A Non-Equilibrium Model of Hemispheric Mean Surface Temperature

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ABSTRACT

A simple mean hemispheric temperature model has been constructed in the form of a differential equation which is a function of three independent variables: carbon dioxide content of the air, volcanic ejecta and anthropogenic particulate pollution. This model appears to simulate the behavior of Northern Hemisphere mean temperatures as well as they are known and gives a different pattern of behavior for the Southern Hemisphere. By more completely accounting for those anthropogenic processes which produce both lower tropospheric aerosols and carbon dioxide, such as fossil fuel burning and agricultural burning, we calculate an expected slight decrease in surface temperature with an increase in CO_2 content. Though an invariant "solar constant" was assumed, an unmistakable 20–25 year periodicity was found in the difference between the calculated and observed direct solar flux reaching the earth's surface, suggesting a definite but small periodic variation in the solar constant.

1. Introduction

There is currently little disagreement with the idea that ultimately the modeling of climate and simulation of climatic change must be done with complete four-dimensional numerical models. While excellent progress has been made toward this goal (Adem, 1965; Mintz, 1964; Bryan, 1969; Gates, 1975; etc.) it is a formidable task that challenges not only the scientific, but the fiscal and computational capabilities of today. Contributory to this effort must be many supportive studies aimed at parameterization of many of the atmospheric processes (e.g., Manabe and Wetherald, 1967, 1975) and identification of critical parameters (Schneider and Dickenson, 1974).

One group of studies of this latter type has focused on the simpler case of modeling the mean temperature of the earth, or a hemisphere, or the mean temperature of each latitude. The present study falls in this class. Following the dictum of William of Ockham, more recently restated by Verner Suomi (personal communication, 1950), "The important thing about science is to know what to set equal to zero," we have attempted to write a differential equation which gives realistic solutions in the time domain while using as few variables as possible. In this work we are preceded by Mitchell (1961) who suggested that variations of volcanic activity and carbon dioxide might explain the pentadal variation of Northern Hemisphere mean temperature as he computed it to be. Recently, Pollack *et al.* (1975) attempted to simulate, from volcanic activity data and the carbon dioxide increase, the course of the hemispheric mean temperature as given by Mitchell (*op. cit.*).

In the model to be presented in the following paragraphs we will follow the leads of Mitchell and Pollack *et al.* (1975) and, as suggested by Budyko (1969), include the effect of anthropogenic particulate load increase as well as anthropogenic carbon dioxide increase.

2. Model assumptions

The design of the present model is predicated on the initial assumption that variations in mean hemispheric temperature are primarily responses to variations in atmospheric transmissivity resulting from processes external to the atmosphere itself. This does not say that if the radiative energy input is changed in absolute value or distribution within the atmosphere-earth system that internal processes might not respond to further change the transmissivity, e.g., by a change in cloud amount or depth or by increased input of deflated dust or sea spray. The validity of this basic assumption will be measured by the success with which the model simulates the actual behavior of the atmosphere.

The external sources of materials which may change the transmissivity are primarily volcanic activity and human activity. Volcanic activity injects particles and gases into the atmosphere sporadically. Some of the gases may later result in the formation of particles, e.g., sulfur dioxide. Volcanic activity meets the criterion of being an extrinsic variable. The long residence time of volcanic ejecta in the atmosphere (Flohn, 1973) plus its sporadic injection implies that its effect should be variations of atmospheric optical properties (such as transmissivity) on the scale of several years to a decade or so.

Human activity more or less continuously injects particles and gases into the atmosphere. While the approximate distribution of human activity on the face of the earth is generally a function of the climatic distribution, the distribution has not changed drastically in the last century; but its intensity has as a result of increased population, increased mechanization and industrialization, and has been accompanied by continuing increases in fossil fuel consumption. The short doubling times of most measures of human activity would suggest that such atmospheric effects that might accrue would be felt on the scale of a decade or so. It will be assumed that the major anthropogenic gaseous contribution is an increase of carbon dioxide above the long-term natural level. The question of anthropogenic particulate input is more complex.

The geologic record shows clearly that there has been a large atmospheric transport of fine particles, especially from arid regions where the soil is exposed to deflation and from the outwash of rock flour associated with continental glaciations (Rex and Goldberg, 1958; Jackson *et al.*, 1973). These sources are clearly related to the state of the atmosphere. Over the past few centuries, a time scale at which large geological movements may be ignored, the quantity of such deflated material resident in the atmosphere may be assumed to have been nearly constant if expressed as decadal averages. The pre-1930 decadal average dustfall of soil material on the high-altitude glaciers of the Caucasus and Altai is only slightly variable according to the data of Davitaya (1965). Human activity may either generate particles *de novo* as in the case of slash-and-burn agriculture or industry, or may increase the deflation of soil material by disturbing desert pavements or surface crusts in arid regions, or by exposing soil that otherwise would be protected from deflation by vegetative cover. These activities should increase the atmospheric particle burden above the "natural" level. The natural level should show slow variations in response to climatic change (intrinsic) with shorter time scale changes superimposed on the natural level as a result of accelerated human activity (extrinsic). Distinctively anthropogenic materials such as DDT or industrial lead may be used as tracers to identify the man-made contribution (Commission on Atmospheric Science, 1973).

It will be assumed that the solar constant is indeed a constant for the purposes of this paper. This does not imply that changes in the spectral composition of solar radiation cannot be observed, or that sunspots and solar flares do not have detectable atmospheric consequences. It is therefore assumed initially that solar variations do not have as large an impact on pentadal or decadal mean hemispheric temperatures as computed by Schneider and Mass (1975) using the Kondratyev-Nikolsky equation for solar variability. The results summarized at the end of this paper appear to the authors to justify this assumption as a first approximation, but not for more sophisticated models.

While the albedo of the earth changes with cloud cover, snow and ice extent and vegetative cover, most of these changes are responses to and feedback into the state of the atmosphere, i.e., albedo responds primarily to intrinsic processes. Man may modify the albedo of the land surface by his land use and perhaps by the production of condensation trails that may somewhat modify the cloud cover. He may also change the albedo of the system by introduction of particulates which increase the backscatter or absorption of sunlight. These are extrinsic variables, not directly dependent on the state of the atmosphere. Anthropogenic hemispheric surface albedo changes are likely to be small since human activity is concentrated in approximately one-tenth of the earth's area, and changes from native surface to other uses usually result in a few percent change in local albedo (Bryson, 1974). It will be assumed that the effect of condensation trails is negligible [see Manabe and Wetherald (1967) for a study of the effects of enhanced cirrus coverage] or proportional to anthropogenic particulates and that surface albedo is effectively constant on the time scale of the past century. Most snow and ice variation is in regions of low insolation. We will assume *initially* that the variations of albedo due to snowcover changes have not been important in the past century, although significant change was observed in the winter of 1971–72 (Kukla and Kukla, 1974).

