the collector drop into its wake region. The disturbed air in the front of the collector drop, in turn, pushes the collecting drop away from it. Because the horizontal distance traveled by the collector drop toward the collecting drop is larger than that traveled by the collecting drop from the collector drop, the pair of drops will collide even though their initial horizontal separation is larger than the sum of their radii. This condition is sometimes referred to as the wake capture phenomenon, which is one of the important processes in precipitation.

The variation of the linear collision efficiency with drop radius ratio for various sizes of collector drops is shown in Fig. 6. It should be noted that the linear collision efficiency is relatively high for drops of approximately equal size. The numerical value increases sub-

Fig. 5. Collision of a pair of water drops.

Fig. 6. Calculated collision efficiency.

stantially when the collector drop \( \geq 30 \mu m \). This value differs from the results of some previous investigators. A comparison of theoretical results with some available experimental data is shown in Fig. 7 for the case of a 75 μm collector drop. It can be seen that the agreement is satisfactory for most of the radius ratio regions except for those drops of approximately equal size. Re-examination of the theoretical results indicates that the vertical distance traveled by a pair of drops of approximately equal size is very long before the collector drop can catch up with the collecting drop. Although this particular situation exists often in nature, conventional laboratory apparatus is limited by its physical dimensions and prevents this phenomenon from being experimentally observed.

Comparison is also made with the analytical results of Klett and Davis (1973) for a collector drop of 70 μm. It is noted that all calculated collisional efficiencies are large for high radius ratios. However, the Klett and Davis results appear to be substantially smaller than available experimental data.
5. Conclusion

The linear collision efficiency of a pair of water drops in the atmosphere has been calculated by using the method of superposition. Based on the flow fields obtained from the numerical solution of the Navier-Stokes equations, collision trajectories between drops of various sizes were determined by considering only gravity, inertia and drag forces. For cases of large collector drops and small collecting drops, the linear collision efficiency is significantly less than the geometric collision efficiency, because the hydrodynamic force upstream of the collector drop pushes the small collecting drops away from its falling axis. For cases of collector and collecting drops of similar sizes, the linear collision efficiency is substantially higher than the geometric collision efficiency, because the low pressure created by the viscous forces downstream of the collecting drop drags the collector drop into its wake. The theoretical result has been compared with existing data from laboratory experiments. Satisfactory agreement between the theoretical and experimental results indicates that the theory is basically sound. In an atmospheric environment, where only inertia, gravity and drag forces are significant, the dominant factor in forming precipitation appears to be the collisional growth of drops of water of approximately equal size. Further investigations to include additional atmospheric parameters, such as electric charges, updraft and turbulence, can be developed by a similar approach for applications to more complicated atmospheric problems.

Acknowledgments. The research reported here was sponsored by the Office of Naval Research under Contract N00014-69A-0141-0006 with the University of Missouri-Rolla. The computer time used in the reported investigation was granted by the National Center for Atmospheric Research which is sponsored by the National Science Foundation.

APPENDIX

Effect of Drag Force on Collision Efficiency

A water drop in the atmosphere is surrounded by air, which produces a drag force when the drop travels with a certain velocity. Calculation of collision efficiency requires a knowledge of the drag force of various drop sizes at different velocities. Consequently, a dimensionless drag coefficient is given as a function of Reynolds

![Fig. 8. Effect of drag equation.](image-url)
numbers. At a Reynolds number region between 0 and 1, it is not absolutely certain that the drag coefficient actually follows the Stokes drag formula or Oseen's drag formula. To examine the actual derivation of the linear collision efficiency caused by the uncertainty of the chosen drag relation, Fig. 8 shows the comparison of the Stokes and Oseen's drag on linear collision efficiency calculations for a 30 μm collector drop. It is obvious that the drag formula does not affect the trend of the linear collision efficiency. A maximum deviation of 10% could be expected in the vicinity of drag radius ratio of 0.75. For larger drops, evaluations of collision efficiencies were made by using Beard and Pruppacher's formulas as well as Lin and Lee's formula based on the same flow field data for a drop radius of 100 μm. Fig. 8 shows that no appreciable difference can be found in linear collision efficiency for radius ratios ≤ 0.7. In the region where the sizes of collecting drops are approaching the size of the collector drop, no drag formula can cause the decrease of collision efficiency as reported in some early investigations. Step changes in the drag formula are the only cause of the irregularities of the collision efficiency curve, and these are not readily explainable in natural phenomena.

REFERENCES