Reply

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The technique which I have outlined in my article for calculating empirical fog droplet size distribution functions, has several striking advantages:

1) The two parameters which characterize the skew and steepness of the distribution, γ and α respectively, may be calculated from four simple, usually well-measured parameters: the radius R_m at which the distribution is a maximum, the small and large radii R_1 and R_2 at which the distribution has dropped to 10^{-n} of its maximum value, and the value of n.

2) The choice of two frequency classes of equal size, $N(a_1)=N(a_2)=10^{-n}N_m$, where the relative sizes are given by $a=R/R_m$, leads to separate equations for γ and α, which can be solved independently of each other.

The greatest possible disadvantage, as Essenwanger points out, is that γ may depend too much on fluctuations in the data. (Though he does not say so, the same, of course, holds true for α.) I have calculated a large number of model distributions for comparison with lidar information and complete results will soon be submitted for publication. I have allowed R_1, R_m and R_2 to vary from one to three orders of magnitude, consistent with reasonable fog droplet size limits. Table 1 shows some results, along with estimates of the fluctuation in γ for 10% fluctuations in a_1 and a_2. The three sets of data shown reflect extremes: case 1 is the case of very small minimum droplet size, case 3 is for very large maximum droplet size, and case 2 is for minimum and maximum close together. ("Minimum" and "maximum" refer, as they did in the 1975 article, to the 10^{-n} points, not to the actual smallest and largest radii.) In cases 1 and 3, the error in γ is comparable to or smaller than the error in a_1 or a_2, even though I have allowed simultaneous 10% errors in a_1 and a_2. The worst case is the narrow distribution, for which the error in γ is 14%. One can expect that, in general, the closer together the minimum and maximum radii, the more susceptible will be the estimator to data fluctuations. One way to remedy this situation is to increase n; i.e., to take a_1 and a_2 further apart, if possible. This will, of course, decrease the absolute values of, and increase the percent of errors in, $N(a_1)$ and $N(a_2)$. A simple estimate shows that, for example, a 10% error in N results in an error of 4% or less in α for $n>1$.

It would be a great advantage indeed if consistent two-order statistics could be applied to each set of data; this, however, would prevent us from using two equal frequency classes, and the equations for γ and α would remain coupled, necessitating iterative rather than direct solution, for questionable gain in quality of the estimator. Probably the best compromise is first to choose a_1 and a_2 very carefully; viz., so that neither is an obviously bad data point, and so that most of the data fall between them (for all of the examples in my first article, 92% or more of the data were included). The next step is, as Essenwanger suggests, to calculate several sets of γ, α and C and to perform a least-squares or minimum χ^2 fit to the total data set.

Essenwanger correctly points out that the distribution may not, in fact, adequately represent some fog data; e.g., multi-modal distributions are possible. As I mentioned, even for some unimodal distributions, the scatter may be so large that one cannot tell if the distribution function is applicable. (Might one

TABLE 1. Estimated error in γ for 10% fluctuations in a_1 and a_2 ($n=1$).

Case	a_1	a_2	γ	Percent error in γ
1	2×10^{-4}	2	5.74	10
2	2×10^{-1}	2	2.58	14
3	2×10^{-1}	2×10^{3}	−1.71	5

speculate whether each of the modes of a multi-modal distribution obeys the empirical distribution law?)

The most useful application of the calculational technique I have described will probably not be the fitting of droplet size data, but rather the systematic construction of model distributions, which may then be used to obtain total Mie scattering cross sections of various fogs, for comparison with light scattering measurements of these cross sections.

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