It will be assumed that latent and sensible heat transfer from the surface to some higher level in the atmosphere occurs primarily within the lowest layer of the atmosphere, and that radiative processes dominate the transfer of energy beyond the lowest level, which will be roughly defined below.

The influence of aerosols on visible and longwave radiation fluxes depends strongly on the composition and optical properties of the particulates. Schneider and Mass (1975), however, point out that recent studies by Coakley and Grams (1976) and Harshvardhan and Cess (1976) "suggest that while infrared effects tend to offset the surface cooling effect of a dust veil, the cooling effect is dominant." We therefore assume that the primary effect of particulates is to reduce the amount of solar energy which reaches and is absorbed by the surface.

Explicitly, in summary, the major model assumptions are as follows:

1) The solar constant is constant.

2) The surface albedo is constant.

3) Cloud cover variations are negligible on the hemispheric scale over the past century.

4) Variations in outgoing terrestrial radiation are mostly proportional to anthropogenic variations in carbon dioxide content.

5) Variations in transmissivity and absorption of the cloud-free atmosphere are primarily due to turbidity variations due to volcanic activity and human activity.

6) The energy input to the earth-atmosphere-ocean system need not be equal to the radiative output from the system over the scale of decades, the differences being reflected in stored heat in the substrate.

7) There is no heat source due to atmospheric or oceanic transport from the opposite hemisphere.

3. Model formulation

Formulation of the model equation begins with the treatment of incoming solar radiation. For this purpose we represent the atmosphere with a model of three layers as shown in Fig. 1. The upper, which we will call the stratosphere, will be assumed cloudless, but containing a variable amount of volcanic dust. The transmissivity of this layer will be designated by a, and will vary only with the aerosol density. The middle layer, at its base, contains clouds of an *equivalent opaque amount* N and has a clean-air transmissivity c. The middle layer will be assumed dust free, and called the upper troposphere. The lowest layer will be cloud free but will contain all of the anthropogenic "dust." This will be called the lower troposphere and its transmissivity will be b. These divisions correspond roughly with the divisions evident in the measurements of Hofmann *et al.* (1975).

The solar beam radiation S penetrating the atmosphere will suffer an attenuation $(1-a)$ in passing through the stratosphere and the beam will emerge with an intensity Sa (Fig. 1). A fraction f_a of the attenuated radiation will be forward-scattered *and forward-radiated* by atmospheric molecules and dust particles, producing a diffuse downward radiation amount of $S(1-a)f_a$.

Both the direct radiation Sa and the diffuse radiation $S(1-a)f_a$ will be further attenuated in the upper troposphere. The direct beam leaving the upper troposphere will be of intensity $(1-N)Sac$. The diffuse downward radiation leaving the upper troposphere will be composed of the forward component of the attenuated diffuse radiation from the stratosphere, $Sf_a(1-a)[c+f_c(1-c)](1-N)$, plus the forward-scattered portion of the radiation attenuated from the direct beam, $Sa(1-c)(1-N)f_c$.

Continuing this cascading of effects, we obtain for the direct plus diffuse radiation absorbed by a ground surface of mean absorptivity $(1-\alpha_0)$:

$$G=S(1-\alpha_0)(1-N)[a+f_a(1-a)][b+f_b(1-b)] \times[c+f_c(1-c)]/[1-\alpha_0 N b^2], \quad (1)$$

where G is the primary downward heat flux to the earth's surface and α_0 the surface albedo. The denominator represents the contribution of multiple reflections of radiation between the surface of the earth and the bases of clouds which completely cover a fraction N of the sky expressed as the sum of an infinite series. This expression is reasonably complete if the scattering coefficients and surface albedo are slightly modified to account for forward reradiation of the incoming flux.

We have used the term "primary input" for the energy flux of solar origin in the forms of direct solar radiation, scattered radiation due to molecules and

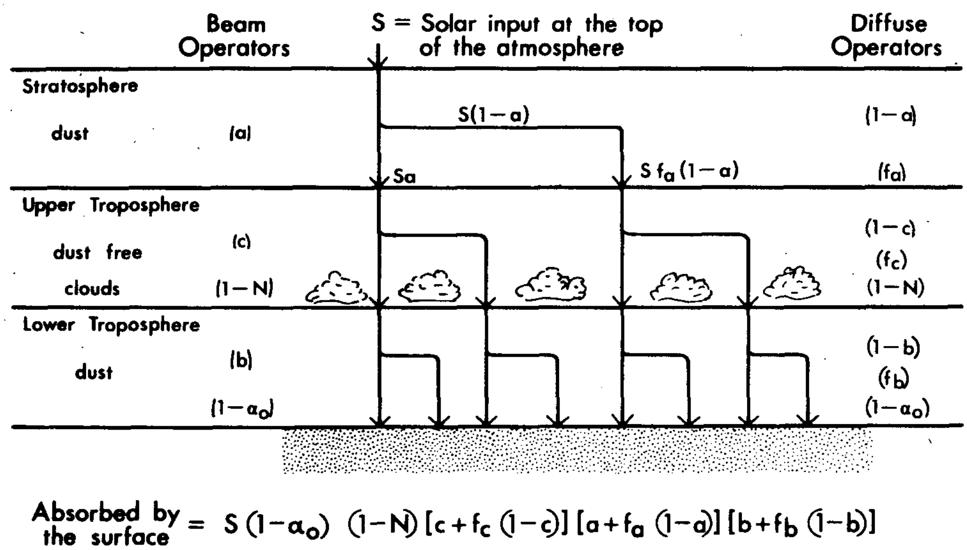

FIG. 1. Schematic diagram of the model. S is the incoming solar radiation at the top of the atmosphere. Operators on beam radiation are on the left in parentheses, operators on diffuse radiation are on the right, also in parentheses. The transmissivities of the stratosphere, upper troposphere and lower troposphere are denoted by a, c and b, respectively. The fraction of the attenuated radiation in the stratosphere that is forward scattered and forward reradiated is f_a. The fractions f_c and f_b are the same for the upper troposphere and lower troposphere, respectively. N is the equivalent opaque cloud amount and α_0 the surface albedo. Downward radiation out of the base of the stratosphere is $S[a+f_b(1-a)]$. Radiation absorbed by the surface is the sum of the components reaching the surface. The effect of multiple reflections between the surface and cloud base has been eliminated in the interest of clarity.

particulates, and solar energy absorbed by dust which is reradiated downward. The latter term is specifically included because climatologically that radiation which is absorbed by particles must be reradiated. We are treating this "primary input" as separate and distinct from the energy flux of terrestrial origin.

