Comments on "The Role of Electrical Forces in Charge Separation by Falling Precipitation in Thunderclouds"

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This note draws attention to a number of errors in a recent paper by Kamra (1975), one of which unfortunately invalidates the results. The paper presented a theoretical treatment of electric field growth in a thundercloud by the precipitation mechanism. In this type of theory negatively charged precipitation drops fall more rapidly than positively charged cloud droplets, thus generating a downward directed electric field. Since the gravitational and electrical forces acting on the negatively charged drops are in opposite directions, their fall velocities are reduced as the electric field increases, so that if the charge per drop remains fixed the charging current carried by them is also reduced. The electrical and gravitational forces act in the same direction on the positively charged cloud droplets, so that their movement always constitutes a discharging current, the magnitude of which increases with the electric field.

Kamra states in the last paragraph of Section 1 that his convention will be that a downward-directed electric field (positive charge above, negative below) is negative. He then goes on to say "consequently, the charging current which brings positive charge up and negative down will be positive." This latter statement appears to be incorrect, for the following reasons: Consider the equation

$$\frac{dE}{dt}=4\pi(QV-\lambda E-i). \tag{1}$$

If E is to be negative, then in order for the field to increase in magnitude the term dE/dt must also be negative. Therefore the charging term $(4\pi QV)$ must be negative, and the conduction terms $[(-4\pi\lambda E)$ and $(-4\pi i)]$ must be positive since they both oppose field growth. This is implicit in Kamra's Eq. (2), which we may rearrange to read

$$\frac{dE}{dt}=-4\pi\sum^{D} N_D Q_D V_D+4\pi\sum^{d} n_d q_d v_d-8\pi\times 10^{-3}E. \tag{2}$$

We note that if E is entered as a negative number the discharging conduction term $(-8\pi\times10^{-3}E)$ will be positive. Assuming that downward-directed velocities are to be positive, we see that the contribution of the positive droplet population in the second term on the right-hand side is also positive, which is consistent with the idea that the motion of the droplets opposes field growth. The charging current due to the falling negative drops is then negative, since from Kamra's equation

$$\bar{Q}_D=K\bar{D}^2, \tag{3}$$

where $K<8.3$, Q_D is entered in the calculation as a positive number, the negative polarity of the charge being accounted for by the minus sign at the front of the first term on the right-hand side of Eq. (2). In view of these arguments it is clear that, contrary to Kamra's statement, charging currents must be negative in his convention. Clearly, changing the sign convention to a self-consistent scheme is simply a matter of conceptual adjustment, and does not affect the validity of the calculations.

Unfortunately, there is a further error in Kamra's treatment which *does* seriously affect the validity of the results. The equations for the terminal velocities of drops ($\bar{V}_D$) and droplets ($\bar{v}_d$) respectively appear as

$$\bar{V}_D=\frac{\bar{m}_D g+\bar{Q}_D E}{3\pi\eta\bar{D}\left(\frac{C_D\,\mathrm{Re}}{24}\right)_{\bar{D}}}, \tag{4}$$

$$\bar{v}_d=\frac{\bar{m}_d g+\bar{q}_d E}{3\pi\eta\bar{d}\left(\frac{C_D\,\mathrm{Re}}{24}\right)_{\bar{d}}}. \tag{5}$$

Recalling that Q_D is entered as a positive number, and E as a negative number, we observe in (4) that the electrical force term $(\bar{Q}_D E)$ is negative, and thus (correctly) opposes the gravitational force. However, in (5) the electrical force term $(\bar{q}_d E)$ is again negative, which is incorrect, since in the downward-directed electric field a positive charge should experience an electrical force in the same direction as the gravitational force. Maintaining Kamra's sign convention, the correction is made by adjusting (5) so that the numerator reads $(\bar{m}_d g - \bar{q}_d E)$.

