operate. This allows for the possibility of stationary growing waves that have characteristics similar to observed blocking events.

Concerning El-Sayed's hypothesis that variability in sensible and latent heat flux off the east coast of Asia can lead to blocking activity over the North Pacific, we spent nearly two years exploring this idea before abandoning it. The reason was that we could not discern any consistent changes in the monthly mean latent and sensible heat fluxes off the east coast of Asia prior or during blocking events. This is reflected in the observations noted in White and Clark (1975) that the average sensible heating (computed from bulk formulas using monthly mean data) across the North Pacific Ocean varies about the climatological mean by no more than $\pm 5\%$.

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A Model for Turbulent Diffusion over Terrain

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ABSTRACT

A statistical model for three-dimensional turbulent diffusion from instantaneous and continuous sources is constructed. It is shown that this diffusion model satisfies the continuity condition and the boundary condition of three-dimensional terrains.

1. Introduction

The statistical properties of particle dispersion from a fixed source were first analyzed by Taylor (1921) by considering the continuous movements of marked fluid particles in a statistically homogeneous, stationary field of turbulence. He found that the mean square of particle displacement may be expressed in terms of the autocorrelation function of particle velocities, and may be used as a measure of particle dispersion. Theoretical models for turbulent diffusion have been developed by many scientists, notably by Sutton (1947), Batchelor (1949) and Frenkiel (1951, 1952). A development of the diffusion formulas may be found in *Meteorology and Atomic Energy* (Slade, 1968). These formulas have been widely used in air pollution meteorology and give good representation of data on atmospheric diffusion. However, these formulas were developed essentially for diffusion of pollutants over flat surfaces. In view of the fact that in many population centers, industrial and power plants are located and planned in terrains and valleys, it is necessary to construct models for turbulent diffusion over terrains. The purpose of this note is to construct such a model.

2. Diffusion from an instantaneous point source

Consider an instantaneous point source of source strength Q emitted at the origin of the coordinate system at $t=0$. It has been shown, with application of the initial condition for the instantaneous point source (Frenkiel, 1952), that the mean concentration distribution of the marked fluid particles takes the form

$$P(x,y,z,t)=\frac{Q}{(2\pi)^{\frac{3}{2}}\sigma_x(t)\sigma_y(t)\sigma_z(t)} \times\exp\left\{-\frac{1}{2}\left[\frac{x^2}{\sigma_x^2(t)}+\frac{y^2}{\sigma_y^2(t)}+\frac{z^2}{\sigma_z^2(t)}\right]\right\}, \quad (1)$$

where $\sigma_x^2(t)$, $\sigma_y^2(t)$, $\sigma_z^2(t)$ are respectively the mean squares of the x, y, z components of the particle displacement, and may be expressed (Taylor, 1921) in terms of the autocorrelation coefficient $R_v(\tau)$ of the Lagrangian turbulent velocity v' at a time lag τ, i.e.,

$$\sigma_{x_i}^2(t)=2\overline{v_i'^2}\int_0^t\int_0^\alpha R_{v_i}(\tau)d\tau d\alpha,\quad i=1,\,2,\,3,$$

where $x_1=x$, $x_2=y$, $x_3=z$, $v_1'=u'$, $v_2'=v'$, $v_3'=w'$.

Let $h(x,y)$ be the height of the terrains which is a function of x and y. It can be shown by assuming total reflection of marked fluid particles at every point on the terrain boundary that the mean concentration for an instantaneous point source in the presence of terrains takes the form

$$P(x,y,z,t)=\frac{Q}{(2\pi)^{\frac{3}{2}}\sigma_x(t)\sigma_y(t)\sigma_z(t)}\times\exp\left\{-\frac{1}{2}\left[\frac{x^2}{\sigma_x^2(t)}+\frac{y^2}{\sigma_y^2(t)}\right]\right\}\left\{\exp\left[\frac{-(z-H)^2}{2\sigma_z^2(t)}\right]+\exp\left[\frac{-[z+H-2h(x,y)]^2}{2\sigma_z^2(t)}\right]\right\},\quad z\geqslant h(x,y),\quad (2)$$

where H is the source height.

It can be shown that (2) satisfies the continuity condition for the conservation of marked fluid particles:

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{h(x,y)}^{\infty}P(x,y,z,t)dzdydx=Q.$$

3. Diffusion from a continuous fixed source

Now let us consider diffusion from a continuous fixed point source located at the origin in a fluid of turbulent motion with a constant mean velocity $\bar{u}$. The solution for the mean concentration may be obtained by integrating (1), and, for sufficiently large distance from the source, can be shown to take the form

$$P(x,y,z)\approx\frac{Q}{2\pi\bar{u}\sigma_y(x)\sigma_z(x)}\exp\left\{-\frac{1}{2}\left[\frac{y^2}{\sigma_y^2(x)}+\frac{z^2}{\sigma_z^2(x)}\right]\right\}.$$

Following the analysis similar to that for the diffusion from an instantaneous point source above a terrain boundary, the solution for the mean concentration for a continuous fixed point source in the presence of a terrain-boundary takes the form

$$P(x,y,z)=\frac{Q}{2\pi\bar{u}\sigma_y(x)\sigma_z(x)}\times\exp\left\{-\frac{y^2}{2\sigma_y^2(x)}\right\}\left\{\exp\left[\frac{-(z-H)^2}{2\sigma_z^2(x)}\right]+\exp\left[\frac{-[z+H-2h(x,y)]^2}{2\sigma_z^2(x)}\right]\right\},\quad z\geqslant h(x,y).\quad (3)$$

It can be shown that (3) satisfies the continuity condition for the conservation of marked fluid particles:

$$\int_{-\infty}^{\infty}\int_{h(x,y)}^{\infty}\bar{u}P(x,y,z)dzdy=Q.$$

If a pollutant is affected by chemical reactions, (5) may be expressed as

$$P(x,y,z)=\frac{Q}{2\pi\bar{u}\sigma_y(x)\sigma_z(x)}\times\exp\left\{-\frac{0.693x}{T_{\frac{1}{2}}\bar{u}}-\frac{y^2}{2\sigma_y^2(x)}\right\}\left\{\exp\left[\frac{-(z-H)^2}{2\sigma_z^2(x)}\right]+\exp\left[\frac{-[z+H-2h(x,y)]^2}{2\sigma_z^2(x)}\right]\right\},\quad z\geqslant h(x,y),\quad (6)$$

where $T_{\frac{1}{2}}$ is the half-life of the pollutant.

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