The losses from the earth's surface will be formulated next; these are net terrestrial radiation, net sensible heat flux and net latent heat flux. We take the upward terrestrial radiation R_T as

$$R_T = \epsilon_e \sigma T_0^4, \tag{2}$$

where ϵ_e is the emissivity of the surface, σ the Stefan-Boltzman constant, and T_0 the mean hemispheric screen temperature. The mean hemispheric screen temperature may differ from the true surface temperature, but will be assumed initially to be representative of the true temperature of emission from the surface.

In order to avoid the complexity of modeling the net terrestrial radiation in the manner of Manabe and Wetherald (1967), we can use a greatly simplified approach. We will let the back radiation from the atmosphere to the surface be proportional to the outward terrestrial radiation consistent with the nearly linear relationship for our temperature range in the data presented by Idso and Jackson (1969). Then

$$\text{net terrestrial radiation} = \epsilon_e \sigma T_0^4 (1-\epsilon), \tag{3}$$

where ϵ is the "emissivity" of the atmosphere and $(1-\epsilon)$ the transmissivity of the atmosphere to infrared radiation.

The emissivity ϵ is assumed to be the sum of contributions from various atmospheric constituents. Then following the additive model of Staley and Jurica (1970)

$$\epsilon = \epsilon_w + \epsilon_c - \epsilon_v + \epsilon_z, \tag{4}$$

where emissivities of the various components are represented by ϵ_w for water vapor, ϵ_c for carbon dioxide and ϵ_z for ozone. The term ϵ_v is the correction for overlap of the water vapor and carbon dioxide bands. In keeping with our original assumptions, all other contributions to ϵ, e.g., particles and trace gases, are assumed to be negligible for the purpose of this paper.

The water vapor content of the air, and thus the water vapor emissivity ϵ_w, is a function of the temperature. However, since the apparent hemispheric mean surface temperature has changed only by about 0.6 K in the last century (Mitchell, 1961) we will assume a constant ϵ_w. Although this amounts to assuming a fixed absolute humidity, it would make little difference for such a small temperature range, whether we assumed a fixed relative humidity or a fixed absolute humidity. This approach is valid as long as we restrict its application to hemispheric mean temperature changes in the last century. The value for ϵ_w is calculated in the manner of Sellers (1973) based on a mean hemispheric relative humidity of 77% (Manabe and Wetherald, 1967). A temperature 288.6 K produces a value of 0.66 for ϵ_w.

For the emissivity of carbon dioxide we used a formulation derived from Staley and Jurica (1972) with a reference level of 1000 mb (their Fig. 1):

$$\epsilon_c = 0.0235 \ln(CO_2) + 0.0537, \tag{5}$$

where CO_2 is in parts per million (ppm).

The correction for band overlap ϵ_v of water vapor and carbon dioxide, although temperature and concentration dependent, is assumed constant since the hemispheric mean temperature varies so little. Its value is 0.12. Similarly, the emissivity ϵ_z of ozone is assumed constant and equal to 0.06.

So far we have been dealing with terrestrial radiation through the clear fraction of the sky $(1-N)$. Under the (opaque) clouds the earth emits a quantity of energy $\epsilon_e \sigma T_0^4 N$ which is attenuated by the subcloud atmosphere before being absorbed by the clouds, and the clouds emit an amount $\sigma T_c^4 N$ which is similarly attenuated before being absorbed by the surface. Note that since N is the equivalent opaque cloud cover, we may use a cloud emissivity of unity.

Thus the net surface terrestrial radiation becomes

$$(1-N)\epsilon_e \sigma T_0^4 (1-\epsilon_w-\epsilon_c+\epsilon_v-\epsilon_z) + N\epsilon_e(\sigma T_0^4 - \sigma T_c^4)(1-\epsilon_{sc}), \tag{6}$$

where T_c is the cloud base temperature and ϵ_{sc} the subcloud atmospheric emissivity. This expression is similar in concept to the two-layer formulation of Sellers (1973).

All calculations in this paper were made with near radiative equilibrium beneath the clouds. Specifically we set T_c 0.5 K colder than T_0 and ϵ_{sc} equal to 20% of ϵ. For the small temperature range of interest the cloud term is essentially a constant.

The heat budget equation for the surface of the hemisphere is then, combining (6) and (1)

$$\begin{aligned} &S(1-N)(1-\alpha_0)[c+f_c(1-c)] \\ &\quad\times[ab+f_a b(1-a)+f_b a(1-b)+f_a f_b(1-a)(1-b)] \\ &\quad\times[1-\alpha_0 N b^2]^{-1}-(1-N)\epsilon_e\sigma T_0^4(1-\epsilon_w-\epsilon_c+\epsilon_v-\epsilon_z) \\ &\qquad -N\epsilon_e(\sigma T_0^4-\sigma T_c^4)(1-\epsilon_{sc})-L=m^*\partial T_0/\partial t, \end{aligned} \tag{7}$$

where L is the net latent and sensible heat flux away from the surface, and m^* the effective heat capacity of the substrate. Note that $m^*\partial T_0/\partial t$ represents variations in storage of heat in the substrate. The model, like the real earth-atmosphere system, is in radiative nonequilibrium on the scale of decades.

In order to apply Eq. (7) it is necessary to obtain values of the various constants as well as the independent variables. Most of the constant parameters may be found in the literature while others must be calculated. Table 1 lists the values used here and the source.

TABLE 1. Parameter values for model calculations. The last column lists the approximate change in computed Northern Hemisphere mean surface temperature due to a $\pm 1\%$ change in each parameter.

Symbol	Value	Source	Sensitivity
		Parameters	
S	340.0 W m^{-2}	Based on 1360 W m^{-2}	± 2.00 K
α_0	0.11	Lettau (1974)	∓ 0.11
N	0.40	Lettau (1974)	∓ 0.40
$[c+f_c(1-c)]$	0.76	Lettau (1974)	± 1.91
f_a	0.91	Budyko (1974)	± 0.06
f_b	0.91	Budyko (1974)	± 0.06
m^*	0.635 W decade $(m^2\ K)^{-1}$	Sellers (1965)	
L	94.1 W m^{-2}	Sellers (1965)	∓ 1.33
ϵ_s	0.98	Lettau (1974)	∓ 0.73
		Variables	
a			± 0.17
b			± 0.19
CO_2			± 0.08

The thermal capacity m^* of the substrate is calculated using (8), i.e.,

$$m^* = A_L \rho_L C_L Z_L + A_S \rho_S C_S Z_S, \tag{8}$$

where A_L and A_S are the fractional area of the hemisphere covered by land or sea, respectively, and Z is the equivalent mixing or thermal response depth. We use an assumed thermocline depth of the sea of 75 m (after Sellers, 1965) and an active layer of 12 m for land. With a land density (ρ_L) of 2×10^3 kg m^{-3}, a specific heat (C_L) of 1040 J $(kg\ K)^{-1}$, m^* becomes 0.635 W decade $(m^2\ K)^{-1}$ for the Northern Hemisphere.