The error in (5) is carried over into Kamra's equation for β:

$$\beta = \left[\frac{144pK^2}{\pi \bar{D}^2 \bar{\rho}^2 g} - \frac{4\bar{n}_d \bar{q}_d^2}{3\bar{d}\eta\left(\dfrac{C_D \,\mathrm{Re}}{24}\right)_{\bar{d}}} + 8\pi \times 10^{-3}\right]. \quad (10)$$

In this expression all the terms in the bracket represent leakage effects, the first being due to the retarding effect of the electrical forces on the precipitation, the second to the accelerating effect of the electrical forces on the droplets, and the third to point discharge and the conductivity of cloudy air. We would expect that all three would have the same sign, since they all oppose field growth. This error appears to have been included throughout the subsequent calculations, as we can see in, for example, the evaluation of β in Kamra's Eq. (14). The correct form of (10) is obtained by making the substitutions as performed by Kamra, but with the corrected form of Eq. (5), to give

$$\beta = \left[\frac{144pK^2 \rho_w}{\pi \bar{D}^2 \bar{\rho}^2 g} + \frac{4\bar{n}_d \bar{q}_d^2}{3\bar{d}\eta\left(\dfrac{C_D \,\mathrm{Re}}{24}\right)_{\bar{d}}} + 8\pi \times 10^{-3}\right], \quad (10a)$$

in which the polarity of the second term in the bracket is now consistent with the actual direction of the electrical force on the droplets. It should also be noted that Kamra's equation

$$\sum^{D} N_D = \frac{p}{\frac{1}{6}\pi \bar{D}^3 \bar{\rho} \bar{V}_D} \quad (7)$$

is dimensionally inconsistent. Deriving the equation from first principles, and appreciating that p refers to the precipitation rate of *melted* water, we find that a factor ρ_w, the density of liquid water, is missing from the numerator in the fraction, which should read $p\rho_w$; the equation is then dimensionally correct. Kamra's equations for α and β are also dimensionally inconsistent, because they include the error in Eq. (7). Consequently, the first terms in the brackets of these equations [Kamra's Eqs. (9) and (10)] must also be multiplied by ρ_w in order to make them dimensionally self-consistent. This has been done in (10a) above, and the first term in the brackets of Kamra's Eq. (9) should be $(24pK\rho_w/\bar{D}\bar{\rho})$, and not $(24pK/\bar{D}\bar{\rho})$. Fortuitously, the numerical calculations are unaffected, since the value of ρ_w is 1 in the units used.

Physically, the error involving the direction of the electrical force on the positively charged droplets is equivalent to treating that part of their motion that is due to the influence of the electrical forces on them as a charging term, when it is in fact a discharging term. This means that the values of E_m, the maximum electric field, computed by Kamra are presumably too high, and the expressions for total current density I_T and the leakage current density I_L are incorrect. The kind of theoretical treatment attempted by Kamra is of considerable interest, and if performed with the correct equations would be of value. Obviously, when the appropriate correction is made, the model will yield lower values of E_m, apparently strengthening Kamra's conclusion that precipitation mechanisms are probably not the dominant means of thundercloud electrification. However, Kamra used an expression for the *ionic* leakage current that *increases* with field. We would draw his attention to the work of Griffiths *et al.* (1974), in which it was shown that because of the trapping of ions by cloud droplets the equilibrium ionic concentration in a thundercloud, and thus the ionic leakage current, is a *decreasing* function of the electric field. It was shown that ionic leakage will be negligibly small compared to precipitation currents in fields of a few kilovolts per centimeter, until the ionic concentration is enhanced by the onset of corona from hydrometeors. It would seem that Kamra has greatly overestimated these ionic leakage effects in the sub-corona region. If a more realistic treatment of this factor were incorporated, the model would yield higher values of E_m, counteracting the effect of correcting the droplet current error. It is not obvious whether the resultant values of E_m would be higher or lower than those already published. Until new calculations are performed it is highly questionable whether any useful conclusions concerning the effectiveness of precipitation models of electrification can be drawn from Kamra's results.

REFERENCES

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