The forward scattering plus forward reradiation fractions f_a and f_b are not standard measures and so must be constructed from other sources. Budyko (1974) indicates that in the presence of stratospheric dust, the decline in total radiation is less than that of the direct radiation by a factor of 0.16 for a global average corresponding to 30° latitude. This implies that 0.84 of the attenuated solar radiation is forward scattered. If we assume for the dust an absorption to backscatter ratio of 7 [comparable to those found by Joseph and Wolfson (1975) for desert aerosols], then for the remaining 0.16 of the attenuated radiation, 0.14 is absorbed by the dust and 0.02 is backscattered. Applying our previous assumption that, climatologically, what is absorbed is reradiated (half downward and half upward), then the downward reradiation fraction is 0.07. The value of f_a is then 0.84 plus 0.07 or 0.91 as shown in Table 1. We also assume as a first approximation that f_b is equal to f_a. These factors are not sensitive to the ratio of absorption to backscatter since a ratio of 1 yields f_a and f_b equal to 0.88. Also, a sensitivity analysis conducted in the manner described below shows that a $\pm 1\%$ change in f_a or f_b produces only a ± 0.06 K change in computed surface temperature.

A good model should be reasonably insensitive to small changes in parameter values. To test our model, then, we perform a sensitivity analysis. Table 1 shows the change in calculated surface temperature due to a $\pm 1\%$ change from the listed value of each parameter. These temperature changes are obtained by differentiating (7) with the storage term set equal to zero. We assume for this purpose that each parameter is independent and use nominal or average values where appropriate.

Changing the solar constant by $\pm 1\%$ produces a ± 2.00 K change in calculated surface temperature. This is slightly larger than other values found in the literature: Schneider and Mass (1975), ± 1.20 K; Manabe and Wetherald (1967), ± 1.52 K; Budyko (1969), ± 1.1 K. Our value is larger because we have accounted for the absorption of solar radiation and its reradiation by particulates.

In this experiment the net latent and net sensible heat flux L is assumed to be constant as a first approximation. Manabe and Wetherald (1975) in studying the effects of doubling the CO_2 content show that the term L varies by about 3% for a temperature change of about 3 K (their Fig. 10 and Table 1). Since the temperature changes being considered here are about ± 0.3 K, the variation of L would be about 0.3% or only about ± 0.3 W m^{-2}.

Let us explore briefly the consequences of a simple feedback mechanism. The most reasonable feedback is to assume that L is proportional to the net radiation R_{net} with a proportionality constant k. Then the heat budget equation becomes $(1-k)R_{net} = m^* \partial T_0/\partial t$ or $R_{net} = [m^*/(1-k)]\partial T_0/\partial t$. Clearly, the main effect is to adjust the effective heat capacity of the substrate. Performing the sensitivity test as before would lead to a zero temperature change for any change in L. However, since the exact form of the feedback mechanism is not known, we will continue to treat L as constant, since that tends to be the least favorable (i.e., most sensitive) case.

Sensitivity to changes in the input variables (a, b and CO_2) is also listed in Table 1. A $\pm 1\%$ change in a or b produces a computed surface temperature change of ± 0.17 or ± 0.19 K, respectively. Since the CO_2 concentration affects the emissivity ϵ of the atmosphere, Eqs. (4) and (5) can be used to calculate the effect of CO_2 on surface temperature.

For a change in the carbon dioxide concentration of $\pm 1\%$ (300 ± 3 ppm) the computed surface temperature sensitivity is ± 0.080 K for our model. Manabe and Wetherald (1967) calculated the effects of halving and doubling the CO_2 concentration. Using their results (their Table 5) for average cloudiness and fixed relative humidity the change in equilibrium temperature of the earth's surface ΔT_e(K) is related to the CO_2 content (ppm) by

$$\Delta T_e = 3.346 \ln(CO_2) - 19.06. \tag{9}$$

Using (9), a change in the CO_2 content of $\pm 1\%$ thus produces an equilibrium surface temperature sensitivity of ± 0.033 K for the Manabe and Wetherald (1967) model. Our value is larger than theirs, but of the same order of magnitude.

However, any calculation of the effect of CO_2 variation, alone, upon the climate is rather unrealistic unless causally related concomitant variations in other parameters are taken into account. The carbon dioxide increase of recent decades is generally acknowledged to be the result of human activities, in particular the burning of fossil fuel (Machta, 1972). Fossil fuel is not burned for the purpose of making carbon dioxide in pure form, but to produce heat and energy for machines. This inevitably produces aerosols as a by-product of the less-than-perfect combustion of a less-than-clean fuel. Carbon dioxide is also a by-product of widespread and increasing agricultural burning in the tropics, which also produces enormous amounts of smoke. Most fossil fuel energized human activities also produce aerosols.

The linear function relating the lower tropospheric transmissivity b and carbon dioxide (derived from Table 2) is

$$b = 1.48 - 0.00163\ CO_2. \tag{10}$$

Substituting this relation in Eq. (7) and repeating the sensitivity calculation for the effect of CO_2, we find a net temperature change of ∓ 0.01 K for a $\pm 1\%$ change in CO_2 and ∓ 4.93 K for doubling CO_2. Therefore, by accounting for processes which produce both CO_2 and lower troposphere aerosols, we find the surface temperature becoming slightly *cooler* instead of warmer with an *increase* in carbon dioxide.

TABLE 2. Input data values used to compute hemispheric mean surface temperature. For the Northern Hemisphere the values listed for CO_2 (from Machta, 1972), and Northern Hemisphere transmissivities a and b were used. For the Southern Hemisphere the values listed for CO_2 and Southern Hemisphere stratospheric transmissivity a were used with b constant and equal to 0.995.

		Transmissivities		
Year	CO_2 concentration (ppm)	a Northern stratosphere	b Northern lower troposphere	a Southern stratosphere
1890	295.0	0.940	0.988	0.947
1895	295.5	0.958	0.994	0.962
1900	296.1	0.953	0.993	0.956
1905	296.9	0.941	0.992	0.952
1910	297.8	0.939	0.992	0.958
1915	298.8	0.941	0.992	0.957
1920	300.0	0.955	0.996	0.950
1925	301.2	0.963	0.992	0.949
1930	302.5	0.968	0.992	0.938
1935	303.9	0.969	0.991	0.930
1940	305.5	0.970	0.990	0.945
1945	307.3	0.968	0.984	0.953
1950	309.2	0.960	0.976	0.955
1955	311.6	0.961	0.972	0.955
1960	314.0	0.954	0.969	0.957

4. Transmissivity evaluation

Though far from satisfactory in spatial coverage there does not seem to be a better historical record of cloudless-sky beam radiation than that of Budyko (1969), and this gives us a rough approximation of the product abc (Fig. 2). Using the hemispheric mean value of c as determined by Lettau (1974), we may calculate ab from Budyko's values of the product abc. We assume c to be constant.

The problem then reduces to finding measures of a and b, functions of the stratospheric and lower tropospheric aerosol loading, respectively. For the stratospheric loading we may use measures of volcanic activity tabulated by Hirschboeck (1976) to provide an index similar to the Dust Veil Index of Lamb (1970) which used a quasi-objective measure of the weighting of the larger eruptions.

Ten-year totals of moderate and great ash volcanic eruptions in latitude bands 0° to 30°N and 30° to 80°N (Table 3) were regressed against Budyko's beam radiation data from 1888–1945, approximately the time period before anthropogenic dust became important. From this regression a weighted sum of eruptions in each category yields an expression for the modified dust veil index V:

$$V = 21.03G_{83} + 4.93G_{30} + 1.14M_{30} - 1.56M_{83} + 8.70. \tag{11}$$

Here G_{83} is the number of great ash eruptions in the latitude band 30° to 80°N in a 10-year period, G_{30} is the same for 0° to 30°N, M_{30} is for moderate ash eruptions from 0° to 30°N and M_{83} is the same for 30° to 80°N. Eruptions categorized as "little or no ash" and all in the Southern Hemisphere were rejected by the principle factor analysis procedure as not being statistically significant. Values of V are shown in Fig. 3.

The lower tropospheric aerosol loading is difficult to establish, but there are at least three measures that may be used to evaluate its variations over the past century. The first two are the dustfall records of Davitaya (1965) obtained from field studies in the high Caucasus and Altai (Fig. 4). Using only the decadal values for particles smaller than 10 μm diameter, these records are assumed proportional to the anthropogenic dust resident in the atmosphere (Bryson, 1972). Another measure is the leadfall on the Greenland glacier (Fig. 4) which may be taken as a tracer of industrial activity. As Murozumi *et al.* (1969) have shown, the natural background leadfall is almost unmeasurably small, and the isotope constitution of the lead indicates an industrial source. It is usually assumed that this dustfall is proportional to the ambient suspended amount with a factor of proportionality such that the ambient amount is scavenged every week or two (Flohn, 1973). These are all scanty and poor measures but there seem to be no others.

Letting B be the clear-sky beam radiation at the surface, and using an approximation of Beer's law,

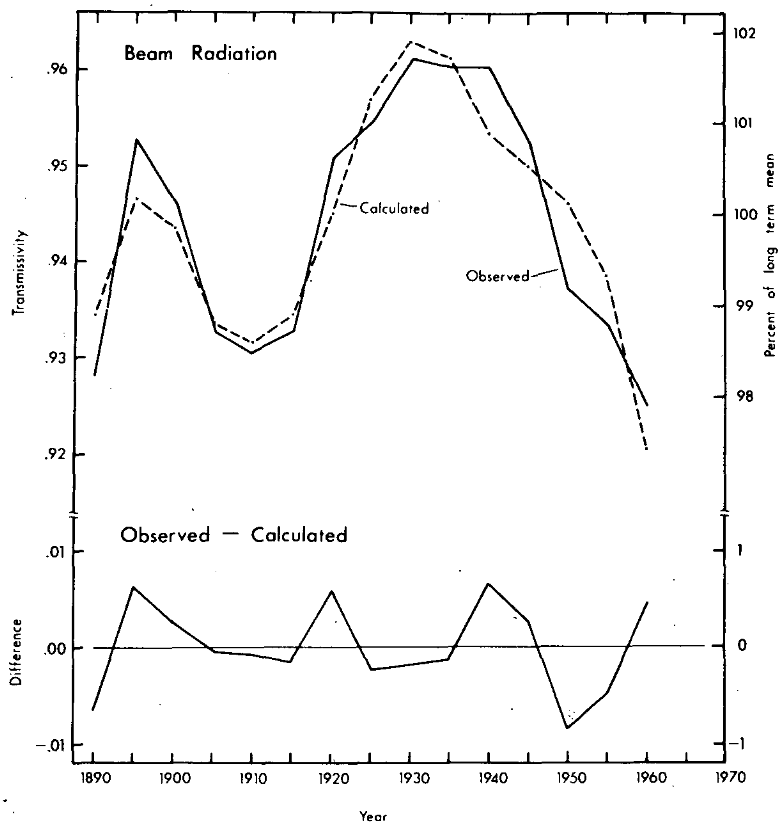

FIG. 2. Observed transmissivity of the cloudless atmosphere (top, solid line) according to Budyko (1969) compared with values calculated (dashed line) as described in the text. The difference between the two curves is shown at the bottom.

we have

$$B=Sabc=S\exp(-A-K_TM-K_sV) \qquad (12)$$

or

$$\ln B=\ln Sc-K_TM-K_sV=\ln S-A-K_TM-K_sV, \qquad (13)$$

where S, a, b, c are as above; A, K_TM, K_sV are the extinction exponents of the total clean air column, man-made aerosol column and volcanic aerosol column, respectively; and M, V are measures of the tropospheric and stratospheric dust.

TABLE 3. Total number of volcanic eruptions in 10 years. Data are listed by center year of each decade, by amount of ash produced and by latitude band. Courtesy of K. Hirschboeck.

Center year of decade	Number of moderate ash eruptions				Number of great ash eruptions			
	80°–30°N	30°N–0°	0°–30°S	30°–80°S	80°–30°N	30°N–0°	0°–30°S	30°–80°S
1885	31	38	73	7	7	1	6	1
1890	37	31	79	15	6	2	4	1
1895	50	29	54	17	4	4	0	0
1900	41	45	60	10	3	9	3	0
1905	41	53	80	14	6	8	3	0
1910	56	36	76	21	8	5	1	0
1915	59	35	54	12	8	3	4	0
1920	49	51	54	4	4	1	4	1
1925	54	54	72	9	2	1	2	1
1930	60	42	78	9	3	1	3	3
1935	56	27	92	5	3	1	6	3
1940	50	18	94	5	5	1	4	0
1945	39	34	74	3	3	1	0	0
1950	41	41	63	10	2	0	3	0
1955	55	28	47	13	4	0	3	1
1960	78	30	32	17	8	1	2	2
1965	77	47	43	12	11	5	3	2

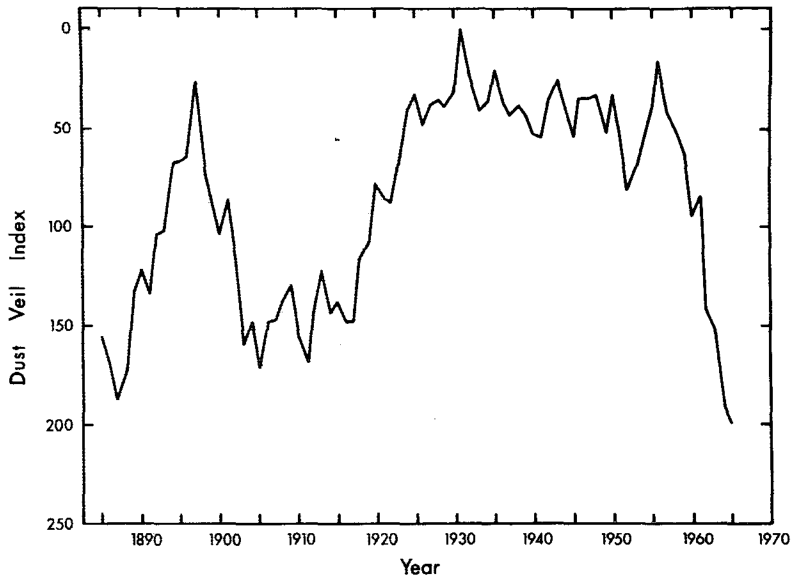

FIG. 3. Modified dust veil index, similar to that of Lamb (1970), calculated from the volcanic eruption chronology of Hirschboeck (1976). Points represent 10-year running means in arbitrary units.

Eq. (13) is a suitable form for evaluating the "extinction coefficients" K_T and K_s. Letting the constant natural background of particulates be subsumed in the term A, which we will assume constant, the variation of $\ln B$ can be attributed to variation of M and V. To preserve the main trends data in Fig. 3 were filtered by taking decadal means. Regressing the decadal values of $\ln B$ against the decadal values of V (filtered) and M (Figs. 3 and 4) gives the values of the extinction coefficients K_T and K_s. We have assumed, as a first approximation, that Budyko's beam radiation data and the measures M and V are representative of the entire hemisphere for the purpose of this experiment. Knowing these extinction coefficients, the time variation of the

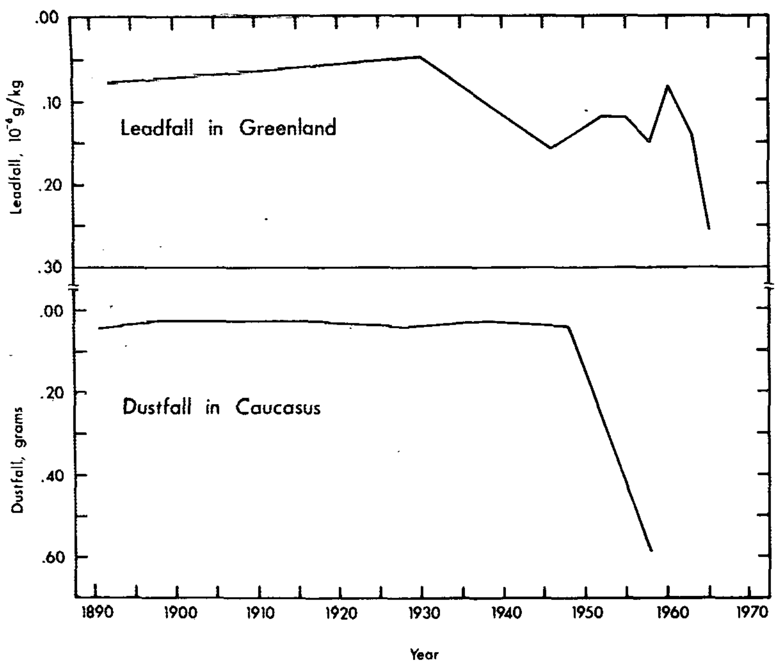

FIG. 4. Industrial lead content of Greenland ice in units of μg of lead per kg of snow from Murozumi *et al.* (1969), and dustfall in the Caucasus expressed as the mass of dust (particles <10 μm in size) found in 10 annual deposits of glacier ice from Davitaya (1965).

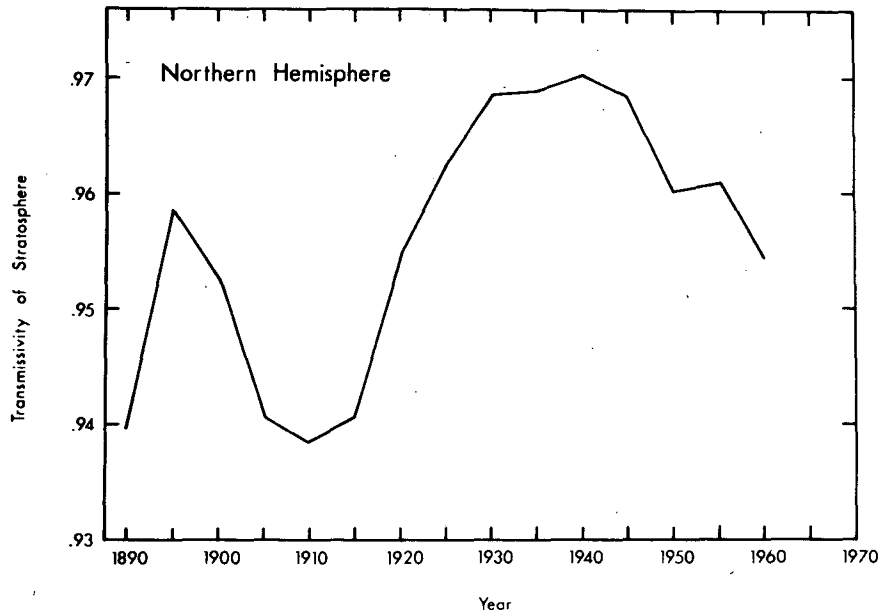

FIG. 5. Time variation of transmissivity in the Northern Hemisphere stratosphere calculated from the modified dust veil index.

transmissivities a, b of the stratospheric and tropospheric aerosol become

$$a = 0.978 \exp(-0.000282V), \quad (14)$$

$$b = \exp(-0.0280 \times \text{dustfall} - 0.111 \times \text{leadfall}), \quad (15)$$

where dustfall and leadfall are in the units of Fig. 4. Comparing the product ab of the computed transmissivities with the observed data of Budyko (1969), we find that the product ab (Fig. 2) explains 87% of the variance. Fig. 2 also shows the difference between Budyko's observations and our calculated ab. There appears to be an unmistakable periodicity on the order of 20–25 years in length, but of small amplitude, which may be related to solar activity. Examination of this phenomenon is reserved for later studies.

Next, in order to produce a more realistic simulation, we use the observed data of Budyko as a basis for correcting the computed values a and b. Specifically, at each point in time we modify a and b such that the ratio of our derived a and b is preserved, and correct the product ab to be consistent with observations. Values of the corrected transmissivities are shown in

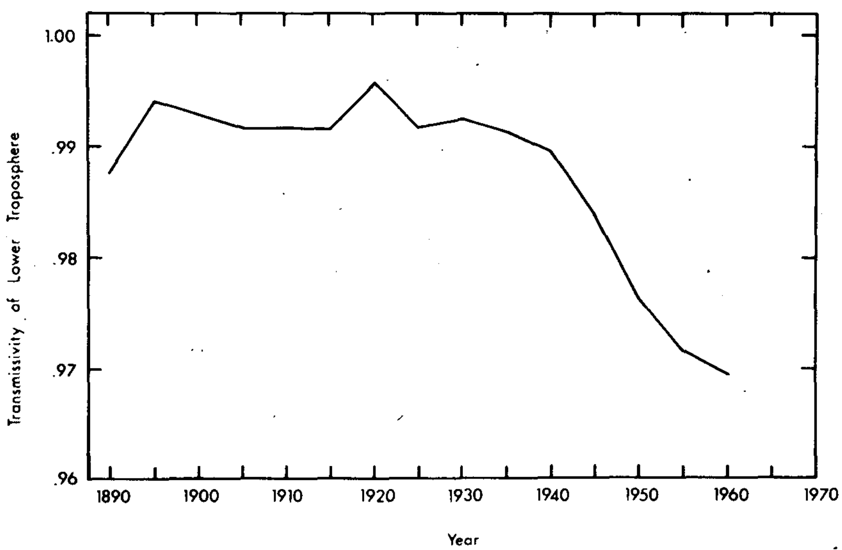

FIG. 6. As in Fig. 5 except in the Northern Hemisphere lower troposphere calculated from leadfall and dustfall on glaciers.

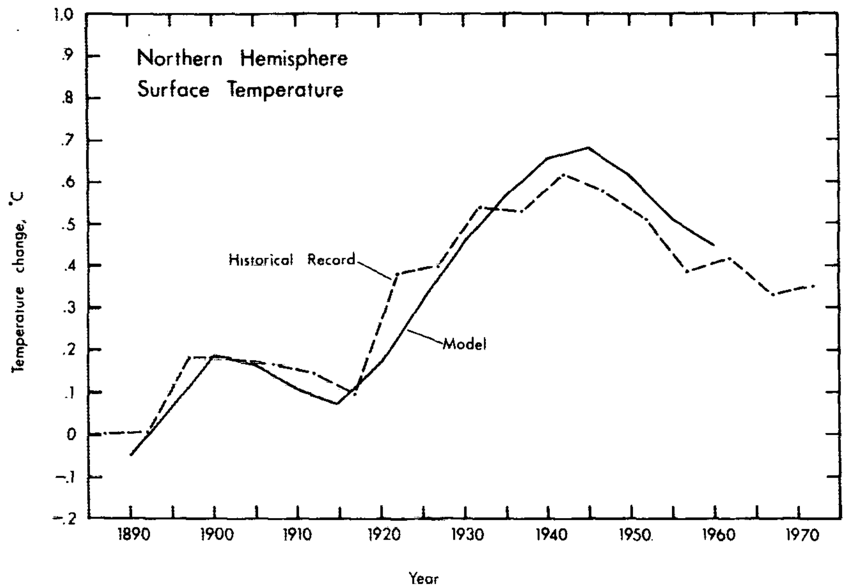

FIG. 7. Calculated Northern Hemisphere mean surface temperature from the model (solid line) compared with the historical record for 0° to 80°N (dashed line) according to Mitchell (1961), as extended by Reitan (1974) and Brinkmann (1976).

Figs. 5 and 6 and are listed in Table 2. In this way we use observed *direct* radiation measurements but allow the *diffuse* radiation terms to respond to dust loading in each layer. In addition the "reflection under the cloud" factor responds to dust loading in the lower troposphere only.

5. Calculated time variation of hemispheric mean surface temperature

Using the pentadal values of transmissivities shown in Figs. 5 and 6, Table 2 and the constants given in Table 1, we evaluated the differential equation given in (7). The solution technique is a simple second-order Runge-Kutta method, and produces results which are relatively insensitive to the size of the time step used.

The solution is given in Fig. 7. In order to evaluate the realism of this simulation we must compare the calculated temperatures with actual observed values. There are two such historical analyses available—that of Mitchell (1961) as extended by Reitan (1974) and Brinkmann (1976) and that of Budyko (1969). Unfortunately the two differ somewhat. The variations in the Budyko curve appear to lead the Mitchell variation by about five years and the minor fluctuations are different, though presumably they are based on nearly the same data. In the one case station averages were used, weighted for area of the latitude zone, and in the other the means were derived from isotherm analyses. Dronia (1967) has also questioned the representativeness of many of the stations used by both authors. Either or neither may be correct. However, we shall assume that both are approximations of the truth. The square of the correlation coefficient between the two curves is 0.82 (see Table 4). The squared correlation coefficient of the computed values from Fig. 7 with Mitchell's values is 0.93 and with Budyko's is 0.78. For comparison the squared correlation coefficient of the computed values from Pollack *et al.* (1975) with Mitchell's data is 0.68.

It would thus appear that (7) describes the essential features of the climatic variation indicated by Budyko and by Mitchell with the use of three extrinsic variables (volcanic ejecta, carbon dioxide and anthropogenic particulate pollution).

6. Extension to the Southern Hemisphere

It may be shown that the CO_2 variation in the Southern Hemisphere parallels that of the Northern Hemisphere (Machta, 1972), while the volcanic activity

TABLE 4. Correlation coefficient squared between the historical analyses of Northern Hemispheric mean surface temperature according to Mitchell (1961) as extended by Reitan (1974) (identified by MR) for 0° to 80°N, Budyko (1969) (BU), the calculated values of Pollack *et al.* (1975) (PT) and those of Bryson and Dittberner (BD). The squared partial correlation coefficients shown provide an estimate of the coefficient of determination to be expected if the influence of the common variable ln (CO_2) is removed.

Source		Correlation coefficient squared			Partial correlation coefficient squared		
		MR	BU	PT	MR	BU	PT
Mitchell (1961) and Reitan (1974)	MR	1			1		
Budyko (1969)	BU	0.82	1		0.65	1	
Pollack *et al.* (1975)	PT	0.68	0.68	1	0.31	0.37	1
Bryson and Dittberner (1976)	BD	0.93	0.78	0.74	0.90	0.63	0.31

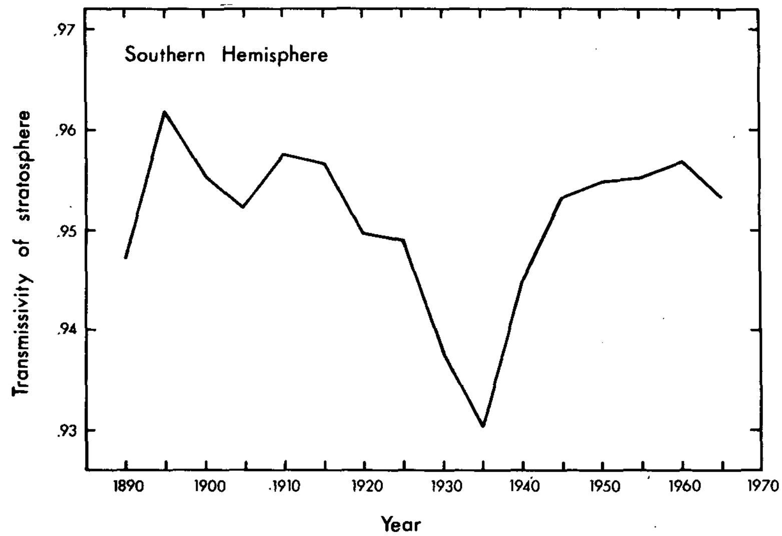

FIG. 8. Time variation of transmissivity in the Southern Hemisphere stratosphere calculated by using only volcanic eruption data for the Southern Hemisphere.

does not (Hirschboeck, 1976). Hofmann *et al.* (1975) found very little lower tropospheric "dust," and it would seem reasonable to assume that the transmissivity of the lower Southern Hemisphere troposphere has remained constant over the last century.

We have, then, the necessary variables to apply our model to the Southern Hemisphere case. A Southern Hemisphere dust veil index was calculated with (11) except that Southern Hemisphere latitude band data was used (Table 3). The transmissivity of stratospheric aerosol (Fig. 8) is then equal to $\exp(K_T V_s)$, where K_T is as before and V_s is the Southern Hemisphere dust veil index. The simulated variation of pentadal mean Southern Hemisphere temperature using an appropriate heat capacity from Eq. (8) and a sensible and latent flux of 95.5 W m^{-2} (after Sellers, 1965) is given in Fig. 9. The simulated course of pentadal mean temperatures is quite different than that for the Northern Hemisphere, being low around 1940 and rising to recent years. Unfortunately there are no reliable observed values for the whole Southern Hemisphere with which to compare these results.

Mitchell (1961) gave an estimate for 0°–60°S, but since the largest variations in the Northern Hemisphere have been at latitude 60° or greater one suspects that 0°–60°S is not a good representation of the whole Southern Hemisphere. It will be necessary to expand the analysis of Southern temperatures in order to assess

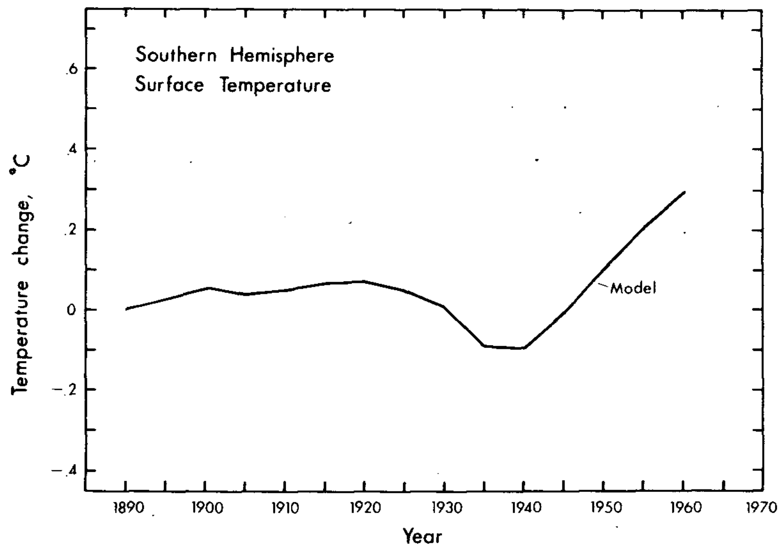

FIG. 9. Calculated Southern Hemisphere mean surface temperature from the model.

the accuracy of this simulation. The simulation is quite similar, however, to mean New Zealand temperatures as given by Trenberth (1975).

7. Summary

In this paper we have reported on an experiment designed to see how few variables could be used to achieve a satisfactory simulation of the pentadal variation of mean hemispheric temperature. The results were better than we expected with only three independent variables. Varying only the transmissivity of the atmosphere to incoming radiation due to suspended particles variation and the terrestrial radiation due only to carbon dioxide variation gave a simulation more like Mitchell's version of the observed temperatures than Budyko's.

We found several interesting, important results: first, by assuming an invariant "solar constant" we found that the difference between our computed atmospheric transmissivities and those observed by Budyko (1969) reveal an unmistakable 20–25 year periodicity, possibly related to solar activity. Further investigation is obviously necessary.

Second, in the process of examining the effect of CO_2 on mean hemispheric surface temperature, we used a simple observed relationship between CO_2 and anthropogenic aerosols. By accounting for this effect, we find a slight net temperature *decrease* with an *increase* in CO_2.

Third, we find that the predominant effect of volcanic and anthropogenic particulates is to reduce the solar flux reaching the surface, leading to cooling. Studies by Harshvardhan and Cess (1976) and Coakley and Grams (1976) are consistent with this result but also demonstrate that for completeness some effect of particulates on terrestrial radiation flux should probably be included.

Finally, it is particularly interesting that such a high degree of success in simulating the course of hemispheric mean surface temperature could be achieved with the assumption of constant mean opaque equivalent cloud cover. However, in retrospect this becomes somewhat easier to understand. Under the clouds the net radiative transfer is nearly zero, and even the average hemispheric sensible and latent heat transfers may be assumed to be dominantly in the cloudless areas. Under these conditions the cloud amount actually cancels from each term of Eq. (7), except for the small storage term $m^*\partial T_0/\partial t$.

In general, then, the results of the experiment suggest an approach to climatic modeling that is primarily thermodynamic with initially simple parameterization of the hydrodynamic processes. Of course, a clear next step might be the utilization of the transmissivities and assumptions contained in this paper in some of the more sophisticated models that are available.

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